# The Real-Time Multi-Objective Vehicle Routing Problem

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# Table of Content

# List of Tables

# List of Figures

# Abstract

One of most important logistics problems in the field of transportation and distribution is the Vehicle Routing Problem (VRP). In general, VRP is concerned with the determination of a minimum-cost set of routes for distribution and pickup of goods for a fleet of vehicles, while satisfying given constraints.

Today, most VRPs are set up with a single objective function, minimizing costs, ignoring the fact that most problems encountered in logistics are multi-objective in nature (maximizing customers' satisfaction and so on), and that for both deterministic and stochastic VRPs, the solution is based on a pre-determined set of routes. Technological advancements make it possible to operate vehicles using the real-time information.

The problem considered in this research is the Real-Time Multi-Objective VRP. In this research, the following objectives will be addressed: (1) Minimizing the total traveling time, (2) Minimizing the number of vehicles, (3) Maximizing customers' satisfaction and (4) Maximizing drivers' satisfaction, while considering constraints such as (1) Time Dependency and (2) Soft time windows.

The first stage in solving the multi-objective vehicle routing problem was to formulate the problem as a mixed integer linear programming problem on a network. This includes the mathematical formulation of both the five objectives as well as the various constraints.

Since VRP is a NP-Hard problem, it cannot be solved to optimality using conventional methods. It is therefore, essential to develop an efficient heuristic algorithm for solving it. Based on literature review, three evolutionary algorithms have been chosen for solving the real-time multi-objective VRP. The three algorithms are an improved version of the vector evaluated genetic algorithm (VEGA), the SPEA2 algorithm and a vector evaluated artificial bee colony based algorithm. For all three algorithms, since a candidate solution to an instance of the VRP must specify the number of vehicles required, the partition of the demands through all these vehicles; the delivery order for each route as well as waiting time at each customer; therefore, solution's representation was considered and described.

A fitness function is a particular type of objective function that is used to summarize, as a single figure of merit, how close a given design solution is to achieving the set aims. Evolutionary algorithms, at each iteration, delete the n worst solutions, and replace them with n new ones. Each solution, therefore, needs to be awarded a figure of merit, to indicate how close it came to meeting the overall specification; this is done using the fitness function.

Sometimes, fitness approximation may be appropriate, especially if (1) fitness computation time of a single solution is extremely high, (2) precise model for fitness computation is missing or (3) the fitness function is uncertain or noisy. In all three algorithms presented, the fitnesses of all five objective functions have to be calculated. Due to the stochastic nature of travel time, to get accurate values from the fitness functions, simulation has to be used. However, simulation is a time-consuming process, while a fast algorithm is necessary when coping with real-time problems, which is the final goal of this study.

Usually, when solving a multi-objective optimization problem, the result is a set of non-dominated solutions, from which, the decision maker has to choose his preferred alternative. However, since the final goal is an automated system, the TOPSIS method, a mechanism for choosing a preferred solution from a set of non-dominated solutions has been implemented. It was shown that the running time of the algorithms can be increased by use an "approximated" fitness function, without influencing their accuracy. Furthermore, when using "approximated" fitness functions, the algorithms converge to the best solution, much faster than when using exact fitness functions.

Other parameters of the algorithms, such as waiting time, and shape of the satisfaction / dissatisfaction functions were also tested.

Finally, the three algorithms were compared using a case study, based on two real-world transportation networks (urban and interurban). The case study was performed using simulation.

The result of the case study shows that in an urban network, when using a linear dissatisfaction function, the VEGA algorithm performs best. When each customer has a different priority, under the same conditions, best results were obtained using either the SPEA2 or the VE-ABC algorithms.

In an urban network and a dissatisfaction function that represents customers who don't like that a supplier is either early or late, and in an interurban network with both types of dissatisfaction network, the results of all algorithms were the same.

From the result, it can be concluded, that the VEGA algorithm when used, although considered old and with inferior results, can provide solutions equal in quality to the solutions obtained from more sophisticated and more recent algorithms. This is important, since the VEGA algorithm has an advantage in the simplicity of implementation and running speed compared with other algorithms.

# 1. Introduction

## 1.1. Background and Motivation

A supply chain is defined as *a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/or information from a source to a customer* (Mentzer et al., 2001). The supply chain encompasses every effort involved in producing and delivering a final product or service, from the supplier's supplier to the customer's customer (Koctas, 2006). Supply-chain management (SCM) refers to the management of materials, information, and funds across the entire supply chain, from suppliers through manufacturing and distributing, to the final consumer. It also includes after-sales services and reverse flows such as handling customer returns and recycling of packaging and discarded products (Pyke & Johnson, 2001).

Supply chain management has generated substantial interest in recent years. Managers in many industries now realize that actions taken by one member of the chain can influence the profitability of all others in the chain (Pyke & Johnson, 2001). Organizations that have achieved supply chain integration success report lower investments in inventory, a reduction in the cash flow cycle time, reduced cycle times, lower material acquisition costs, higher employee productivity, increased ability to meet customer requested dates (including short-term increases in demand), and lower logistics costs (Lummus & Vokurka, 1999).

While supply chain planning has attracted significant attention due to its critical impact on customer service, cost effectiveness, and, thus, competitiveness in increasingly demanding global markets (Giaglis, Minis, Tatarakis & Zeimpekis, 2004), supply chain execution has received less attention, at least as far as real-time decision making and risk management are concerned. Processes such as stock control and warehouse management have been thoroughly investigated and supported; improvement opportunities still lie in the area of distribution management (Ehrgott, 2005; Gendreau & Potvin, 1998; Ichoua, Gendreau & Potvin, 2003). The importance of distribution management has motivated intense theoretical work and the development of efficient models and algorithms. The most important model in distribution management is the vehicle routing problem (VRP).

In general, VRP concerns the determination of a minimum-cost assignment of a number of vehicles to deliver goods to (or pick up goods from) a set of *n* customers while satisfying given constraints. Each of the vehicles is assigned to a route, which specifies an ordered subset of the customers, with each route starting and ending at a fixed point called the depot (Administration, 2004).

VRPs are frequently used to model real cases. However, they are often set up with the single objective of minimizing the cost of the solution, despite the fact that the majority of the problems encountered in industry, particularly in logistics, are multi-objective in nature. In real-life, for instance, there may be several costs associated with a single tour. Moreover, the objectives may not always be limited to cost. In fact, numerous other aspects, such as balancing workloads (time, distance ...), can be taken into account simply by adding new objectives (Jozefowiez, Semet & Talbi, 2008).

Traditionally, vehicle routing plans are based on deterministic information about demands, vehicle locations and travel times on the roads. What is likely to distinguish most distribution problems today from equivalent problems in the past, is that information that is needed to come up with a set of good vehicle routes and schedules is dynamically revealed to the decision maker (Psaraftis, 1995). Until recently, the cost of obtaining real-time traffic information was deemed too high in comparison with the benefits of real time control of the vehicles. Furthermore, some of the information needed for real time routing was impossible to acquire. Advancement of the technology in communication systems, the geographic information system (GIS) and the intelligent transportation system (ITS) make it possible to operate vehicles using the real-time information about travel times and the vehicles' locations (Ghiani, Guerriero, Laporte & Musmanno, 2003).

While traditional VRPs have been thoroughly studied, limited research has to date been devoted to multi-objective, real-time management of vehicles during the actual execution of the distribution schedule, in order to respond to unforeseen events that often occur and may deteriorate the effectiveness of the predefined and static routing decisions. Furthermore, in cases when traveling time is a crucial factor, ignoring travel time fluctuations (due to various factors, such as peak hour traveling time, accidents, weather conditions, etc.) can result in route plans that can take the vehicles into congested urban traffic conditions. Considering time-dependent travel times as well as information

regarding demands that arise in real time in solving VRPs can reduce the costs of ignoring the changing environment (Haghani & Jung, 2005).

## 1.2. Problem Statement

The problem considered in this research is the Real-Time Multi-Objective VRP. The Real-Time Multi-Objective VRP is defined as a vehicle fleet that has to serve customers of fixed demands from a central depot. Customers must be assigned to vehicles, and the vehicles routed so that the a number of objectives are minimized/maximized (Malandraki & Daskin, 1992). The travel time between two customers or a customer and the depot depends on the distance between the points and the time of day, and it also has stochastic properties.

This research attempts to adjust the vehicles' routes at certain times in a planning period. This adjustment considers new information about the travel times, current location of vehicles, and new demand requests (that can be deleted after being served or added, since they arise after the initial service began) and more. This results in a dynamic change in the demand and traveling time information as time changes, which has to be taken into consideration in order to provide optimized real-time operation of vehicles.

According to the literature review (presented later), we believe that the following objectives should be addressed: (1) **Minimizing the total traveling time** (e.g. (Malandraki & Daskin, 1992)) - Minimizing the total traveling time can reduce the cost of an organization among other things, for the following reasons: (a) the less time a driver spends driving the less chances there are for being involved in a car accident (b) maintenance has to be performed less often. (2) **Minimizing the number of vehicles** (e.g., (Corberan, Fernandez, Laguna & Mart, 2002)) - Since in a real world, the fixed cost of using additional vehicles is much more than the routing operations costs, we can reduce the total cost by minimizing the number of vehicles in service. (3) **Maximizing customers' satisfaction** (e.g. (Sessomboon, Watanabe, Irohara & Yoshimoto, 1998)) - Customers who are not satisfied with the level of service may switch to a different provider, which results in a reduction of manufacturing and delivery. (4) **Maximizing drivers' satisfaction** (e.g. (Lee & Ueng, 1998)) - In a similar manner, drivers who are not satisfied with their work schedule may feel frustrated, which may affect their work, which in turn may influence customers' satisfaction. (5) **Minimizing the arrival time of**

**the last vehicle** – each vehicle, on its return back to the depot, can be assigned to a new route (meaning more routes with fewer vehicles). Minimizing the arrival time of the last vehicle arriving at the depot, ensures that all other vehicles are present at the depot before the arrival of the last vehicle, and therefore, can be assigned to new routes.

Besides the regular constraints of VRP, the following constraints should be satisfied as well: (1) **Time Dependency** – since we are interested in minimizing traveling time, we should consider that in the real world, traveling time is dependent on both the distance between two customers and the time of day, and that ignoring the fact that for some routes, the traveling time changes throughout the day, we may get solutions that are far from optimal. (2) **Soft time windows** – soft time windows allow vehicles to arrive at the demand point before or after the required service time; however, in such cases, a penalty is incurred.

## 1.3. Research Objective and Scope

The major goals of this research are to formulate the real-time multi-objective vehicle routing problem as described in section 1.2 and to find a proper solution algorithm for it. In order to achieve this goal, the following objectives will be pursued:

- Developing a model for the real-time multi-objective vehicle routing problem stated in Section 1.2.
- Study of various dynamic VRPs, and the methods used for solving them.
- Study of various methods known in the literature for solving multi-objective optimization problems.
- Incorporating methods used for solving dynamic VRPs and multi-objective optimization problems and developing an algorithm for the real-time multi-objective vehicle routing problem. The main idea here is that this algorithm must find a reasonable solution for the problem at hand within a reasonable time, so that it can be used in a dynamic real-time situation.
- Collecting real travel time information, and generating transportation networks based on this information.
- Apply the algorithm on the generated networks, and perform a sensitivity analysis.

## 1.4. Research Approach

The first step of this research is to formulate the problem described in section 1.2. This formulation step is one of the most important parts of this research, because a good formulation with fewer variables and constraints can reduce the calculation time for the exact solution.

Next, based on knowledge gathered from work on dynamic VRPs and multi-objective problems, a proper heuristic method for the real-time multi-objective VRP is developed. As it is well known, vehicle routing problems are NP-hard, and therefore, an exact solution cannot be found. Moreover, because of the real-time nature of the problem as well as being a multi-objective problem, general heuristic methods may not be very efficient. The soft time windows constraint and the penalty from the time windows violations make this problem even more complicated. In this study, three evolutionary algorithms (EAs) are proposed as the heuristic method for the problem formulated in this research. In designing the algorithm two objectives were carefully considered, the calculation time as well as the accuracy of the results. These two objectives are important since in a real-time problem, decisions regarding vehicle control have to be made efficiently and within a reasonable time.

The third step is algorithm calibration. In each of the algorithms presented, there are several parameters that may affect the algorithm performance. In the third step, the influences of these parameters are tested, and the best option is chosen.

The fourth step, involves model testing by comparing the results of three EAs. The proposed EAs are applied on a network built using real-world data, with an attempt to mimic a real-world situation. The last part is the case study that involves a whole day simulation where the network situation and the demand information change dynamically.

## 1.5. Organization of the Dissertation

The organization of this dissertation is as follows.

Chapter 1 introduces the background and the motivation for this research. It also presents the problem statement and the research approach.

Chapter 2 discusses other research in vehicle routing problems. The review is focused on the basic capacitated VRP, for which it reviews some exact methods such as branch-

and-bound, set-covering and column generation, branch-and-cut and dynamic programming. It also reviews some heuristics, such as the Saving algorithm, Sweep algorithms and the Fisher and Jaikumar algorithm, as well as some meta-heuristics algorithms, such as simulated annealing, Tabu search, genetic algorithms and more. Some of the most common extensions to the basic VRP, such as the split delivery VRP and VRP with time windows are also reviewed. This chapter also provides an extended review on multi-objective VRP and Real-Time VRP.

Chapter 3 presents the proposed formulation of the real-time multi-objective vehicle routing problems as a mixed integer linear programming model.

Chapter 4 provides an overview of some of the common and most recent methods for handling dynamic VRPs.

Chapter 5 provides an overview of some of the common and most recent methods for solving multi-objective optimization problems.

Chapter 6 presents an overview of the evolutionary algorithms, including general background and general structure of evolutionary algorithms. This chapter also presents the proposed algorithms, which were developed especially to solve the problem presented in Chapter 3. It describes the representations used to describe the problem accurately, the algorithm methods for selection and replacement, and some other operators developed for the purposes of this research.

Chapter 7 deals with issues regarding the fitness functions and convergence of the algorithm.

Chapter 8 deals with the calibration of the wait-time parameter present in the formulation presenter in chapter 3, and used by the algorithm presenter in chapter 6.

Chapter 9 describes some customers' satisfaction functions based on information supplied by logistics managers.

Chapter 10 describes the case study for the whole day simulation. It discusses the time dependent shortest path algorithm that is developed based on Dijkstra's algorithm. It also compares the whole day case study results from the five different strategies used.

Finally, Chapter 11 presents the summary, conclusions and recommendations for future research.

# 2. Theoretical Background

The Vehicle-Routing Problem (VRP) is a common name for problems involving the construction of a set of routes for a fleet of vehicles. The vehicles start their routes at a depot, call at customers, to whom they deliver goods, and return to the depot. The objective function for the vehicle-routing problem is to minimize delivery cost by finding optimal routes, which are usually the shortest delivery routes (Boding, 1983). The basic VRP consists of designing a set of delivery or collection routes, such that (1) each route starts and ends at the depot, (2) each customer is called at exactly once and by only one vehicle, (3) the total demand on each route does not exceed the capacity of a single vehicle, and (4) the total routing distance is minimized. It is common to address the basic VRP as Capacitated Vehicle-Routing Problem (CVRP).

Since the VRP was first introduced formally by Dantzig and Ramser (1959), the problem has been extensively discussed and a large number of algorithms, based on exact methods, heuristics and meta-heuristics, have been developed for solving it. We start with a formal definition, as a graph theoretic model, of the basic problems of the vehicle routing class.

Let $G=(V,E)$ be a complete graph, where $V=\{0,\ldots,n\}$ is the vertex set and $E$ is the edge set. Each vertex $i \in V\backslash\{0\}$ represents a customer, having a non-negative demand $d_i$, whereas vertex 0 corresponds to the depot. Each edge $e \in E = \{(i,j) : i,j \in V, i \neq j\}$ is associated with a nonnegative cost, $C_{ij}$, which represents the travel cost spent to go from vertex $i$ to vertex $j$. Generally, the use of the loop edges, $(i,i)$, is not allowed (this is imposed by defining $c_{ii}=+\infty$ for all $i \in V$). A fixed fleet of $M$ identical vehicles, each of capacity $Q$, is available at the depot. The VRP calls for the determination of a set of no more than $M$ routes whose total travel cost is minimized and such that: (1) each customer is visited exactly once by one route; (2) each route starts and ends at the depot, (3) the total demand of the customers served by a route does not exceed the vehicle capacity $Q$, and (4) the length of each route does not exceed a preset limit $L$. (It is common to assume constant speed so that distances, travel times and travel costs are considered as synonymous.) A solution can be viewed as a set of $M$ cycles sharing a common vertex at the depot (Cordeau, Laporte, Savelsbergh & Vigo, 2005).

If $G$ is a directed graph, the cost matrix $C$ is asymmetric, and the corresponding problem is called asymmetric CVRP (ACVRP). Otherwise, we have $c_{ij}=c_{ji}$ for all $(i,j)\in E$, the problem is called symmetric CVRP (SCVRP). (Toth & Vigo, 2001b)

$$\min \sum_{i\in V}\sum_{j\in V} C_{ij}x_{ij} \tag{2.1}$$

subject to

$$\sum_{i\in V} x_{ij} = 1 \qquad \forall j \in V \setminus \{0\} \tag{2.2}$$

$$\sum_{j\in V} x_{ij} = 1 \qquad \forall i \in V \setminus \{0\} \tag{2.3}$$

$$\sum_{i\in V} x_{i0} = N \tag{2.4}$$

$$\sum_{j\in V} x_{0j} = N \tag{2.5}$$

$$\sum_{i\notin S}\sum_{j\in S} x_{ij} \geq r(S) \qquad \forall S \subseteq V \setminus \{0\},\ S \neq \varnothing \tag{2.6}$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in V \tag{2.7}$$

The in-degree and out-degree constraints (2.2) and (2.3) impose that exactly one edge enters and leaves each vertex associated with a customer, respectively. Analogously, constraints (2.4) and (2.5) impose the degree requirements for the depot vertex.

Constraint (2.6), capacity-cut constraints (CCCs), impose both the connectivity of the solution and the vehicle capacity requirements. In fact, they stipulate that each cut ($V\setminus S$,$S$) defined by a customer set $S$ is crossed by a number of edges not smaller than $r(S)$ (minimum number of vehicles needed to serve set $S$). The value of $r(S)$ may be determined by solving an associated Bin Packing Problem (BPP - the bin packing problem is a combinatorial NP-hard problem, in which objects of different volumes must be packed into a finite number of bins of capacity $Q$ in a way that minimizes the number of bins used) with an item set $S$ and bins of capacity $Q$.

This model can be easily adapted to the symmetric problem. To this end, it should be noted that in SCVRP the routes are not oriented (i.e., the customers along a route may be visited indifferently clockwise or counterclockwise). Therefore, it is not necessary to know in which direction edges are traversed by the vehicles, and for each undirected edge

$(i,j) \in E$, $i,j \neq 0$, only one of the two variables $x_{ij}$ and $x_{ji}$ must be used, for example, that with $i<j$. Note that when single-customer routes are not allowed, the edges incident to the depot can be traversed at most once. When, instead, a single-customer route is allowed for customer $j$, one may either include in the model both binary variables $x_{0j}$ and $x_{j0}$ or use a single integer variable, which may take value $\{0,1,2\}$. In this latter case, if $x_{0j}=2$, then a route including the single customer $j$ is selected in the solution. In the following models we assume that single-customer routes are allowed. The symmetric version of model VRP1 then reads

$$\min \sum_{i \in V \setminus \{n\}} \sum_{j>i} C_{ij} x_{ij} \tag{2.8}$$

subject to

$$\sum_{h<i} x_{hi} + \sum_{j>i} x_{ij} = 2 \qquad \forall i \in V \setminus \{0\} \tag{2.9}$$

$$\sum_{j \in V \setminus \{0\}} x_{0j} = 2k \tag{2.10}$$

$$\sum_{i \in S} \sum_{\substack{j<i \\ h \notin S}} x_{hi} + \sum_{i \in S} \sum_{\substack{j>i \\ j \notin S}} x_{ij} \geq 2r(S) \qquad \forall S \subseteq V \setminus \{0\}, S \neq \varnothing \tag{2.11}$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in V \setminus \{0\}, i < j \tag{2.12}$$

$$x_{0j} \in \{0,1,2\} \qquad \forall j \in V \setminus \{0\} \tag{2.13}$$

The degree constraints (2.9) and (2.10) impose that exactly two edges are incident into each vertex associated with a customer and that $2K$ edges are incident into the depot vertex, respectively. The CCCs (2.11) impose both the connectivity of the solution and the vehicle capacity requirements by forcing that a sufficient number of edges enter each subset of vertices. Constraints (2.10)-(2.12) may be adapted to SCVRP in a similar way.

# 2.1. Exact Methods for CVRP

## 2.1.1. Branch-and-bound algorithms

Branch-and-bound is a general algorithm for finding optimal solutions of various optimization problems, especially in discrete and combinatorial optimization. It consists of a systematic enumeration of all candidate solutions, where large subsets of fruitless

candidates are discarded, by using upper and lower estimated bounds of the quantity being optimized.

The branch-and-bound method has been used extensively in recent decades to solve the CVRP and its main variants. In many cases, these algorithms still represent the state of the art with respect to the exact solution methods. In their extensive survey of exact methods, Laporte and Nobert (1987) provide a complete and detailed analysis of the branch-and-bound algorithms proposed until the late 1980s. Recently, more sophisticated bounds have been developed, mainly those based on Lagrangean relaxations or on the additive bounding procedure, which have substantially increased the size of the problems that can be solved to optimality.

Many different elementary combinatorial relaxations were used in early branch-and-bound algorithms. A first family of relaxations is obtained from the integer programming formulations of these problems by dropping the connectivity and capacity constraints. The first branch-and-cut algorithm, proposed by Laporte, Mercure and Nobert (1986), which used this relaxation, was developed for solving asymmetrical CVRP (ACVRP). In the asymmetric case, the relaxed problem is the well-known *transportation problem*, calling for a min-cost collection of circuits of $G$ visiting once all the vehicles in $V\backslash\{0\}$, and $K$ times vertex 0, which may be transformed into an assignment problem (AP) by introducing copies of the depot. The counterpart, for the symmetric case, is the so-called $b$-matching relaxation, which requires the determination of a min-cost collection of cycles covering all the vertices and such that the degree of each vertex $i$ is equal to $b_i$, where $b_i=2$ for all the customer vertices, and $b_0=2K$ for the depot vertex. This relaxation was used by Miller (1995), after the development of efficient algorithms for the $b$-matching problem (see e.g.,(Miller & Pekny, 1995)). The relaxed problems may then be solved in polynomial time (see e.g., (Miller & Pekny, 1995) and (Dell'Amico & Toth, 2000)).

The second family of relaxations is based on degree-constrained spanning trees. These relaxations extend the well known $k$-tree relaxation proposed by Held and Karp (1971) for the TSP. The earliest branch-and-bound algorithm based on this relaxation, proposed by Christofides, Mingozzi and Toth (1981a), can only solve relatively small instances. More recently, Fisher (1994) presented another tree- based relaxation requiring the determination of a $k$-tree, defined as a minimum cost set of $n+k$ edges spanning the graph.

The approach used by Fisher is based on formulation VRP3 (Toth & Vigo, 2001a) with the additional assumption that single-customer routes are not allowed. This is imposed by defining as binary all the variables associated with edges incident into the depot. However, as Fisher observed, in many cases this assumption is not constraining. Fisher modeled the SCVRP as the problem of determining a $k$-tree with degree equal to $2k$ at the depot vertex, and with additional constraints imposing the vehicle capacity requirements and the degree of each customer vertex, which must be equal to 2.

The previously described basic combinatorial relaxations, for both ACVRP and SCVRP, are of poor quality, and, when used within branch-and-bound approaches, they only allow for the optimal solution of small instances. Therefore, different improved bounding techniques were proposed, which considerably increased the size of the instances solvable by branch-and-bound algorithms.

Two relaxations were introduced by Fischetti, Toth and Vigo (1994), who embedded them into overall additive bounding procedures. The additive approach proposed by Fischetti and Toth (1989) allows for the combination of different lower bounding procedures, each exploiting different substructures of the problem under consideration. The first relaxation is based on a disjunction on infeasible arc subsets, and the second lower bound is a projective bound based on a min-cost flow relaxation of ACVRP. The resulting branch-and-bound approach is able to solve randomly generated instances containing up to 300 vertices and four vehicles. Fisher (1994) proposed a way of extending to the asymmetric CVRP the Lagrangean bound based on $m$-trees. No computational testing for this bound was presented by Fisher.

Hadjiconstantinou, Christofides and Mingozzi (1995a) proposed a branch-and-bound algorithm where the lower bound is computed by heuristically solving the dual of the linear programming relaxation of the Set-Partitioning (SP) formulation of the SCVRP.

Almoustafa, Hanafi and Mladenovic (2011) suggested a new Branch and Bound algorithm for solving ADVRP. In this algorithm, the lower bounds are obtained by relaxation of sub-tour elimination and maximum distance constraints. Thus the Assignment problem (AP) is solved in each node of the B&B tree. A best-first-search strategy and adapted tolerance based rules are used for branching. That is, the next node in the tree is one with the smallest relaxed objective function value. In case of a tie, two tie-breaking rules are used: (1) the last one in the list; (2) the random one among them.

Computational results show that the algorithm can provide exact solutions for instances with up to 1000 nodes.

### 2.1.2. Set-Covering and Column Generation Algorithms

A classical method, first suggested by Balinski and Quandt (1964), for solving the CVRP is based on formulating the problem as a set-covering problem. The idea is as follows: Enumerate all feasible routes, where a feasible route is one that starts and ends at the depot and picks up a total load not exceeding Q. Let the index set of all feasible routes be $\mathcal{R} = \{1, 2, ..., R\}$. Let $c_r$ be the cost (e.g., length) of route *r*, and let $S_r \subseteq V$ denote those customers appearing in route *r* for all $r \in \mathcal{R}$. $\alpha_{ir}$ is defined as 1, if customer *i* is served in route *r*, and 0 otherwise, for each customer $i \in V$ and each route $r \in \mathcal{R}$. Also, for every $r \in \mathcal{R}$, let $y_r = 1$ if route *r* is in the optimal solution and 0 otherwise.

In the set-covering formulation of the CVRP, the objective is to select a minimum-cost set of feasible routes such that each customer is included in some route. It is

$$(P) \quad \min \sum_{r \in \mathcal{R}} c_r y_r \tag{2.14}$$

subject to

$$\sum_{r \in \mathcal{R}} \alpha_{ir} y_r \geq 1 \quad \forall i \in V \tag{2.15}$$

$$\sum_{r \in \mathcal{R}} y_r \leq K \tag{2.16}$$

$$y_r \in \{0,1\} \quad \forall r \in \mathcal{R} \tag{2.17}$$

Constraint (2.15) requires that each customer appear in at least one route, while constraint (2.16) imposes that at most *K* routes be used. Constraints (2.15) is written as inequality constraints instead of equality constraints. The formulation with equality constraints is equivalent, since it is assumed that the distance matrix $\{t_{ij}\}$ satisfies the triangle inequality, and therefore each customer will be visited exactly once in the

optimal solution. The formulation with inequality constraints is used here since it turns out to be easier to work with for implementation.

This mathematical programming formulation was used successfully by Cullen, Jarvis and Ratliff (1981) to design heuristic methods for the VRP. Exact algorithms based on this method were developed by Agarwal, Mathur and Salkin (1989) and later by Bixby and Adviser-Coullard (1999) and Hadjiconstantinou, Christofides and Mingozzi (1995b).

Column generation is an efficient algorithm for solving larger linear programs. The overarching idea is that many linear programs are too large to consider all the variables explicitly. Since most of the variables will be non-basic and assume a value of zero in the optimal solution, only a subset of variables need to be considered in theory when solving the problem. Column generation leverages this idea to generate only the variables which have the potential to improve the objective function - that is, to find variables with negative reduced cost (assuming without loss of generality that the problem is a minimization problem).

The problem being solved is split into two problems: the master problem and the subproblem. The master problem is the original problem with only a subset of variables being considered. The subproblem is a new problem created to identify a new variable. The objective function of the subproblem is the reduced cost of the new variable with respect to the current dual variables, and the constraints require that the variable obey the naturally occurring constraints.

The process works as follows. The master problem is solved - from this solution, we are able to obtain dual prices for each of the constraints in the master problem. This information is then utilized in the objective function of the subproblem. The subproblem is solved. If the objective value of the subproblem is negative, a variable with negative reduced cost has been identified. This variable is then added to the master problem, and the master problem is re-solved. Re-solving the master problem will generate a new set of dual values, and the process is repeated until no negative reduced cost variables are identified. If the subproblem returns a solution with non-negative reduced cost, we can conclude that the solution to the master problem is optimal.

To solve the linear programming relaxation of problem *P*, described earlier, without enumerating all the routes, column generation technique can be used. A detailed explanation of this method is given below, but the general idea is as follows: A portion of

all possible routes is enumerated, and the linear relaxation with this partial route set is solved. The solution to this linear program is then used to determine if there are any routes not included in the formulation that can further reduce the objective function value. This is the column generation step. Using the values of the optimal dual variables (with respect to the partial route set), we solve a simpler optimization problem where we identify if there is a route that should be included in the formulation. Then the linear relaxation of this expanded problem is resolved. This is continued until no additional routes are found that can reduce the objective function value. In that case, we can show that an optimal solution to the linear program is found, one that is optimal for the complete route set.

Specifically, we first enumerate a partial set of routes $\mathcal{R}' \in \mathcal{R}$ and formulate the corresponding linear relaxation of the set-covering problem with respect to this set:

$$(P') \ \min \sum_{r \in \mathcal{R}'} c_r y_r \tag{2.18}$$

subject to

$$\sum_{r \in \mathcal{R}'} \alpha_{ir} y_r \geq 1 \quad \forall i \in V \tag{2.19}$$

$$\sum_{r \in \mathcal{R}'} y_r \leq K \tag{2.20}$$

$$y_r \geq 0 \quad \forall r \in \mathcal{R}' \tag{2.21}$$

Let $\bar{y}$ be the optimal solution to problem *P*', and let $\bar{\pi} = \{\bar{\pi}_1, \bar{\pi}_2, ..., \bar{\pi}_n\}$ be the corresponding optimal dual variables associated with constraints (2.19). Let $\bar{\theta}$ be the optimal dual variable associate with constraint (2.20). We would like to know whether $\bar{y}$ (or, equivalently, $(\bar{\pi}, \bar{\theta})$) is optimal for the linear relaxation of problem *P* (respectively, the dual of the linear relaxation of problem *P*). To answer this question, observe that the dual of the linear relaxation of problem *P* is

$$(P_D)\ \max \sum_{i \in V} \pi_i - K\theta \tag{2.22}$$

subject to

$$\sum_{i \in V} \alpha_{ir} \pi_i - \theta \le c_r \quad \forall r \in \mathcal{R}, \forall i \in V, \pi_i \ge 0, \theta \ge 0 \tag{2.23}$$

Clearly, if $(\bar{\pi}, \bar{\theta})$ satisfies constraint (2.23), then it is optimal for problem $P_D$ and therefore $\bar{y}$ is optimal for the linear programming relaxation of problem *P*. The vector $(\bar{\pi}, \bar{\theta})$ is not feasible in problem $P_D$ if there exists a single constraint, *r*, such that

$$\sum_{i \in V} \alpha_{ir} \bar{\pi}_i > c_r + \theta \tag{2.24}$$

Consequently, if there exists a column *r* that minimizes the quantity $c_r - \sum_{i \in V} \alpha_{ir} \bar{\pi}_i$ and this quantity is less than $-\bar{\theta}$, then a violated constraint is found. In that case the current vector $(\bar{\pi}, \bar{\theta})$ is not optimal for problem $P_D$. The corresponding column just found can be added to the formulation of problem *P*, which is solved again. The process repeats itself until no violated constraint (negative reduced cost column) is found; in this case the optimal solution to the linear relaxation of problem *P* (the vector $\bar{y}$) and the optimal solution to problem $P_D$ (the vector $(\bar{\pi}, \bar{\theta})$) is found.

The column-generation problem is to identify a feasible route $r \in \mathcal{R}$ that satisfies (2.24) . Define $\bar{c}_r$ to be the reduced cost of column *r*, i.e., $\bar{c}_r = c_r + \bar{\theta} - \sum_{i \in S_r} \bar{\pi}_i$ for each $r \in \mathcal{R}$. Also define $d(s) = \sum_{i \in S} d_i$ for any $S \subseteq V$. The task is then to solve the column generation problem, which is

$$(CG)\ \ \min \{\bar{c}_r : d(S_r) \le C\} \tag{2.25}$$

It is not clear how this column-generation problem, CG, should be solved. Problem CG is itself NP-hard since, even given $S_r$, evaluating $\bar{c}_r$ (or $c_r$) requires solving the Traveling Salesman Problem (TSP) with respect to vertex set $S_r \cup \{0\}$.

In summary, the column-generation algorithm for solving the linear relaxation of problem *P* can be described as follows:

1. Generate an initial set of columns $\mathcal{R}'$.
2. Solve problem *P*' and get optimal primal variables, $\bar{y}$, and optimal dual variables, $(\bar{\pi}, \bar{\theta})$.
3. Solve problem CG, or identify routes $r \in \mathcal{R}$ satisfying $\bar{c}_r < 0$.
4. For every $r \in \mathcal{R}$ with $\bar{c}_r < 0$ add the column *r* to $\mathcal{R}$ and go to 2.
5. If no routes *r* have $\bar{c}_r < 0$, i.e., $\bar{c}_{\min} \geq 0$, then stop.

The procedure produces a vector $\bar{y}$ which is the optimal solution to the linear relaxation of problem *P*. The objective function value $\sum_{r \in \mathcal{R}'} c_r \bar{y}_r$ is then a lower bound on the optimal solution value to the CVRP, i.e., the optimal integer solution value to *P*.

The column generation step (step 3) usually turns out to be the most time consuming. To reduce the computation time of this step, the following additional features can be implemented. First, it is important to generate a good set of initial routes in step 1. To do this, a large number of quick heuristics for the CVRP can be used. In fact, if a good dual solution is available, then it can be used to help generate routes with low reduced cost (with respect to this dual solution). Several methods for estimating good dual variables were given by Agarwal et al. (1989) and Hadjiconstantinou et al. (1995b). It is also important that in each iteration of step 3 a number of routes with negative reduced cost be generated, not just one. In addition, it is particularly helpful to generate sets of new columns that are disjoint (as in an integer solution).

### 2.1.3. Branch-and-cut algorithms

Branch-and-cut is a method of combinatorial optimization for solving integer linear programming problems, where some or all the unknowns are restricted to integer values. The method is a hybrid of branch and bound and cutting plane methods.

Branch-and-cut has been very successful in solving many combinatorial optimization problems (see (Caprara & Fischetti, 1997)); however, there are situations in which it may perform poorly. This unpleasant situation happens, for example, when (1) we do not have a good algorithm with which to perform the cutting plane phase, (2) the number of iterations of the cutting plane phase is too high, (3) the linear program becomes unsolvable because of its size, or (4) the tree generated by the branching procedure becomes too large and termination seems unlikely within a reasonable amount of time.

Nevertheless, branch-and-cut algorithms currently constitute the best available exact approach for the solution of the CVRP. However, the amount of research effort spent to solve CVRP by this method is not comparable with what has been dedicated to the TSP, and is still quite limited and most of it is not yet published.

The use of branch-and-cut for the CVRP is rooted in the exact algorithm of Laporte, Nobert and Desrochers (1985). Augerat et al. (1995) developed the first complete branch-and-cut approach for the CVRP. They described several heuristic separation procedures for the classes of valid inequalities proposed by Cornuejols and Harche (1993), as well as four new classes of valid in-equalities. Separation procedures were further investigated by both Augerat (1995) and Augerat, Belenguer, Benavent, Corberan and Naddef (1998).

Augerat et al. (1998) presented a computational study that uses a branch-and-cut algorithm that makes use of many of the separation procedures and strategies. The algorithm, developed by three groups of researchers, was not done with the purpose of being efficiently implemented. Rather than state-of-the-art software, the code is a kind of experimental environment that can easily accommodate various separation routines and algorithmic strategies, with the purpose of making comparison testing readily available.

The main drawback of such a code is the lack of several components that are common to most branch-and-cut codes, like, among others, pool management and the possibility of having only subsets of variables active in the solution of the linear programs. In addition, the visit of the enumeration tree is done using the depth-first, which is the easiest to

implement but also the least effective. Last but not least, the algorithm was implemented via independent pieces of code communicating through files written in the mass storage. Such an algorithmic design provided some flexibility to the developers but has, of course, a price in terms of efficiency.

Due to these facts, the computational results and the performance indicators reported by Augerat (1995) are not to be taken as reliable evidence of the actual potential of the technique. Nevertheless, it is the first algorithm which found an optimal solution (and proved its optimality) for two instances of 135 nodes proposed by Fisher (1994). To the best of our knowledge, these are still the largest instances for which a certified optimal solution has been computed.

Lysgaard, Letchford and Eglese (2004) developed new separation procedures for most of the families of valid inequalities proposed so far. Their overall branch-and-cut approach is able to solve within moderate computing times previously solved instances and three new medium size ones.

In a further computational study, Ralphs T. K. , Kopman, Pulleyblank and Trotter (2003) (see also (Ralphs T. K., 1995) and (Kopman, 1999)) have presented a parallel branch-and-cut algorithm for the CVRP in which an exact separation of valid *m*-TSP inequalities is used in addition to heuristic separation of capacity inequalities. Such an algorithm is able to find optimal solutions (and prove their optimality) and improve the best known solutions for some of the test problems.

Another, more recent, study was reported by Blasum and Hochstattler (2000), who developed an algorithm using the same branching strategy and separation procedures as in Augerat (1995) with some modifications. For example, they developed a heuristic procedure for separating the rounded capacity inequalities based on their algorithm for the separation of the multi-star inequalities. However, they used the state-of-the-art branch-and-cut framework ABACUS, developed by Junger and Thienel (1998). It is remarkable, however, that the algorithm can solve two difficult 76-node problems to optimality with computing times considerably shorter than previous algorithms.

Fukasawa et al. (2006) proposed a successful branch-and-cut-and-price algorithm combining branch-and-cut with the *q*-routes relaxation of Christofides et al. (1981a). This method produces tighter bounds than other branch-and-cut algorithms and is capable of solving several previously unsolved instances with up to 75 customers. Baldacci, Bodin

and Mingozzi (2006) have used their set partitioning algorithm to solve difficult CVRP instances. Their approach yields bounds whose quality is comparable to those of Fukasawa et al. (2006), but seems much quicker.

Other branch-and-cut algorithms are described in Achuthan, Caccetta and Hill (1996), Achuthan, Caccetta and Hill (2003) and Blasum and Hochstattler (2000). The polyhedral structure of the special case of CVRP, where all the customers have a unit demand, is described in Campos, Corberan and Mota (1991) and by Araque, Hall and Magnanti (1990). Branch-and-cut algorithms for this problem are presented by Araque G, Kudva, Morin and Pekny (1994) and by Ghiani, Laporte and Semet (2006).

### 2.1.4. Dynamic Programming

Dynamic programming (Bellman, 1954; Bertsekas, Bertsekas, Bertsekas & Bertsekas, 1995; Dreyfus & Law, 1977) was first proposed for VRPs by Eilon, Watson-Gandy and Christofides (1971). Consider a VRP with a fixed number *m* of vehicles. Let $c(S)$ denote the cost (length) of a vehicle route through vertex 1 and all vertices of a subset *S* of $V\setminus\{1\}$. Let $f_k(U)$ be the minimum cost achievable using k vehicles and delivering to a subset U of $V\setminus\{1\}$. Then the minimum cost can be determined through the following recursion:

$$f_k(k)=\begin{cases} c(U) & k=1 \\ \min\limits_{U^*\subset U\subseteq V\setminus\{1\}}\left[f_{k-1}\left(U\setminus \mathrm{U}^*\right)+c\left(U^*\right)\right] & k>1 \end{cases} \tag{2.26}$$

The solution cost is equal to $f_m\left(V\setminus\{1\}\right)$ and the optimal solution corresponds to the optimization subsets $U^*$ in (2.26).

It is apparent that if $f_k(U)$ has to be computed for all *k* and for all subsets *U* of $V\setminus\{1\}$, the number of computations required is likely to be excessive in most problems. Efficient use of dynamic programming requires a substantial reduction of the number of states by means of a relaxation procedure, or by using feasibility or dominance criteria.

Christofides, Mingozzi and Toth (1981b) introduced a *state-space* relaxation, which is an efficient way of reducing the number of states. It provides a longer bound on the cost

of the optimal solution. The optimum can then be reached by embedding the bounding procedure in an enumerative scheme. The method can be summarized as follows: Consider the general dynamic programming recursion

$$f_{0,i} = \min_{k \in \Delta^{-1}(j)} \left[ f_{0,i-1}(0,k) + c_i(k,j) \right] \tag{2.27}$$

where $f_{0,i}(0,j)$ is the least cost of going from state 0 at stage 0 to state $j$ at stage $i$, $\Delta^{-1}(j)$ is the set of all possible states from which state $j$ can be reached directly, and $c_i(k,j)$ is the cost of going from state $k$ at stage $i$-1 to state $j$ at stage $i$. Let $g(\cdot)$ be a mapping from state space $S$ associated with (2.27) to a state space $T$ of smaller cardinality, and let $F^{-1}(g(i))$ be a set satisfying

$$k \in \Delta^{-1}(g) \Rightarrow g(k) \in F^{-1}(g(j)). \tag{2.28}$$

Recursion (2.27) then becomes

$$f_{0,i}(g(0), g(j)) = \min_{t \in F^{-1}(g(j))} \left[ f_{0,i-1}(g(0), t) + \overline{c}_i(t, g(j)) \right] \tag{2.29}$$

where

$$\overline{c}_i(t, g(j)) = \min \left[ c_i(k,l) : g(k) = t, g(j) = g(l) \right]. \tag{2.30}$$

It results that

$$f_{0,i}(g(0), g(i)) \leq f_{0,i}(0,i). \tag{2.31}$$

This relaxation is useful only if (1) $F^{-1}(\cdot)$ can easily be determined, this will be so if $g(\cdot)$ is *separable*, so that given $g(U)$ and $r$, $g(U \setminus \{r\})$ can be computed; (2) $g(\cdot)$ is

such that the optimization of (2.30) is over a small domain or that a good lower bound on $\overline{c}_i\left(t, g\left(j\right)\right)$ can be computed.

Christofides et al. (1981b) used the following relaxation for CVRPs. Let $f_k\left(U, r\right)$ be the least cost of supplying a set *U* of vertices, using *k* vehicles, where the last vehicle of the *k* corresponding routes belong to $\left\{2,...,r\right\}$ $(k \le r \le n)$. Let $c\left(U, r\right)$ be the cost of the TSP solution through $U \cup \left\{1\right\}$, where the last vertex before the depot is *r*. The recursion is then

$$f_k\left(U, r\right) = \begin{cases} \min\left[ f_k\left(U, r-1\right), \min\limits_{U^* \in U}\left\{ f_{k-1}\left(U \setminus \mathrm{U}^*, r-1\right) + c\left(U^*, r\right)\right\}\right] & k, r > 1 \\ c\left(U, r\right) & k = 1 \end{cases} \tag{2.32}$$

subject to

$$\sum_{i \in V \setminus \{1\}} d_i - \left(m-k\right)D \le \sum_{i \in U} d_i \le kD \quad k = 1,...,m\,. \tag{2.33}$$

For this problem, is given by

$$g\left(U\right) = \sum_{i \in U} d_i \cdot \tag{2.34}$$

Recursion (2.29) then becomes

$$f_k\left(g\left(U\right), r\right) = \min\left[ f_k\left(g\left(U\right), r-1\right), \min_{p}\left\{ f_{k-1}\left(g\left(U\right) - p, r-1\right) + \overline{c}\left(p, r\right)\right\}\right] \tag{2.35}$$

subject to

$$g\left(V\right) - \left(m-1\right)D \le p \le \min\left(g\left(U\right), D\right) \tag{2.36}$$

Using this and other relaxations, lower bounds on optimal VRP solutions were obtained for problems with up to 25 vertices. The ration "lower bound / optimum" varied between

93.1% and 100%. In a later study, Christofides (1985) reported that CVRPs with up to 50 vertices can be solved systematically with this approach.

## 2.2. Heuristic Methods

An impressive number of heuristics have been proposed for the VRP. In this chapter, there is a description of the most important heuristic methods.

### 2.2.1. The Savings Algorithm

The Savings heuristic, proposed by Clarke and Wright (1964), is based on the concept of saving. If $i$ is the last customer of a route and $j$ is the first customer of another route, the associated saving is defined as $S_{ij}=C_{i0}+C_{0j}-C_{ij}$. An initial solution is defined as a set of routes, each of which starts at the depot, visits one customer and returns to the depot. An iterative process merges all routes that can be feasibly merged into a single route, when in each step the two routes with the highest non negative value of savings are merged.

This algorithm naturally applies to problems for which the number of vehicles is a decision variable, and it works equally well for directed or undirected problems. However, according to Vigo (1996) the behavior of the method worsens considerably in the directed case, although the number of potential route merges is then halved.

One drawback of the original Savings algorithm is that it tends to produce good routes at the beginning but less interesting routes toward the end, including some circumferential routes. To remedy this, Gaskell (1967) and Yellow (1970) proposed generalized savings of the form $S_{ij}=C_{i0}+C_{0i}-\lambda C_{ij}$ where $\lambda$ is a route shape parameter. The larger the $\lambda$, the more emphasis is put on the distance between the vertices to be connected. Golden, Magnanti and Nguyen (1977) reported that using $\lambda=0.4$ or 1.0 yields good solutions, taking into account the number of routes and the total length of the solution.

### 2.2.2. The Sweep Algorithm

The sweep algorithm, proposed by Gillett and Miller (1974), is a two-phase process applied to planar VRP instances. Assume each vertex $i$ is represented by its polar coordinates $(\theta_i,l_i)$, where $\theta_i$ is the angle and $l_i$ is the ray length. The algorithm starts with

an arbitrary customer and then sequentially assigns the remaining customers to the current vehicle by considering them in order of increasing polar angle with respect to the depot and the initial customer. As soon as the current customer cannot be feasibly assigned to the current vehicle, a new route is initialized with it. Once all customers are assigned to vehicles, each route is separately defined by solving a TSP.

Some implementations include a post-optimization phase in which vertices are exchanged between adjacent clusters, and routes are re-optimized. To our knowledge, the first mentions of this type of method are found in a book by Wren (1971) and in a paper by Wren and Holliday (1972).

### 2.2.3. The Fisher and Jaikumar algorithm

The Fisher and Jaikumar (1981) algorithm, as the Swap algorithm, is a two-phase process in which feasible clusters of customers are first created by solving a generalized assignment problem (GAP). The GAP is solved either optimally or heuristically. The final routes are determined by solving a TSP on each cluster.

To formulate the GAP, it is first necessary to determine a seed for each route from which customer distances are computed. Since the GAP is NP-hard, it is usually solved by means of a Lagrangian relaxation technique. Fisher and Jaikumar (1981) provided integer solutions values without providing the rounding or truncating rule. Their solutions cannot be verified, which makes the assessment of the algorithm difficult.

Bramel and Simchi-Levi (1995) optimized the choice of seeds in the Fisher and Jaikumar algorithm by solving a capacitated location problem. Their results on the seven CMT instances containing only capacity constraints show a significant average deviation (3.29%) from the best known results.

## 2.3. Meta-heuristics Algorithms

Several meta-heuristics have been applied to the VRP. With respect to classical heuristics, they perform a more thorough search of the solution space and are less likely to end with a local optimum. These can be broadly divided into three classes: (1) local search, including simulated annealing, deterministic annealing, and tabu search; (2) population search, including genetic algorithm and adaptive memory procedures; (3) learning mechanisms, including neural networks and ant colony optimization. The best

heuristics often combine ideas borrowed from different meta-heuristic principles. Recent surveys of VRP meta-heuristics can be found in (Gendreau, Laporte & Potvin, 2001), (Cordeau & Laporte, 2004), and (Cordeau, Gendreau, Hertz, Laporte & Sormany, 2005).

### 2.3.1. Simulated Annealing

Simulated annealing (Kirkpatrick, Gelatt & Vecchi, 1983) is a generic probabilistic meta-heuristic for a global optimization problem of applied mathematics, namely locating a good approximation to global minimum (or maximum) of a given function in a large search space.

A simulated annealing algorithm consists of a discrete-time inhomogeneous Markov chain $x(t)$, whose evolution is the following:. At $t=0$, a random feasible solution is chosen. Let $x(t)=i$, choose a neighbor $j$ of $i$ at random. Once $j$ is chosen, the next state $x(t+1)$ is determined as follow: (1) if $C(j) \leq C(i)$ then $x(t+1)=j$, (2) if $C(j)>C(i)$ then $x(t+1)=j$ with probability $exp[-(C(j)-C(i))/T(t)]$, otherwise $x(t+1)=i$, where C is a cost function and $T$ is a non increasing function (Bertsimas Dimitris & Tsitsiklis, 1993).

A limited number of simulated annealing heuristics for the CVRP were proposed in the early 1990s. Two early implementations are those of Robuste, Daganzo and Souleyrette (1990), and Alfa, Heragu and Chen (1991). Robuste et al. (1990) tested their algorithm on four instances ($n=80, 100, 120, 500$), but no comparisons with alternative methods are available. The algorithm proposed by Alfa et al. (1991) was applied to three instances ($n=30, 50, 75$) and did not produce competitive results.

Osman's implementation (Osman, 1993) is the most involved and also the most successful. This algorithm succeeded in producing good solutions, but was not competitive with the best tabu search implementations available at the same period.

### 2.3.2. Tabu Search

The roots of tabu search go back to the 1970's. However, it was first presented in its current form by Glover (1986) and by Hansen M. P. (1986).

The first step ($k=0$, where $k$ is the iteration counter) of tabu search is choosing a feasible solution, $i$. Let $i^*$ denote the best solution found so far, at $k=0$, $i^*$ is equal to $i$. The following iterative procedure is carried out: At each iteration a subset $V^*$ of solution in $N(i,k)$ is generated, where $N(i,k)$ denotes the neighborhood of solution $i$, in which some

recently visited solutions are removed (the tabu list). Then, $i$ is assigned with the best solution found in $V^*$, with respect to a cost function $C$. If $C(i)>C(i^*)$ (for maximization problems), $i^*$ is set to $i$. The tabu list is updated with the new solution. The iterative process is repeated until a stopping condition is met.

A large number of tabu search algorithms have been produced over the past twenty years. The first known implementation is of Willard (1989), but it was soon superseded by more powerful algorithms, including those of Osman (1993), Taillard E. (1993), and Gendreau, Hertz and Laporte (1994). To this day, Taillard's algorithm remains one of the most successful tabu search implementations for the CVRP.

Deterministic annealing was first applied to the VRP by Golden, Wasil, Kelly and Chao (1998) and more recently by Li F., Golden and Wasil (2005).

A limited number of heuristics based on learning mechanisms have been proposed for the VRP. None of the known neural networks based methods is satisfactory, and the early ant colony based heuristics could not compete with the best available approaches. However, recently Reimann, Doerner and Hartl (2004) have proposed a well-performing heuristics called *D*-ants.

### 2.3.3. Genetic Algorithms

A genetic algorithm (GA) is a randomized global search technique used in computing to find exact or approximate solutions to optimization and search problems by imitating processes observed during natural evolution, such as mutations and crossover. GAs are categorized as global search heuristics, and are a particular class of evolutionary algorithms (EA) that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover. GAs were first introduced by Holland (1975). Basically, a GA evolves a population, also known as chromosomes, where each chromosome encodes a solution to a particular instance. This evolution takes place through the application of operators that mimic natural phenomena observed in nature (e.g., reproduction, mutation). For more information about GAs refer to (Mitchell, 1996).

Homberger and Gehring (1999) described two GAs for the VRPTW. Starting from a population with $\mu$ individuals, subsets of individuals are randomly selected and recombined to yield a total of $\lambda>\mu$ offspring. Each offspring is then subjected to a mutation operator, and the $\mu$ fittest are selected to form the new population. In the first

algorithm, new individuals are generated directly through mutations and no recombination takes place. In the second algorithm, offspring are generated through a two-step recombination procedure in which three individuals are involved. In both algorithms, the fitness of an individual depends first on the number of vehicles used and second on the total distance traveled.

In a later work, Gehring and Homberger (2002) proposed a two-phase meta-heuristic in which the first phase uses GA to minimize the number of vehicles, while the second one minimizes the total distance through tabu search.

Berger and Barkaoui (2003) developed a GA that concurrently evolves two distinct populations pursuing different objectives under partial constraint relaxation. The first population aims to minimize the total distance traveled while the second one focuses on minimizing the violations of the time window constraints. The maximum number of vehicles imposed in the first population is equal to $k_{min}$ whereas the second population is allowed only $k_{min}-1$ vehicles, where $k_{min}$ refers to the number of routes in the best known feasible solution. Whenever a new feasible solution emerges from the second population, the first population is replaced with the second and the value of $k_{min}$ is updated accordingly. Two recombination operators and five mutation operators are used to evolve the populations. This approach has proved to be rather efficient in minimizing the number of vehicles used.

More recently, Mester and Bräysy (2005) developed an iterative meta-heuristic that combines guided local search and evolution strategies. An initial solution is first created by an insertion heuristic. This solution is then improved by the application of a two-stage procedure. The first stage consists of a guided local search procedure in which 2-opt* and Or-opt exchanges are performed together with 1-interchanges. This local search is guided by penalizing long arcs appearing often in local minima. The second stage iteratively removes a selected set of customers from the current solution and reinserts the removed customers at minimum cost. These two stages are themselves repeated iteratively until no further improvement can be obtained. Very good results are reported by the authors on large-scale instances. According to Bräysy and Gendreau (2005b), the three approaches just described seem to produce the best results among genetic algorithms. Other such

algorithms have also been proposed by a number of researchers including Potvin and Bengio (1996), Thangiah and Petrovic (1998) and Tan K., Lee, Zhu and Ou (2001).

Wilck and Cavalier (2012) developed two hybrid genetic algorithms, each with a different fitness approach, for the SDVRP for which they provided computational results for thirty-two data sets from previous literature. Of the two fitness approaches, the second fitness approach, ration of demand unit vs. distance unit, performed better than the first fitness approach, shortest route, for most of the 32 data sets in terms of solution quality. Neither fitness approach was better than the other in solution time.

Shen Y. and Murata (2012) presented a genetic algorithm (GA) for the basic vehicle routing problem with two-dimensional loading constraints, which is a combination of the Bin Packing Problem and the Vehicle Routing Problem (2L-CVRP). In the field of combinatorial optimization, loading and routing problems have been studied intensively but separately. 2L-CVRP is a generalization of the Capacitated Vehicle Routing Problem, in which customer demand is formed by a set of rectangular, weighted items.

A GA developed for solving the problem performs well, although it does not equal the mathematic model that ran by MIP in terms of solution quality, but it only takes 8.92% on average of the computing time. By changing the Two-point crossover to a One-point crossover, the algorithm was able to get an acceptable result that can accurately calculate the vehicle numbers up to 75 customers, while consuming only 4.31% cost on average and greatly reducing the computing time (8.23%).

### 2.3.4. Ant Systems Algorithms

Ant systems (AS) is a probabilistic technique for solving optimization problems, which can be reduced to finding good paths through graphs. AS methods are inspired by an analogy with real ant colonies foraging for food. In their search for food, ants mark the paths they travel by leaving an aromatic essence called pheromone. The quantity of pheromone left on a path depends on the length of the path and the quality of the food source. This pheromone provides information to other ants that are attracted to it. With time, paths leading to the more interesting food sources, i.e., those close to the nest and with large quantities of food become more frequented and are marked with larger amounts of pheromone. Overall, this process leads to an efficient procedure for procuring food by ant colonies.

This observation led Colorni, Dorigo and Maniezzo (1991) to propose a new class of meta-heuristics for solving combinatorial problems based on the following correspondences: Artificial ants searching the solution space simulate real ants exploring their environment, objective function values are associated with the quality of food sources, and values recorded in an adaptive memory mimic the pheromone trails.

The number of papers with an application of AS to the VRP is very limited. Kawamura, Yamamoto, Mitamura, Suzuki and Ohuchi (1998) proposed a complex hybrid variant of AS that involves 2-opt improvement procedures and probabilistic acceptance rules reminiscent of simulated annealing. The method was applied to two geometric instances of 30 and 60 customers, and it identified the optimal solution in both cases.

Bullnheimer, Hartl and Strauss (1997) developed a hybrid AS in which each vehicle route produced in a given iteration is improved by the 2-opt heuristic before the trail update. This algorithm also uses terms related to vehicle capacity and distance savings with respect to the depot when selecting the next vertex to be visited. In the trail update step, they use a number of "elitist ants" to account for the best solution found so far (these ants are assumed to always travel on this best solution). Their computational experiments on the 14 problems of Christofides, Mingozzi and Toth (1979) indicate that the addition of a 2-opt step and the use of elitist ants are clearly beneficial. The best results obtained over 30 distinct runs range from 0 to 14.09% above the best known solutions to the problems with an average error of 4.43%.

In a later paper (Bullnheimer, Hartl & Strauss, 1999), the authors refined their algorithm in several ways: (1) the capacity term previously used in the vertex selection rule, which was quite expensive to compute, is dropped, and the saving term is incorporated directly in the visibility term in a parametric fashion, (2) Only the $\lfloor \eta/4 \rfloor$ nearest neighbors of any vertex are considered when choosing the next customer to visit, (3) Only the five best solutions found in each iteration are used for trail update, and the pheromone quantity laid is further weighted according to the solution's rank. These various changes have led to shorter run times and improved solutions. The computational results obtained on the 14 benchmark problems are quite good with an average error of only 1.51% above the best known solutions and CPU times that are very reasonable.

Montemanni, Gambardella, Rizzoli and Donati (2003) studied a dynamic vehicle routing problem, and proposed an Ant Colony System based algorithm for solving it. The algorithm proposed is based on three main elements: (1) Event manager, which collects new orders and keeps track of the already served orders and of the current position of each vehicle. (2) Ant Colony System (ACS) algorithm. The information collected by the event manager is used to construct a sequence of static VRP-like instances, which are solved heuristically by the an ACS algorithm. (3) Pheromone conservation procedure, which is strictly connected with the ACS algorithm, and is used to pass information about characteristics of good solutions from a static VRP to the following one.

The method has been tested on a set of benchmarks defined starting from a set of widely available problems. Computational results confirm the effectiveness and the efficiency of the strategy proposed.

Yu, Yang and Yao (2009) proposed an improved ant colony optimization (IACO), which possesses a new strategy to update the increased pheromone, called ant-weight strategy, and a mutation operation; the mutation operator is designed to conduct customer exchanges in a random fashion, to solve VRP. The computational results of 14 benchmark problems reveal that the proposed IACO is effective and efficient.

Erfianto and Indrawan (2012) considered a new parameter of road congestion level as an obstacle to the VRP, and presented the Multiobjective Ant Colony System (MOACS). In this modified algorithm, the level of congestion affects the probability of route selection in MOACS. MOACS itself has been used to solve the VRP, and is based on Ant Colony System with Pareto approach (Baran & Schaerer, 2003).

The idea of this algorithm is to construct a feasible solution using the vehicle as much as needed. Having obtained a feasible solution, the algorithm then performs checking of the feasibility of such solutions to get into the Pareto set. This is done repeatedly until the entire solution generated from one generation has been checked to obtain a Pareto set Solution.

Simulation results showed that a higher level of road congestion can affect the value of the probability of the road path to be selected. A congestion level of 0.9 makes the probability of selecting a road (33-22) decrease to 0.013, from 0.117 in conditions without the congestion.

Variations in the number of congested roads with a congestion level of 0.9 on the best roads from the a state without congestion, can affect the total travel time required by a vehicle. The more the number of congested roads the greater the total travel time required by a vehicle. In a congestion scenario, MOACS that involve the level of congestion can produce solutions with a total route travel time better than the original MOACS algorithm.

### 2.3.5. Neural Networks

Neural networks are computational models composed of units that are richly interconnected through weighted connections, like neurons in the human brain: a signal is sent from one unit to another along a connection and is modulated through the associated weight. Although superficially related to their biological counterpart, artificial neural networks exhibit characteristics related to human cognition. In particular, they can learn from experience and induce general concepts from specific examples through an incremental adjustment of their weights. These models were originally designed for tasks associated with human intelligence and where traditional computation has proven inadequate, like artificial vision and speech comprehension. More recently, they have been applied to combinatorial problems as well, starting with the pioneering work of Hopfield and Tank (1985). The TSP, in particular, has been the subject of many investigations with the Hopfield-Tank model, the elastic net (EN) (Durbin & Willshaw, 1987), and the self-organizing map (SOM) (Kohonen, 1988). The EN and SOM models are quite remote from classical neural networks, but they have proved to be more effective on the TSP than the Hopfield-Tank model. However, neither of these methods is yet competitive with other meta-heuristics (Potvin, 1993).

## 2.4. Important Variants of the Vehicle Routing Problem

As research developed a number of researchers developed extensions to the basic VRP. The goal was to develop more realistic models, to adapt to the larger number of constraints of the real world. The following is a description of the most common variants.

### 2.4.1. Split Delivery Vehicle Routing Problem

In the CVRP, one of the constraints states that each customer is serviced by exactly one visit of a single vehicle. In the split delivery VRP (SDVRP), a customer can be serviced by more than one vehicle, that is, a customer's demand can be split among several vehicles. By allowing split deliveries, the potential exists to use fewer vehicles and to reduce the total distance traveled by the fleet. However, we should note that in general, for both the VRP and SDVRP, using fewer vehicles does not necessarily reduce the total distance.

Research such as Burrows (1988) and Dror and Trudeau (1989) showed that by allowing split deliveries, the total length of routes can be smaller by up to 10% compared to CVRP.

A Savings based algorithm for solving SDVRP was introduced by Burrows (1988).

Wilck and Cavalier (2012) developed two hybrid genetic algorithms, each with a different fitness approach, for the SDVRP for which they provided computational results for thirty-two data sets from previous literature. Of the two fitness approaches, the second fitness approach, ration of demand unit vs. distance unit, performed better than the first fitness approach, shortest route, for most of the 32 data sets in terms of solution quality. Neither fitness approach was better than the other in solution time.

### 2.4.2. Vehicle Routing Problem with Time Windows

The VRP with Time Windows (VRPTW) is an important extension of the CVRP in which service at every customer $i$ must start within a given time window $[a_i,b_i]$. A vehicle is allowed to arrive before $a_i$ and wait until the customer becomes available, but arrivals after $b_i$ are prohibited. Time windows constraints are hard constraints when a route is not feasible, if the service of a customer either starts before the earliest time or ends after the latest time of the day established by the time window. These are Vehicle Routing Problems with Hard Time Windows. In other cases, both lower and upper bounds of the time window need not be satisfied, but can be violated at a penalty. These are Vehicle Routing Problems with Soft Time Windows.

The VRPTW has numerous applications in distribution management. Common examples are beverage and food delivery, newspaper delivery, and commercial and

industrial waste collection (see e.g., (Golden, Assad & Wasil, 2002)). Solomon's work (Solomon, 1987) was one of the first studies done on VRPTW. In his work, Solomon presented a number of extensions to existing algorithms, such as the Savings algorithm, for solving VRPTW.

Optimal solutions for small instances of the VRPTW, in which the single objective of minimizing the total travel distance is considered, can be obtained using exact methods (Desrochers, Desrosiers & Solomon, 1992). Current state of the art exact algorithms are proposed by Chabrier (2006), Irnich and Villeneuve (2003), Jepsen, Petersen, Spoorendonk and Pisinger (2006), Jepsen, Petersen, Spoorendonk and Pisinger (2008) and Kallehauge, Larsen and Madsen (2006). To date, 45 out of 56 instances in Solomon's benchmarks have been solved to optimality (Jepsen et al., 2006) using exact methods.

Although exact methods can guarantee the optimality of the solution, they require considerable computer resources in terms of both computational time and memory. As a result, research on the VRPTW has concentrated on heuristics and meta-heuristics. For an extensive list of studies of different heuristics and meta-heuristics for solving VRP, as well as a comparison of the results obtained, the reader is referred to Bräysy and Gendreau (2005a), Bräysy and Gendreau (2005b), Cordeau, Desaulniers, Desrosiers, Solomon and Soumis (2001) and Golden, Raghavan and Wasil (2008).

As of today, state-of-the-art heuristics for the VRPTW consist of evolution strategies (Homberger & Gehring, 2005; Mester & Bräysy, 2005), large neighborhood searches (Bent & Van Hentenryck, 2004; E., G. & M., 2009; Pisinger & Ropke, 2007), iterated local searches (Ibaraki et al., 2008; Ibaraki, Kubo, Masuda, Uno & Yagiura, 2001) and multi-start local searches (Ibaraki et al., 2001; Lim & Zhang, 2007). It should be noted that in all of these algorithms the hierarchical objective is considered and therefore these state-of-the-art heuristics are all based on a two-stage approach, where the number of routes is minimized in the first stage and the total travel distance is then minimized in the second stage, allowing us to independently develop algorithms for the route, and for the distance, minimization.

### 2.4.3. Multi-Depot Vehicle Routing Problem

Whereas the CVRP has been studied widely, the multi-depot VRP (MDVRP) has attracted less attention. In the MDVRP, customers must be serviced by one of several

depots. As with the CVRP, each vehicle must leave and return to the same depot and the fleet size at each depot must range between a specified minimum and maximum.

Early branch and bound algorithms were proposed by Laporte, Nobert and Arpin (1984) for the case where the travel time matrix is symmetric, and by Laporte, Nobert and Taillefer (1988) for the asymmetric case. The largest problems reported solved to optimality involved 50 customers in the first case and 80 customers in the second case. The first heuristics were proposed by Tillman (1969), Tillman and Hering (1971), Tillman and Cain (1972), Wren and Holliday (1972), Gillett and Johnson (1976), Gillett and Miller (1974), Golden et al. (1977) and Raft (1982), all using adaptations of standard VRP procedures. Chao, Golden and Wasil (1993) proposed a better approach based again on the record-to-record method. Renaud, Laporte and Boctor (1984) described a tabu search heuristic yielding highly competitive results. It constructs an initial solution by assigning each customer to its nearest depot and solving the resulting VRPs by means of the Improved Petal heuristic (Renaud, Boctor & Laporte, 1996).

### 2.4.4. Time Dependent Vehicle Routing Problem

In the real world, especially in urban areas, the travel time is dependent on both the distance between two customers and the time of day. Ignoring the fact that for some routes the travel time changes throughout the day, we may obtain solutions that are far from optimal. The Time-Dependent VRP (TDVRP) was developed in order to avoid just such a mistake. Whereas most VRP variants look for the shortest paths in terms of length, the TDVRP seeks the shortest paths in terms of travel time.

There has been limited research related to time-dependent vehicle routing compared to other VRP models (Ichoua et al., 2003; Ji & Wu, 2011).

The time dependent VRP was first formulated by Malandraki (1989) and Malandraki and Daskin (1992), using a mixed integer linear programming formulation. Malandraki and Daskin treated travel time as a function of both distance and the time of the day resulting in a piecewise constant distribution of the travel time. Although they only incorporated the temporal component of traffic-density variability, they acknowledged its importance. They developed two algorithms for solving the time-dependent vehicle-routing problem. The first algorithm was a greedy nearest-neighbor algorithm (three

variants of the algorithm were introduced), and the second was a branch and bound-based algorithm that provided better solutions, but was suitable only for small problems.

Hill and Benton (1992) considered a time dependent VRP (without time windows) and proposed a model based on time dependent traveling speeds that alleviates both the data collection and data storage problems inherent in time-dependent travel speed vehicle scheduling models. They also discussed the issue of developing algorithms to find near-optimal vehicle schedules with time-dependent travel speeds .Computational results for one vehicle and five customers were reported. Ahn and Shin (1991) discussed modifications to the Savings, insertion, and local improvement algorithms to better deal with TDVRP. In randomly generated instances, they reported computation time reductions as a percentage of "unmodified" Savings, insertion, and local improvement algorithms. Malandraki and Dial (1996) proposed a "restricted" dynamic programming algorithm for the time dependent traveling salesman problem, i.e. for a fleet of just one vehicle. A nearest-neighbor type heuristic was used to solve randomly generated problems. Although it is argued that many different types of travel time functions can be handled by this algorithm, results are only reported for step functions.

An important property for time dependent problems is the First In - First Out (FIFO) principal (Ahn & Shin, 1991; Ichoua et al., 2003). A model with a FIFO property guarantees that if two vehicles left the same location for the same destination (and traveled along the same path), the one that left first would never arrive later than the other. This is an intuitive and desirable property though it is not present in all models. Earlier formulations and solutions methods (Geiger, 2001; Held & Karp, 1971; Hill & Benton, 1992; Hong & Park, 1999; Malandraki, 1989; Malandraki & Daskin, 1992; Malandraki & Dial, 1996) do not guarantee the FIFO property.

Ichoua et al. (2003) introduced a model that guarantees the FIFO principle. This model is satisfied by working with step-like speed distributions and adjusting the travel speed whenever a vehicle crosses the boundary between two consecutive time periods. The algorithms that they developed, which were based on tabu-search meta-heuristics, provided better solutions for most test scenarios.

Fleischmann, Gietz and Gnutzmann (2004) utilized route construction methods already proposed in the literature, savings and insertion, to solve un-capacitated time dependent VRP with and without time windows. Fleischmann et al. (2004) assume travel times to be

known between all pairs of interesting locations and constant within given time slots. Neighbor slots with similar travel times are joined to reduce memory requirements, and the transitions between slots are smoothed to ensure a FIFO property on travel times. Fleischmann et al. (2004) tested their algorithms in instances created from Berlin travel time data. Time dependent VRP with time windows was also addressed by Hashimoto, Yagiura and Ibaraki (2008), who proposed an iterated local search algorithm.

Jung and Haghani (2001) and Haghani and Jung (2005) proposed a genetic algorithm to solve time dependent problems. By formulating the problem as a mixed integer linear programming problem, they obtain lower bounds by relaxing most of the integer requirements. The lower bounds are compared with the primal solutions from the genetic algorithm to evaluate the quality of the solutions. Using randomly generated test problems, the performance of the genetic algorithm was evaluated by comparing its results with exact solutions.

Van Woensel, Kerbache, Peremans and Vandaele (2008) used a tabu search to solve CVRP with time dependent travel times (with no time windows). Approximations based on queuing theory and the volumes of vehicles in a link were used to determine the travel speed. Donati, Montemanni, Casagrande, Rizzoli and Gambardella (2008) proposed an algorithm based on the ant colony heuristic approach and a local search improvement approach. The algorithm was tested using a real life network in Padua, Italy, and some variations of the Solomon problem set.

Ji and Wu (2011) proposed a revised scheme for the Artificial Bee Colony algorithm (a new population-based metaheuristic approach proposed by Karaboga Dervis (2005), inspired by the intelligent foraging behavior of a honeybee swarm), with improved performance for solving Capacitated VRP with Time-dependent Travel Times. Using a set of instances of different size, Ji and Wu (2011) showed that the ABC algorithm is improved in terms of better solution achieved, greater robustness and higher computational efficiency.

### 2.4.5. Stochastic Vehicle Routing Problem

A stochastic VRP arises when at least one of the problem's variables is random (Gendreau, Laporte & Seguin, 1996). Over the years, different solution frameworks have been developed for solving the problem (Shen Z., Ordónez & Dessouky, 2006). A

taxonomy of these frameworks classifies them into dynamic or static types (Secomandi & Margot, 2009).

Stochastic VRP can be divided into the following classes (Li X., Tian & Leung, 2010): (1) VRP with stochastic demand (VRPSD), in which the vehicles serve a set of customers with stochastic and uncertain demands (Bertsimas D. J., 1992; Gendreau, Laporte & Seguin, 1995; Stewart, 1983; Tillman, 1969). (2) VRP with stochastic customers (VRPSC), in which each customer has a deterministic demand and a probability p of being present. (3) VRP with stochastic customers and demands, a combination of VRPSD and VRPSC. For a detailed survey of the SVRP, one may refer to Gendreau et al. (1996).

A stochastic model is usually modeled in two stages. In the first stage, a planned *a priori* route is determined, followed by a realization of the random variables. In the second stage, corrective action is applied to the solution of the first stage base on actual information.

Methods modeled in two stages include a branch-and-bound method based on the integer L-shaped algorithm for solving VRP with stochastic demands, proposed by Laporte, Louveaux and Van Hamme (2002). In a more recent work, Rei, Gendreau and Soriano (2007) tackled the single VRPSD (SVRPSD), a variant where only one route is to be designed. Their method consists of using local branching to generate optimality cuts on an integer L-shaped algorithm. Although successful, these approaches are limited to solving instances of up to 100 customer nodes.

Stochastic travel times were introduced into the vehicle-routing problem by Laporte, Louveaux and Mercure (1992), who presented a CCP model. Their aim was to find a set of paths that had a travel time that was no longer than a given constant value. The problem was solved optimally by means of an Integer *L*-shaped algorithm for $10 \leq n \leq 20$ and two to five travel time scenarios (each scenario corresponds to a different travel speed for the entire network).

In VRP with Stochastic Travel Times (VRPSTT), vehicles follow their planned routes and may incur a penalty if the route duration exceeds a given deadline. It is natural to make this penalty proportional to the elapsed route duration in excess of the deadline (Laporte et al., 1992). Another possibility is to define a penalty proportional to the uncollected demand within the time limit, as is the case in a money collection application

studied by Lambert, Laporte and Louveaux (1993). Wang X. and Regan (2001) have proposed models for this class of problems in the presence of time windows.

In a more recent study, Kenyon and Morton (2003) have investigated properties of VRPSTT solutions and have developed bounds on the objective function value. They have developed two models for the stochastic VRP with random travel and service times and an unknown distribution. The first model minimizes the expected completion time, and the second model maximizes the probability that the operation is complete prior to a pre-set target time *T*. Both models are based on a heuristic that combines branch-and-cut and Monte-Carlo simulation which, if run to completion, terminates with a solution value within a preset percentage of the optimum. Using small instances (9-nodes and 28-nodes) Kenyon and Morton showed that using their models' solutions to VRPSTT can be significantly better than solutions obtained by solving the associated mean-value model.

## 2.5. Multi-Objective Vehicle routing

VRPs are frequently used to model real cases. However, they are often set up with the single objective of minimizing the cost of the solution, despite the fact that the majority of the problems encountered in industry, particularly in logistics, are multi-objective in nature. For instance, in real life there may be several costs associated with a single tour. In these cases, it is possible to transform them into a single objective, using a common factor, such as cost value. However, objectives may not always be limited to cost. In fact, numerous other aspects, such as balancing of workloads (time, distance ...), can be taken into account simply by adding new objectives (Jozefowiez et al., 2008).

Multi-objective routing problems are used mainly in three ways: (1) to extend classic academic problems in order to improve their practical application, (2) to generalize classic problems, and (3) to study real-life cases in which the objectives have been clearly identified by the decision-maker and are dedicated to a specific real-life problem or application.

### 2.5.1. Extending Classic Academic Problems

Multi-objective optimization is one possible way to study other objectives other than the one initially defined, which is often related to solution cost. In this context, the problem definition remains unchanged, and new objectives are added. The purpose of such

extensions is often to enhance the practical applications of the model by recognizing that logistics problems are not only cost driven. As an example of such an objective, we can consider the following: (1) *Driver workload* – an extension to VRP in which the balance of tour lengths is considered. This balance objective was added to increase the fairness of the solution (Jozefowiez, Semet & Talbi, 2002; Jozefowiez N., Semet F. & Talbi E. G., 2006b; Lee & Ueng, 1998). (2) *Customer Satisfaction* – an objective added to VRP with time windows in order to improve customer satisfaction with regard to delivery dates (Sessomboon et al., 1998). (3) *Commercial Distribution* – an extension of the periodic VRP that takes into account diverse objectives, including cost, balancing, and marketing issues (Ribeiro, Louren o & Fargas, 2001).

Some multi-objective routing problems do not share common objectives with classic routing problems at all. For example, Jozefowiez, Semet and Talbi (2009) proposed a meta-heuristic method based on an evolutionary algorithm for solving a bi-objective vehicle routing problem in which the total length of routes is minimized as well as the balance of routes, i.e. the difference between the maximal route length and the minimal route length.

Chitty and Hernandez (2004) define a dynamic VRP in which the total mean transit time and the total variance in transit time are minimized simultaneously. Likewise, Murata and Itai (2005, 2007) define a bi-objective VRP which seeks to minimize both the number of vehicles and the maximum routing time of those vehicles (makespan).

### 2.5.2. Generalizing Classic Problems

Another way to use multi-objective optimization is to generalize a problem by adding objectives instead of one or several constraints and/or parameters. In the literature, this strategy has notably been applied to VRP with time windows constraints, where the time windows are replaced by one or several objectives (Baran & Schaerer, 2003; Gendreau & Laporte, 1997; Hong & Park, 1999; Ombuki, Ross & Hanshar, 2006; Rahoual, Kitoun, Mabed, Bachelet & Benameur, 2001; Tan K. C., Chew & Lee, 2006a).

Feillet, Dejax and Gendreau (2005) have described a class of problems, called traveling salesman problems with profits (TSPP), which belong to this category. In these problems, a profit, associated with each customer, can be collected when the customer is visited, but

it is not necessary to visit all customers. The two conflicting objectives in such problems are: (1) Maximize the profit by visiting the maximum number of customers, thus increasing the length of the solution. (2) Minimize the length of the solution by visiting fewer customers, thus decreasing the profit generated by the solution. An attempt to address the traveling salesman problem with profits in its explicitly multi-objective form was made by Keller C.P. (1985), and later by Keller C. P. and Goodchild (1988), who referred to the problem as the multi-objective vending problem.

Another example of routing problem generalization is the bi-objective covering tour problem (CTP) (Jozefowiez, Semet & Talbi, 2007a), which generalizes the covering tour problem (Gendreau & Laporte, 1997). In the CTP, the goal is to find a tour on a network subset, such that certain nodes are a given distance *c* from visited nodes. In the bi-objective generalization, the parameter *c* is removed and replaced by an objective to optimize the cover. The cover of the solutions is then computed according to the visited nodes.

The traveling purchaser problem consists of determining a route through a subset of markets in order to collect a set of products, while simultaneously minimizing the traveling distance and the purchasing cost. This problem is usually solved as a single-objective problem in which a single composite function is obtained by adding the traveling distance and the purchasing cost. Riera-Ledesma and Salazar-Gonzalez (2005) formulated the problem as a bi-objective mixed-integer linear program.

### 2.5.3. Studying Real-Life Cases

Multi-objective routing problems are also studied for a specific real-life situation, in which decision makers define several clear objectives that they would like to see optimized. As an example we can consider the two following real-life situations.

Bowerman, Hall and Calamai (1995) looked at schoolbus route planning for urban areas, which is more complex than the classic VRP. The problem is to find a collection of schoolbus routes that will ensure fair distribution of services to all eligible students, who are located in different areas. The authors proposed a multi-objective mathematical model with four objectives: (1) the minimization of the total route length, (2) the minimization of the total student walking distance, (3) the fair distribution of the load, and (4) the fair division between the buses of the total distance traveled.

Similarly, Gupta, Singh and Pandey (2010) presented a case study with the overall goal of developing a plan for the Jain University bus service to be able to serve all customers in the most efficient way. In this study four objectives were considered: (1) the minimization of the total route length, (2) the minimization of the fleet size, (3) the maximization of average grade of customer satisfaction, and (4) the minimization of total waiting time over vehicles.

In Lacomme, Prins and Sevaux (2006), trash had to be collected in the streets of the town of Troyes (France) and delivered to a waste treatment facility. This problem was modeled as an arc-routing problem. The trucks left the factory at 6 a.m. and had to return to the factory before a given hour since the workers had to sort the waste afterwards. The authors considered two objectives: (1) the minimization of the total route length and (2) the minimization of the longest route.

Motivated by the case of Lantmannen, a large distributor operating in Sweden, Wen, Cordeau, Laporte and Larsen (2010) proposed a model to solve the dynamic multi-period vehicle routing problem (DMPVRP). In the DMPVRP, customers place orders dynamically over a planning horizon consisting of several periods. Each request specifies a demand quantity, a delivery location and a set of consecutive periods during which delivery can take place. The distributor must plan its delivery routes over several days so as to (1) minimize the total travel time and (2) customer waiting, and (3) to balance the daily workload over the planning horizon.

Faccio, Persona and Zanin (2011) studied the problem of municipality solid waste collection optimization considering real time input data, homogeneous and variable fleet size based in a single depot. In the study three objective functions were addressed: (1) the minimization of the number of vehicles, (2) the minimization of travel time and (3) the minimization of total distance covered.

Anbuudayasankar, Ganesh, Lenny Koh and Ducq (2011) addressed the bi-objective vehicle routing problems with forced backhauls, in which the optimization of the process of replenishing money in ATMs is considered as a bi-objective problem which minimizes the total routing cost and the span of travel tour.

## 2.5.4. Most Common Objectives

The different objectives studied in the literature can be presented and classified according to the component of the problem with which they are associated. The following is a summary of the most common objectives. (1) **Objectives related to the tour**: (a) **Cost**: Minimizing the cost of the solutions generated is the most common objective. Generally speaking, minimizing cost is linked to an economic criterion; however, other motivations are possible. For instance, in studies by Park and Koelling (1986; 1989), the distance traveled must be minimized to avoid damaging the product being transported. (b) **Makespan**: Corberan et al. (2002) and Pacheco and Marti (2005) presented a VRP in which the makespan is minimized (i.e., to minimize the length of the longest tour). This choice was motivated by the environment: a rural area in Spain, where due to the large distances between pick-up locations, the bus routes tend to be long and the bus never full. Minimizing the makespan ensures some fairness in terms of time spent on the bus by the first student picked up, compared to the time spent by the last one. Minimizing the makespan was also an objective for Lacomme et al. (2006), because the trash collection had to be finished as soon as possible so that the workers would have time to sort the trash. (b) **Balance**: Some objectives are designed to even out disparities between the tours. Such objectives are often introduced in order to bring an element of fairness into play. Lee and Ueng (1998) incorporated balance to enhance fairness between drivers' assignments. Balance was also an issue for Ribeiro et al. (2001). In their study, the tour workload was equal to the volume transported during the period. (2) **Objectives related to node/arc activity**: Most of the studies dealing with objectives related to node/arc activity involve time windows (Augerat et al., 1998; Cordeau, Laporte, et al., 2005; Deb, 2001; Gendreau et al., 1994; Knowles J. D. & Corne, 2000). In such studies, the time windows are replaced by an objective that minimizes either the number of violated constraints (Rahoual et al., 2001), the total customer and/or driver's wait time due to earliness or lateness (Baran & Schaerer, 2003; El-Sherbeny, 2001; Hong & Park, 1999), or both objectives at the same time (Geiger, 2001, 2008). (3) **Objectives related to resources**: The main resources encountered in the literature are vehicles and goods. One objective that often appears is the minimization of the number of vehicles. For VRP with time windows, the classic model has two objectives that are treated lexicographically

(mining, in order of their importance). First, the number of vehicles is minimized, and then the length of the solution is minimized for that given number of vehicles. The existing research on multi-objective VRPs with time windows assigns the same level of priority for both objectives, rather than considering them lexicographically. Other vehicle-related objectives can be used to maximize vehicle cost-effectiveness in terms of time (El-Sherbeny, 2001; Sessomboon et al., 1998) or capacity (Sutcliffe & Board, 1990), while goods-related objectives can be introduced to take the nature of the goods into account (merchandise is perishable and we want to avoid its deterioration (Park Y. & Koelling, 1986; Park Y. B. & Koelling, 1989)).

| *Authors* | *Problem* | *Objectives* |
|---|---|---|
| Current and Schilling (1994) | Median tour problem, Maximal covering tour problem | (1) Min. the total length; (2) Max. the accessibility of the nodes not included on the tour |
| Doerner, Focke and Gutjahr (2007) | Mobile healthcare facility tour planning | (1) Min. the ineffectiveness of the personnel; (2) Min. the average distance for an inhabitant to walk; (3) Max. the size of the population covered |
| Lee and Ueng (1998) | VRP | (1) Min. the traveled distance; (2) Optimize the balance of the load (length) |
| Park Y. and Koelling (1986); Park Y. B. and Koelling (1989) | VRP | (1) Min. the travel distance; (2) Max. the total fulfillment of emergent services and conditions; (3) Min. the total deterioration of goods |
| Sessomboon et al. (1998) | VRPTW | (1) Min. the traveled distance; (2) Max. the customer satisfaction; (3) Min. the number of vehicles; (4) Min. the vehicle waiting times |
| Sutcliffe and Board (1990) | VRP | (1) Min. the traveled distance; (2) Min. travel and boarding time; (3) Max. the equalization of the vehicle travel times; (4) Max. the equalization of the number of unused places in each vehicle; (5) Max. the equalization of the use of the ambulance to carry trainees in wheelchairs. |
| Hansen M. P. (2000) | Multi-Objective TSP | (1) Min. the total costs (each objective is associated with a different cost matrix) |
| Ribeiro et al. (2001) | Multi-period VRP | (1) Min. the traveled distance; (2) Optimize the balance (number of visited customers); (3) Marketing: driver/customer relationship |
| (Jozefowiez et al., 2002, 2006b; Jozefowiez, Semet & Talbi, 2007b; Jozefowiez et al., 2009) | VRP with tour balance | (1) Min. the traveled distance; (2) Optimize the balance of the tours (length) |

| *Authors* | *Problem* | *Objectives* |
|---|---|---|
| Borges and Hansen (2002) | Multi-objective TSP | (1) Min. the total costs |
| Paquete and Stutzle (2003) | Multi-objective TSP | (1) Min. the total costs |
| Zhenyu, L., Lishan and Guangming (2003) | Multi-objective TSP | (1) Min. the total costs |
| Angel, Bampis and Gourv's (2003) | Multi-objective TSP | (1) Min. the total costs |
| Chitty and Hernandez (2004) | Dynamic VRP | (1) Min. the total mean transit time; (2) Min. the total variance in transit time |
| Li W. (2005) | Bi-objective TSP | (1) Min. the total costs |
| Murata and Itai (2005, 2007) | Multi-objective VRP | (1) Optimize makespan; (2) Min. the number of vehicles |
| Keller C.P. (1985), Keller C. P. and Goodchild (1988) | TSP with profit | (1) Min. the tour length; (2) Max. the profit |
| Hong and Park (1999) | VRPTW | (1) Min. the total travel time; (2) Min. the total customer waiting times |
| Geiger (2001) | VRPTW | (1) Min. the total distance; (2) Min. the time window violation; (3) Min. number of violated time windows; (4) Min. the number of vehicles |
| Rahoual et al. (2001) | VRPTW | (1) Min. the traveled distance; (2) Min. the number of violated constraints; (3) Min. the number of vehicles |
| Baran and Schaerer (2003) | VRPTW | (1) Min. the total time; (2) Min. the total delivery times; (3) Min. the number of vehicles |
| Tan K. C., Chew and Lee (2006b) | VRPTW | (1) Min. the total length; (2) Min. the number of vehicles |
| Jozefowiez, Semet and Talbi (2004); Jozefowiez et al. (2007a) | Covering tour problem | (1) Min. the length; (2) Min. the cover |
| Riera-Ledesma and Salazar-Gonzalez (2005) | Traveling purchaser problem | (1) Min. the length; (2) Min. the purchasing cost |
| Ombuki et al. (2006) | VRPTW | (1) Min. the total length; (2) Min. the number of vehicles |
| Bowerman et al. (1995) | Urban school bus routing | (1) Min. the total length; (2) Min. the load balance; (3) Min. the length balance; (4) Min. the student walking distance |
| Giannikos (1998) | Location and routing for hazardous waste transportation and treatment | (1) Min. of the total cost; (2) Min. of the total perceived risk; (3) Equitable distribution of risk among population centers; (4) Equitable distribution of the disutility caused by the operation of the treatment facilities |
| El-Sherbeny (2001) | VRP adapted to the case of a Belgian Transportation company | (1) Min. the total time; (2) Optimize the balance (length); (3) Max. the flexibility; (4) Min. the waiting times; (5) Min. the number |

| *Authors* | *Problem* | *Objectives* |
|---|---|---|
| | | of trucks; (6) Min. the number of covered trucks; (7) Min. the number of uncovered trucks; (8) Min. the unused working hours |
| Corberan et al. (2002) | Rural school bus routing problem | (1) Min. the makespan; (2) Min. the number of vehicles |
| Lacomme et al. (2006) | Capacitated arc routing problem | (1) Min. the total length; (2) Min. the makespan |
| Pacheco and Marti (2005) | Rural school bus routing problem | (1) Min. the makespan; (2) Min. the number of vehicles |
| Zografos and Androutsopoulos (2004) | VRPTW for hazardous product transportation | (1) Min. the travel time; (2) Min. the transportation risk |
| Tan K. C. et al. (2006a) | Truck and trailer VRP | (1) Min. the total length; (2) Min. the number of vehicles |
| Mourgaya (2004) | Multi-period VRP | (1) Optimize the regionalization (clustering) of the customers; (2) Optimize the balance of the load |
| Gupta et al. (2010) | Urban school bus routing | (1) Min. the total length; (2) Min. fleet size; (3) Max. average customer satisfaction; (4) Min. total waiting time |
| Wen et al. (2010) | Dynamic multi-period VRP | (1) Min. total travel time; (2) Min. customers waiting time; (3) Max. the balance of the daily workload over the planning horizon. |
| Faccio et al. (2011) | Municipality solid waste collection | (1) Min. number of vehicles; (2) Min. total travel time; (3) Min. total travel length |
| Anbuudayasankar et al. (2011) | Bi-objective VRP with forced backhauls | (1) Min. total routing cost; (2) Min. maxspan |

**Table 2.1 – Summary of recent multi-objective VRP and related problems**

| *Objective* | *Authors* |
|---|---|
| Min. the total costs (each objective is associated with a different cost matrix) | Hansen M. P. (2000), Borges and Hansen (2002), Paquete and Stutzle (2003), Zhenyu et al. (2003), Angel et al. (2003), Li W. (2005), Keller C.P. (1985), Keller C. P. and Goodchild (1988), Riera-Ledesma and Salazar-Gonzalez (2005) |
| Max. The profit | Keller C.P. (1985), Keller C. P. and Goodchild (1988) |
| Min. The purchasing cost | Riera-Ledesma and Salazar-Gonzalez (2005) |

**Table 2.2 – Summary of objectives found in recent multi-objective TSP**

| *Objective* | *Authors* |
|---|---|
| Marketing: driver/customer relationship | Ribeiro et al. (2001) |
| Max. the equalization of the use of the ambulance to carry trainees in wheelchairs. | Sutcliffe and Board (1990) |

| *Objective* | *Authors* |
|---|---|
| Max. The customer satisfaction | Sessomboon et al. (1998); Gupta et al. (2010) |
| Max. the equalization of the number of unused places in each vehicle | Sutcliffe and Board (1990) |
| Max. the equalization of the vehicle travel times | Sutcliffe and Board (1990) |
| Max. The flexibility | El-Sherbeny (2001) |
| Max. the total fulfillment of emergent services and conditional | Park Y. and Koelling (1986); Park Y. B. and Koelling (1989) |
| Min. The number of covered trucks | El-Sherbeny (2001) |
| Min. The number of uncovered trucks | El-Sherbeny (2001) |
| Min. The number of vehicles | Sessomboon et al. (1998); Murata and Itai (2005, 2007); Geiger (2001); Rahoual et al. (2001); Baran and Schaerer (2003); El-Sherbeny (2001); Tan K. C. et al. (2006a); Ombuki et al. (2006); Tan K. C. et al. (2006b); Gupta et al. (2010); Faccio et al. (2011) |
| Min. the number of violated constraints/time windows | Rahoual et al. (2001); Geiger (2001) |
| Min. The total deterioration of goods | Park Y. and Koelling (1986); Park Y. B. and Koelling (1989) |
| Min. (customers) waiting times | Hong and Park (1999); El-Sherbeny (2001); Sessomboon et al. (1998); Gupta et al. (2010); Wen et al. (2010) |
| Min. The total delivery times | Baran and Schaerer (2003) |
| Min. The total length | Tan K. C. et al. (2006b); Ombuki et al. (2006); Tan K. C. et al. (2006a); Lee and Ueng (1998); Park Y. and Koelling (1986); Park Y. B. and Koelling (1989); Sessomboon et al. (1998); Sutcliffe and Board (1990); Ribeiro et al. (2001);(Jozefowiez et al., 2002, 2006b, 2007b, 2009); Geiger (2001); Rahoual et al. (2001); Gupta et al. (2010); Faccio et al. (2011) |
| Min. The total mean transit time | Chitty and Hernandez (2004) |
| Min. The total travel time | Hong and Park (1999); Baran and Schaerer (2003); El-Sherbeny (2001); Zografos and Androutsopoulos (2004); Wen et al. (2010); Faccio et al. (2011) |
| Min. The total variance in transit time | Chitty and Hernandez (2004) |
| Min. The transportation risk | Zografos and Androutsopoulos (2004) |
| Min. the unused working hours | El-Sherbeny (2001) |
| Min. travel and boarding time | Sutcliffe and Board (1990) |
| Optimize makespan | Murata and Itai (2005, 2007); Anbuudayasankar et al. (2011) |
| Optimize the balance (number of visited customers) | Ribeiro et al. (2001) |
| Optimize the balance of the load (length) | Lee and Ueng (1998); Mourgaya (2004) |

| *Objective* | *Authors* |
|---|---|
| Optimize the balance of the tours (length) | Jozefowiez et al. (2002, 2006b); Jozefowiez, Semet and Talbi (2007c); Jozefowiez et al. (2009); El-Sherbeny (2001); Wen et al. (2010) |
| Optimize the regionalization (clustering) of the customers | Mourgaya (2004) |
| Min. total route costs | Anbuudayasankar et al. (2011) |

**Table 2.3 – Summary of objectives found in recent multi-objective VRP**

## 2.5.5. Multi-Objective Optimization Algorithms

Over the last several years, many techniques have been proposed for solving multi-objective problems. These strategies can be divided into three general categories: (1) scalar methods, (2) Pareto methods, and (3) methods that belong to neither the first nor the second category.

The most popular scalar method is weighted linear aggregation. However, this method has several disadvantages. Nevertheless, this technique is relatively simple to implement and can be used with any of the single-objective heuristics or meta-heuristics described in the literature. For multi-objective routing problems, weighted linear aggregation has been used with specific heuristics (Blasum & Hochstattler, 2000; Haghani & Jung, 2005; Murata & Itai, 2005), local search algorithms (Paquete & Stutzle, 2003; Ribeiro et al., 2001), and genetic algorithms (Ombuki et al., 2006).

Being a population-based approach, GAs are well suited to solve multi-objective optimization problems. A generic single-objective GA can be modified to find a set of multiple non-dominated solutions in a single run. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems with non-convex, discontinuous, and multi-modal solutions spaces. The crossover operator of GA may exploit structures of good solutions with respect to different objectives to create new non-dominated solutions in unexplored parts of the Pareto front. In addition, most multi-objective GAs do not require the user to prioritize, scale, or weigh objectives. Therefore, GAs have been the most popular heuristic approach to multi-objective design and optimization problems. Jones, Mirrazavi and Tamiz (2002) reported that 90% of the approaches to multi-objective optimization aimed to approximate the true Pareto front for the underlying problem. A majority of

these used a meta-heuristic technique, and 70% of all meta-heuristics approaches were based on evolutionary approaches.

Pareto methods use the notion of Pareto dominance directly. This approach was mainly introduced by Goldberg (1989) for genetic algorithms. Though it does not allow one compromise to be favored over another, it can be a useful aid for the decision-makers. In multi-objective vehicle routing problems, the Pareto concept is frequently used within an evolutionary framework. Many authors (Doerner et al., 2007; Geiger, 2001; Jozefowiez et al., 2002, 2004; Jozefowiez N., Semet F. & Talbi E .G., 2006a; Jozefowiez et al., 2007a; Lacomme et al., 2006; Murata & Itai, 2005, 2007; Ombuki et al., 2006; Rahoual et al., 2001; Sessomboon et al., 1998; Tan K., Lee, Chew & Lee, 2003; Tan K. C. et al., 2006b; Zhenyu et al., 2003) have used evolutionary algorithms with Pareto methods to solve multi-objective routing problems. Some of them have proposed hybrids based on evolutionary algorithms and local searches, heuristics, and/or exact methods for the considered problem (Doerner et al., 2007; Jozefowiez et al., 2002; Jozefowiez et al., 2006a; Jozefowiez et al., 2007a, 2007b; Lacomme et al., 2006; Sessomboon et al., 1998; Tan K. et al., 2003; Tan K. C. et al., 2006a, 2006b).

Some studies do not employ either scalar or Pareto methods to solve multi-objective routing problems. In this case, these non-scalar and non-Pareto methods are based on genetic algorithms, lexicographic strategies, ant colony mechanisms, or specific heuristics. Doerner et al. (2007) proposed using VEGA (Vector Evaluated Genetic Algorithm) to solve their problem. In VEGA, at each iteration, the population is divided into $n$ subpopulations, where $n$ is the number of objectives, which are mixed together to obtain a smaller population to which genetic operators are applied. In lexicographic methods, the objectives are each assigned a priority value, and the problems are solved in order of decreasing priority. When one objective has been optimized, its value cannot be changed and it becomes a new constraint for the problem. Such a lexicographic approach has been used by Keller C.P. (1985), Keller C. P. and Goodchild (1988) and Current and Schilling (1994). Baran and Schaerer (2003) do not use a standard multi-objective approach, but rather consider the multi-objective nature of the problem via mechanisms in the ant colony system they propose. Chitty and Hernandez (2004) also use an ant colony system. The ant colony paradigm is adapted to the bi-objective situation by using two types of pheromones, one for the total mean transit time and one for the variance in

the transit time. Alternatively, to solve their problem, Doerner et al. (2007) treat the location and routing aspects simultaneously by means of P-ACO (Pareto Ant Colony Optimization). P-ACO is a multi-objective meta-heuristic that generalizes the Ant Colony Optimization (ACO) meta-heuristic to the case of several objective functions, determining approximations to the set of optimal Pareto solutions.

## 2.6. Real-Time Vehicle routing

Due to recent advances in information and communication technologies, vehicle fleets can now be managed in real-time. When jointly used, devices like geographic information systems (GIS), global positioning systems (GPS), traffic flow sensors and cellular telephones are able to provide relevant real-time data, such as current vehicle locations, new customer requests and periodic estimates of road travel times (Brotcorne, Laporte & Semet, 2003). If suitably processed, this large amount of data can, in principle, be used to reduce cost and improve service level. To this end, revised routes have to be timely generated as soon as new events occur.

In recent years, three main developments have contributed to the acceleration and quality of algorithms relevant in a real-time context: (1) Increase in computing power due to better hardware. (2) Development of powerful meta-heuristics whose main impact has been on solution accuracy, even if this gain has sometimes been achieved at the expense of computing time. (3) Development has arisen in the field of parallel computing. The combination of these three features has yielded a new generation of powerful algorithms that can effectively be used to provide real-time solutions in dynamic contexts (Ghiani et al., 2003).

A real-time VRP, also known as dynamic VRP, due to the nature of information needed to come up with a set of good vehicle routes and schedules, which is dynamically revealed to the decision maker, can be either deterministic or stochastic (Powell W. B., Jaillet & Odoni, 1995). In deterministic-dynamic problems, all data are known in advance and some elements of information depend on time. For instance, the VRP with time windows reviewed in (Cordeau et al., 2001) belongs to this class of problems. In stochastic-dynamic problems uncertain data are represented by stochastic processes. For instance, user requests can behave as a Poisson process (as in (Bertsimas D. & Van Ryzin, 1990)). Since uncertain data are gradually revealed during the operational interval,

routes are not constructed beforehand. Instead, user requests are dispatched to vehicles in an on-going fashion as new data arrive (Psaraftis, 1988).

The events that lead to a plan modification can be (1) arrival of new user requests (or cancellations), (2) arrival of a vehicle at a destination and (3) update of travel times. Every event must be processed according to the policies set by the vehicle fleet operator. When a new request is received, one must decide whether it can be serviced immediately, delayed or rejected. If the request is accepted, it is assigned to a position in a vehicle route. If another event occurs, the request might be assigned to a different position in the same vehicle route, or even dispatched to a different vehicle. At any time each driver needs to know his next stop. Hence, when a vehicle reaches a destination it has to be assigned a new destination. Due to advances in communication technologies, route diversions and reassignments are now a feasible option and should take place if this results in a cost saving or in an improved service level (Gendreau & Potvin, 1998; Ichoua, Gendreau, Potvin, op?rationnelle & Universit? de, 2000). Finally, if an improved estimation of vehicle travel times is available, it may be useful to modify the current routes or even the decision of accepting a request or not.

Most solution approaches to the VRP are in practice implemented in a centralized computer resource, producing a daily plan to be provided to the vehicles before the beginning of the distribution execution. Some of these approaches have been implemented in commercial systems successfully used by numerous transportation, logistics and manufacturing companies over the last 20 years.

These systems have not, however, been designed to address the case in which the execution of delivery cannot follow the plan as prescribed, due to some unforeseen event. When there is need for real-time intervention, it may be necessary to re-compute the plan using new input data. If a typical VRP approach is used for re-planning (i.e. re-planning the whole schedule from scratch), many vehicle schedules may be affected, thus causing significant performance inefficiencies (high overhead, nervousness, errors, and high costs).

Thus, re-planning based on classical VRP solution methods may not be a realistic option. In the absence of algorithms capable of “isolating” the part of the VRP affected by the unexpected event in order to minimize the disturbance to the overall schedule, interventions are typically performed manually (for example, through voice

communication between drivers and the logistics manager) and the quality of decisions taken is naturally affected.

As pointed out by Psaraftis (1988) and Psaraftis (1995), real-time VRPs possess a number of unique features, some of which have just been described.; The remaining characteristics are: (1) *Quick response* - Real-time routing and dispatching algorithms must provide a quick response so that route modifications can be transmitted timely to the fleet. To this end, two approaches can be used, simple policies (like the first-come first served (FCFS) policy (Bertsimas D. & Van Ryzin, 1990)), or more involved algorithms running on parallel hardware (like the tabu search heuristics described in (Gendreau, Guertin, Potvin & S?guin, 2006; Gendreau, Guertin, Potvin & Taillard, 1999)). The choice between them depends mainly on the objective, the degree of dynamism and the demand rate. For each real-time problem, it is important to specify the time horizon (also know as planning horizon) – a fixed point in time at which certain processes will be evaluated - in order to determine how long to wait before taking action; in other words, we have to define the term "quick response." (2) *Denied or deferred service* - In some applications it is valid to deny service to some users, or to forward them to a competitor, in order to avoid excessive delays or unacceptable costs. For instance, in Gendreau et al. (1999) requests that cannot be serviced within a given time window are rejected. When no time windows are imposed, some user requests can be postponed indefinitely because of their unfavorable location. This phenomenon can be avoided by imposing dummy time windows, or by adding a non-linear delay penalty to the objective function. (3) *Congestion* - If the demand rate exceeds a given threshold, the system becomes saturated, i.e., the expected waiting time of a request grows to infinity.

## 2.7. Summary

The Vehicle-Routing Problem (VRP) is a common name for problems involving the construction of a set of routes for a fleet of vehicles. The vehicles start their routes at a depot, call at customers, to whom they deliver goods, and return to the depot. The objective function for the vehicle-routing problem is to minimize delivery cost by finding optimal routes, which are usually the shortest delivery routes.

The basic VRP consists of designing a set of delivery or collection routes, such that (1) each route starts and ends at the depot, (2) each customer is called at exactly once and by

only one vehicle, (3) the total demand on each route does not exceed the capacity of a single vehicle, and (4) the total routing distance is minimized. It is common to address the basic VRP as Capacitated Vehicle-Routing Problem (CVRP).

VRP has been solved optimally using Branch-and-Bound algorithms, Set-Covering and Column Generation algorithms, Branch-and-Cut algorithms, Dynamic algorithms and more.

Since VRP is an NP-Hard problem, many heuristics have been developed for solving it. The classic algorithms include, among others, the Savings algorithms, Swap algorithm and the Fisher and Jaikumar algorithm. Meta-heuristics algorithms, such as Simulated Annealing, Tabu Search, Genetic Algorithms, Ant Systems Algorithms and Neural Networks are also used in solving VRPs.

As research developed a number of researchers developed extensions to the basic VRP. The goal was to develop more realistic models, to adapt to the larger number of constraints of the real world. The following is a description of the most common variants. Such extensions include the Split Delivery Vehicle Routing Problems (a customer can be served by more than one vehicle), Vehicle Routing Problems with Time Windows (a customer must be served within a given time window), Multi-Depot Vehicle Routing Problems, Time Dependent Vehicle Routing Problems (traveling time may change during the day), Stochastic Vehicle Routing Problems, Multi-Objective Vehicle Routing problems and Real-Time Vehicle Routing Problems.

# 3. Problem Formulation

This chapter describes the formulation of the real-time multi-objective vehicle routing problem stated in chapter 1. The problem is formulated as a mixed integer linear programming problem on a network. This chapter is divided into four sections. The first section explains the assumptions and limitations of the model. The second section deals with the notations and variables used in the model formulation. The third section introduces the objective function. The fourth section deals with the constraints of the formulation. We summarize this chapter in the last section.

## 3.1. Assumptions and Limitations

### 3.1.1. Demand Characteristics

The problem formulated in this research considers a system with dynamic conditions. These conditions include the real-time variation in travel times between the depot and the customers as well as between the customers themselves and real-time service requests. Since there is no end of service time constraint, demands can be requested at any time during the day.

When rerouting during the day, demands can be divided into two groups, recent demands and longstanding demands. The demands already assigned to the vehicles in the last routing process are the longstanding demands, and demands that are requested after the last routing process are the recent demands. All demands, longstanding and recent, have to be served. New demands can be assigned to any of the vehicles without any restriction, as long as the remaining capacity of the vehicles allows it; otherwise, a new vehicle has to be delivered.

All demands have specified service times and service time intervals (generally time windows are described with intervals such as [a, b]). We consider soft time windows for service around the desired service time because soft time windows are more realistic and more flexible than hard time windows. A soft time windows formulation can have solutions in cases where a hard time windows formulation fails.

Each customer is assigned with a satisfaction or penalty function. These functions describe customers' satisfaction/dissatisfaction when customers' demands are not fulfilled on time as a function of the deviation from the customer's time window.

### 3.1.2. System Characteristics

- Network: In this study a transportation network is considered. While there may exist more than one trajectory between two nodes (customers and the depot) with shortest traveling times, each for a different time of the day, this case is not considered (however, this should not be a problem if a large number of customers are considered). For every two nodes the shortest path is calculated. Based on distance between every two nodes and information about traveling time, the average driving speed between every two nodes can be calculated. Time-dependent travel time information is supplied in the form of a matrix of travel times between the demand nodes.
- Travel Time: Travel times are lognormally distributed. Using lognormal distribution is more suitable than normally distributed because (1) the positive skew shape (i.e., right skewed) is more suitable for travel time description; that is, a higher probability exists for long travel time than for short travel time, and (2) the range [0, ∞) of the distribution is more natural than a truncated normal distribution (the probability distribution of a normally distributed random variable whose value is either bounded below, above or both), because negative travel times are impossible.

  According to Law and Kelton (1991), let $x \sim Log\mathcal{N}(\mu_x, \sigma_x)$, and $y \sim \mathcal{N}(\mu_y, \sigma_y)$, such that the following relations exist:

$$y = \ln(x) \tag{3.1}$$

$$f(x) = \frac{1}{x\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\sigma_y}\right)^2} \tag{3.2}$$

$$\mu_x = e^{\mu_y + \frac{1}{2}\sigma_y} \tag{3.3}$$

$$\sigma_x^2 = e^{2\mu_y + \sigma_y^2}\left(e^{\sigma_y^2} - 1\right) \tag{3.4}$$

$$\mu_y = \ln\left(\frac{\mu_x^2}{\sqrt{\mu_x^2 + \sigma_x^2}}\right) \tag{3.5}$$

$$\sigma_y^2 = \ln\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right) \tag{3.6}$$

Equation (3.1) describes the relation between the two random variables, (3.2) describes the lognormal probability distribution function, and Equations (3.3), (3.4), (3.5), and (3.6) describe the conversion between the expectancies and variances.

In order to combine correlations within the travel time distribution, the multivariate lognormal distribution was used (Mood, Graybill & Boes, 1974). Let *X* be a multivariate lognormal distribution with the mean vector $\mu = (\mu_1, \mu_2, ..., \mu_d)$ and covariance matrix $\Sigma$ with $(i, j)$th entry $\sigma_{ij}$, so the correlation coefficients are $\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}$ and Equations (3.7) and (3.8) define the covariance and correlation coefficient.

$$\operatorname{cov}\left(x_i, x_j\right) = \left(e^{\sigma_{ij}} - 1\right)\exp\left(\mu_i + \mu_j + \frac{\sigma_{ii} + \sigma_{jj}}{2}\right) \tag{3.7}$$

$$\operatorname{co}r\left(x_i, x_j\right) = \left(\frac{e^{\sigma_{ij}} - 1}{\sqrt{\left(e^{\sigma_{ii}} - 1\right)\left(e^{\sigma_{jj}} - 1\right)}}\right) \tag{3.8}$$

If for two vehicles, the travel time distribution is defined as a bivariate lognormal distribution with μ = (μ, μ) and $\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}$, then, according to Mood et al. (1974), the conditional distribution of $x_2$ given $x_1 = x$ is lognormal with

$$\mu_2 = \mu_2 + \frac{\rho\sigma_2}{\sigma_1}\left(x - \mu_1\right) \tag{3.9}$$

and

$$\sigma_2^2 = \sigma_2^2\left(1 - \rho^2\right) \tag{3.10}$$

With one additional parameter (ρ), it is straightforward to estimate the parameter as a constant or as a function of the time gap between the vehicles.

- Vehicles: All vehicles in the proposed model have the same capacity.
- Information System: We assume that there is a real-time communication system between the vehicles and the control center. The control center has information about the location of all vehicles as well as real-time traveling time information. All vehicles are equipped with route guidance systems.
- Number of depots: The real-time multi-objective VRP in this dissertation is a single depot problem.

## 3.2. Variables and Problem Definition

The following is a list of variables used in the problem formulation presented next, the objective functions and the constraints.

$V$ Set of nodes, including the depot and the demand nodes

$E$ Set of edges

$N$ Number of customers (customer number 0 denotes the depot)

$d_i^t$ Demand of demand node *i* requested at time *t*.

$D_i^t$ The total demand of customer $i$ at time $t$. $D_i^t$ is defined as $\sum_{t=0}^{t_S} d_i^t - \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1} d_j^t \overline{x}_{ij}^{mt}$, which means that the demand of a customer equals the sum of all customers' demands received between time interval of $t=0$ to time $t=t_S$ minus the demands that already have been served. For customer 0, which is the depot, $d_i^{t_S}=0$ for all $t_S$.

$x_{ij}^{mt}$ A decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t \geq t_s$, and 0, otherwise.

$\overline{x}_{ij}^{mt}$ Known decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t < t_s$, and 0, otherwise.

$t_S$ Time of routing plan. The time of routing plan can start at $t_s=0$ and end at $t_S=T$.

$C_{ij}^t$ A stochastic time-dependent nonnegative cost function, which represents the

travel cost from vertex *i* to vertex *j* starting at time *t*.

$\overline{C}_{ij}^{\,t}$ Known cost for traveling from node *i* to node *j* at time *t*, where $t < t_s$

$M$ The maximum number of vehicles available

$Q$ capacity of a vehicle (all vehicles have the same capacity)

$t_S^m$ The last departure time of vehicle *m* from the depot. $t_S^m$ is defined as $t_S^m = \max t \in \{0,...,t_S\}$ which satisfies $\sum_{j=0}^{N} \overline{x}_{0j}^{mt} = 1$, and there is no $\hat{t} \in \{0,...,t_S\}$, such that $\hat{t} > t$ and $\sum_{j=0}^{N} \overline{x}_{j0}^{m\hat{t}} = 1$. If such $t_S^m$ does not exist, then $t_S^m = t_S$.

$v_i^{t_S}$ Does a customer require a visit at time $t_S$. $v_i^{t_S}$ is defined as $v_i^{t_S} = \begin{cases} 1 & d_i^{t_S} > 0 \\ 0 & otherwise \end{cases}$.

$TW_i^S$ Start of time window of customer *i*

$TW_i^E$ End of time window of customer *i*

$EET_i$ Endurable earliness time - the earliest service time that customer *i* can endure when a service starts earlier than $TW_i^S$.

$ELT_i$ Endurable lateness time - the latest service time that customer *i* can endure when a service starts later than $TW_i^E$.

$ST_i$ Service time at customer *i*.

$\overline{ST}_i$ Known service time at customer *i*.

$WT_i$ Waiting time at customer *i*.

$\overline{WT}_i$ Known waiting time at customer *i*.

Using the previously described variables, the real-time multi-objective VRP can be defined as follows.

At time $t_s$, let $G = (V, E)$ be a complete graph, where $V = \{0,1,...,n\}$ is the nodes set and $E$ is the edge set. Each node $i \in V \setminus \{0\}$ represents a customer (or a demand node), having a non-negative (0 or higher) demand $D_i^{t_S}$, whereas vertex 0 corresponds to the depot. Each edge $e \in E = \{(i, j) : i, j \in V, i \neq j\}$ is associated with a stochastic time-

dependent nonnegative cost, $C_{ij}^{t}$, which represents the travel cost (equal to the travel time) spent to go from vertex *i* to vertex *j* starting at time *t*. The use of the loop edges, (*i*,*i*), is not allowed (this is imposed by defining $c_{ii}^{t} = +\infty$ for all $i \in V$). A fixed fleet of *M* identical vehicles, each of capacity *Q*, is available at the depot.

The real-time multi-objective VRP calls for the determination of a set of at most *M* vehicles that optimizes the following objectives:

(1) Minimizing the total traveling time.

(2) Minimizing the number of vehicles used.

(3) Maximizing customers' satisfaction, or minimizing customers' dissatisfaction.

(4) Maximizing tour's balance

satisfying the following constraints:

(1) Each customer, whose demand at time $t_s$ equals 0 is not visited at all.

(2) Each customer, whose demand at time $t_s$ is higher than 0, is visited exactly once by one route.

(3) For every driving vehicle at time $t_s$, whose destination node is a customer, there must exist a corresponding route, which starts at the vehicle's destination node.

(4) For every vehicle whose current location, at time $t_s$, is a demand node, there must exist a corresponding route, which starts at the vehicle's current location.

(5) All other routes start at the depot.

(6) All routes end at the depot.

(7) The total demand of the customers, known at time $t_s$, served by a route does not exceed the vehicle capacity *Q*.

## 3.3. Objectives

Since this is a multi-objective problem, several objective functions are considered.

### 3.3.1. Minimizing the Travel Time

The first objective considered is minimizing the total travel time, and is described in equation (3.11).

$$\min \quad Z = \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + \overline{ST}_j + \overline{WT}_j\right]\bar{x}_{ij}^{mt} + \\ + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt} \tag{3.11}$$

The time passed since a vehicle left node *i* (either the depot or a customer) towards node *j* and the time it left node *j* can be calculated in the following way.

Consider the traveling cost (time) from node *i* to node *j* when leaving node *i* at time *t*. If by leaving node *i* at time *t* a vehicle reaches node *j* before its time window's start time (meaning $t + C_{ij}^{t} < TW_j^S$), then the vehicle has to wait until the beginning of the time window in order to start serving. Otherwise, it starts serving upon arrival. The time between the time the vehicle left node *i* towards node *j*, denoted as *t*, and the time it starts serving node *j* can be formulated as $\max\left(C_{ij}^{t}, TW_j^S - t\right)$. If node *j* is a customer, then both service time at customer *j*, $ST_j$, and waiting time at customer *j*, $WT_j$, have to be added to the traveling time. But, if node *j* is the depot, then both service time, $ST_j$, and waiting time, $WT_j$, equal 0. Therefore, the time passed since a vehicle left node *i* towards node *j* and the time it left node *j* can be defined as $\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j$.

For each edge $e \in E = \left\{(i, j) : i, j \in V, i \neq j\right\}$, there exists a decision variable $x_{ij}^{mt}$, defined as 1 if edge *e* was traveled at time *t* by vehicle *m*, otherwise it is 0. Multiplying the above notation, $\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j$, by the decision variable $x_{ij}^{mt}$, gives us the time passed since vehicle *m* left node *i* towards node *j* at time *t* and the time it left node *j*, if such vehicle exists, otherwise it is 0.

Let's refer to the time passed since vehicle *m* left node *i* towards node *j* at time *t* and the time it left node *j* multiplied by the decision variable, $\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt}$, as the true travel time from node *i* to node *j*. By summing all possible true travel times, $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{T}\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt}$, we get the total travel time, which is to be minimized.

The total travel time can be decomposed into two parts, the known travel time and the unknown travel time. If the planning time $t_s$ is not equal to 0, then we are not at the beginning of the day, and some vehicles have already been sent to customers. In this case, information regarding traveled edges, travel costs, service time and waiting time is already known for every edge traveled and for every customer visited before $t_s$.

Let $\bar{C}_{ij}^{t}$ denote the known cost from traveling from node *i* to node *j* at time *t*, where $t < t_s$. Similarly, let $\bar{x}_{ij}^{mt}$ denote the known decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t < t_s$, and 0, otherwise. The known traveled cost can be defined as $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + \overline{ST}_j + \overline{WT}_j\right]\bar{x}_{ij}^{mt}$.

The unknown traveling cost can be defined as $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt}$, therefore, the total traveling cost is the sum of the known traveling cost and the unknown traveling cost, $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + \overline{ST}_j + \overline{WT}_j\right]\bar{x}_{ij}^{mt} +$

$+\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt}$

### 3.3.2. Minimizing Number of Vehicles

Since in the real world, the fixed cost of using additional vehicles is higher then the routing operation costs, we can reduce the total cost by minimizing the number of vehicles in service.

$$\min \quad Z = \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t=t_s-1}\bar{x}_{0j}^{mt} + \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=t_s}^{T}x_{0j}^{mt} \tag{3.12}$$

The total number of vehicles can be defined as the number of vehicle leaving the depot.

### 3.3.3. Maximizing Customers' Satisfaction

In a traditional VRPTW, a feasible solution must satisfy all time windows. When a customer is served within his specified time window, the supplier's service level is

satisfactory; otherwise, it is not. Hence, a customer's satisfaction level (also the supplier's service level) can be described using a binary variable. The customer satisfaction level takes 1 if the service time falls within the specified time window, and 0 if it does not. The service level function of the customer can be described by Figure 3.1.

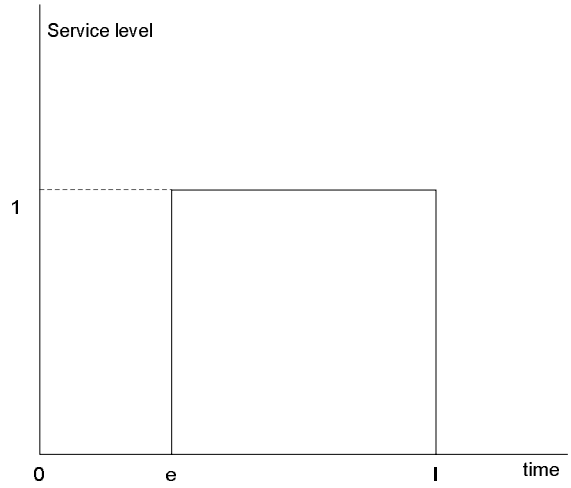



**Figure 3.1 – The service level function of a hard time window**

Time windows may sometimes be violated for economic and operational reasons. However, there exist certain bounds on the violation (earliness or lateness) that a customer can endure. The following two concepts are introduced to describe these bounds.

Let $EET_i$ denote endurable earliness time, the earliest service time that customer *i* can endure when a service starts earlier than $TW_i^S$, and let $ELT_i$ denote endurable lateness time, the latest service time that customer *i* can endure when a service starts later than $TW_i^E$.

An example is given to describe the relationship of $TW_i^S$, $TW_i^E$, $EET_i$ and $ELT_i$. A factory needs some kind of raw material for its daily production. Every day, the factory opens at 8:00 and production starts at 10:00. The raw material is shipped from an upstream supplier and the process of unloading the raw material requires 30 min. The factory specifies its preferred delivery time window to be [8:30, 9:00], because materials delivered within that time window can be directly moved to the workshop without any tardiness. However, the factory is not operating in a just-in-time mode; the delivery can

be a little earlier or later than the specified time window. A reasonable combination of EET and ELT could be [8:00, 9:30]. If the materials are delivered within [8:00, 8:30], then instead of being moved directly into the workshop, they must be moved into the warehouse to wait due to limited space in the workshop. This is of course not what the manager of the factory wants to see, but he/she can accept it. If the materials are delivered within [9:00, 9:30], no inventories have to be held; however, this demands that the execution of the production plan have higher accuracy, which reduces the robustness of the production operations in the factory. Since the factory opens at 8:00, deliveries before 8:00 must wait outside the factory; since the production procedure starts at 10:00, delivery after 9:30 is totally unacceptable because of the 30-min unloading process. Simply put, although the manager of the factory will be happiest to be served within [8:30, 9:00], he/she will also be reasonably satisfied if served within [8:00, 8:30] or [9:00, 9:30]; however, the consequence is that the customer's satisfaction will go down, and deliveries made before 8:00 or after 9:30 are not acceptable. Similar scenarios also appear in dial-a-ride problems.

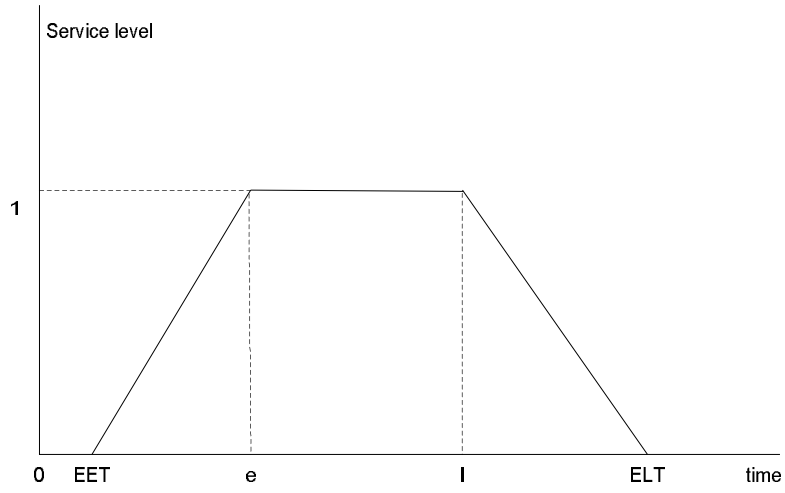



**Figure 3.2 – The service level function of fuzzy time windows**

As discussed above, the service may start outside the time window $\left[TW_i^S, TW_i^E\right]$, and the bounds of acceptable earliness and lateness are described by $EET_i$ and $ELT_i$, respectively. Obviously, the earliness and lateness are closely related to the quality of service of the supplier. The response of a customer satisfaction level to a given service

time may not be simply “good” or “bad”; instead, it may be between “good” and “bad”. For example, the customer might say, “it’s all right” to be served within $\left[EET_i, TW_i^S\right]$ or $\left[TW_i^E, ELT_i\right]$. In either case, the service level cannot be described by only two states (0 or 1).

For problems involving personal human feelings, fuzzy set theory is a strong tool. Intuitively, with the concepts of $EET_i$ and $ELT_i$, the supplier’s service level for each customer can be described by a fuzzy membership function:

$$S_i(t) = \begin{cases} 0, & t < EET_i \\ f_i(t), & EET_i \le t < TW_i^S \\ 1, & TW_i^S \le t < TW_i^E \\ g_i(t), & TW_i^E \le t < ELT_i \\ 0, & ELT_i \le t \end{cases} \tag{3.13}$$

when in most recent research, $f_i(t)$ is defined as

$$f_i(t) = \frac{t - EET_i}{TW_i^S - EET_i} \tag{3.14}$$

and $g_i(t)$ is defined as

$$g_i(t) = \frac{ELT_i - t}{ELT_i - TW_i^E} \tag{3.15}$$

However, since customer’s satisfaction level, as a function of the deviation from the customer’s time window, in most cases cannot be described as a linear function, the following function, which better describes customer's satisfaction, is used.

$$f_i(t) = \frac{\sum_{j=1}^{n} \beta_i^j \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^{\alpha_i}}{\sum_{j=1}^{n} \beta_i^j} \tag{3.16}$$

$$g_i(t) = \frac{\sum_{j=1}^{m} \delta_i^j \left( \frac{ELT - t}{ELT - TW_i^E} \right)^{\gamma_i}}{\sum_{j=1}^{m} \delta_i^j} \tag{3.17}$$

Assuming that each customer has his own satisfaction function, $S_i(t)$, and that the service provider assigns an *importance* factor, $\sigma_i$, to each customer that states how important it is to satisfy customer *i* compared to all other customers, the maximizing customers' satisfaction objective can be described as

$$\min \quad Z = \sum_{i=0}^{n} \sigma_i S_i \left( \sum_{j=0}^{N} \sum_{m=1}^{M} \left( \sum_{t=0}^{t_S - 1} \left( \left( t + \bar{C}_{ji}^t \right) \bar{x}_{ji}^{mt} \right) + \sum_{t=t_S}^{T} \left( \left( t + C_{ij}^t \right) x_{ji}^{mt} \right) \right) \right) \tag{3.18}$$

### 3.3.4. Maximizing the Balance of the Tours

This objective function seeks to balance the work between vehicles. The idea of making a multi-objective model that balances the work and minimizes cost has been explored in the work of Lee and Ueng (1998). Ribeiro et al. (2001) balanced the work by minimizing the difference between the work of each vehicle and the work of the vehicle with the lowest work level.

In this model, the work of a single vehicle is defined as the total length the vehicle traveled in a single day.

The standard deviation of the work of each vehicle at the end of the period is minimized. The model allows one vehicle to work more than another on a given route, as long as the total work of a vehicle at the end of the day ($t$=$T$) is balanced.

$$w_m = \sum_{j=0}^{N} \sum_{m=1}^{M} \left( \sum_{t=0}^{t_S - 1} \left( \bar{C}_{ji}^t \bar{x}_{ji}^{mt} \right) + \sum_{t=t_S}^{T} \left( C_{ij}^t x_{ji}^{mt} \right) \right) \tag{3.19}$$

$$\min\{StdDev\} = \min\ Z = \left\{ \sqrt{\frac{\sum_{m=1}^{M} w_m^2}{M} - \left( \frac{\sum_{m=1}^{M} w_m}{M} \right)^2} \right\}$$

### 3.3.5. Minimizing the Arrival Time of the Last Vehicle

The last objective considered in this study is minimizing the arrival time of the last vehicle. Each vehicle starts its route and ends its route at the depot. While the start time of each vehicle is known, the end time is unknown and is due to change, mainly because of the stochastic nature of the travel time.

By minimizing the arrival time of the last vehicle, we guarantee two things: (1) Maximum availability of vehicle for unscheduled deliveries and (2) that there are no too long routes.

$$\min\ Z = Max\left(t : x_{i0}^{mt} = 1 \quad \forall i \in N, m \in M, t \in T\right) \tag{3.20}$$

## 3.4. Constraints

### 3.4.1. Vehicle constraints

There are several constraints related to the vehicles. Every unused vehicle starts at the depot and returns to the depot. The used vehicles start from the nodes where the vehicles are located at the time of rerouting. Also vehicles are limited by capacity constraint and must return to the depot before the end of the day. These constraints are expressed in the following equations.

- Vehicle cannot drive from one node to itself. This constraint if defined by equation (3.21).

$$x_{ii}^{mt} = 0 \quad \forall i \in N,\ m \in M,\ t \in T \tag{3.21}$$

- All vehicles start their routes at the depot. This constraint is defined by equation (3.22).

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt}+\sum_{t=t_S}^{T}x_{0j}^{mt}\right)\le 1 \quad \forall m\in M \tag{3.22}$$

Each vehicle, *m*, is associated with a start time, $t_s^m$, equal to the latest time vehicle *m* left the depot. Mathematically, $t_S^m$ can be defined as $t_S^m=\max t\in\{0,...,t_S\}$ which satisfies $\sum_{j=0}^{N}\overline{x}_{0j}^{mt}=1$, and there is no $\hat{t}\in\{0,...,t_S\}$, such that $\hat{t}>t$ and $\sum_{j=0}^{N}\overline{x}_{j0}^{m\hat{t}}=1$. If such $t_S^m$ does not exist, then $t_S^m=t_S$.

Each vehicle *m* can belong to one of the following states: (1) it has never been assigned to any route, (2) it has been assigned to one route or more in the past and is currently waiting at the depot and (3) it is currently assigned to a route. If vehicle *m* has never been assigned to a route, and is therefore waiting at the depot, then $t_S^m=t_S$, and therefore $\sum_{j=1}^{N}\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt}=0$ and $\sum_{j=1}^{N}\sum_{t=t_S}^{T}x_{0j}^{mt}=0$ as well.

If vehicle *m* is assigned to a route, then there are two options. First, vehicle *m* left the depot before $t_S$ ($t_S>t_S^m$). In this case, $\sum_{j=1}^{N}\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt}=1$ and $\sum_{j=1}^{N}\sum_{t=t_S}^{T}x_{0j}^{mt}=0$. Second, vehicle *m* left the depot at $t_S$ or later ($t_S\le t_S^m$). In this case, $\sum_{j=1}^{N}\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt}=0$ and $\sum_{j=1}^{N}\sum_{t=t_S}^{T}x_{0j}^{mt}=0$.

From the above we can conclude, that in any case the sum $\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt}+\sum_{t=t_S}^{T}x_{0j}^{mt}\right)$ is either 0 or 1.

- All vehicles end their routes at the depot. This constraint is defined by equation (3.23) .

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}{}^{mt}+\sum_{t=t_S}^{T}x_{0j}{}^{mt}\right)=\sum_{i=1}^{N}\sum_{t=t_S}^{T}x_{i0}{}^{mt}\quad\forall m\in M \tag{3.23}$$

As defined in equation (3.22), each vehicle *m* can either be assigned to a route or not. If vehicle *m* is assigned to a route then the term $\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}{}^{mt}+\sum_{t=t_S}^{T}x_{0j}{}^{mt}\right)$ is equal to 1, otherwise it is equal to 0. For a vehicle *m* that is not assigned to a route, the term $\sum_{i=1}^{N}\sum_{t=t_S}^{T}x_{i0}{}^{mt}$ is equal to 0. However, for a vehicle *m* that is assigned to a route, the term $\sum_{i=1}^{N}\sum_{t=t_S}^{T}x_{i0}{}^{mt}$ is equal to 1 if the vehicle ends its route at the depot, and 0 otherwise. By defining that $\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}{}^{mt}+\sum_{t=t_S}^{T}x_{0j}{}^{mt}\right)=\sum_{i=1}^{N}\sum_{t=t_S}^{T}x_{i0}{}^{mt}$ for every vehicle *m*, we ensure that every route which starts at the depot also ends at the depot.

### 3.4.2. Demand Constraints

The demand constraints ensure that only one visit is made to each demand node by only one vehicle.

- All customers require a visit, are visited exactly once, while all other customers are not visited. This constraint is defined by equations (3.24) and (3.25).

$$\sum_{i=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt}+\sum_{t=t_S}^{T}x_{ij}{}^{mt}\right)=v_j^{t_S}\quad\forall j\in N,\ i\neq j \tag{3.24}$$

$$\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt}+\sum_{t=t_S}^{T}x_{ij}{}^{mt}\right)=v_i^{t_S}\quad\forall i\in N,\ i\neq j \tag{3.25}$$

Constraint (3.24) states that every customer, who's demand at time $t_S$ is higher than 0, is visited by a vehicle arriving from either the depot (*i*=0) or another customer ($i \in N, i \neq 0$).

Let $v_i^t$ denote whether customer *i* requires a visit at time *t*, when $v_i^t = 1$ states that customer *i* requires a visit at time *t*, and $v_i^t = 0$ states that he/she doesn't. The value of $v_i^t$ is implied from the total demand of customer *i* at time *t*, $D_i^t$. $D_j^t$, the total demand of demand node *j* is defined as the sum of all demands made by customer *j* from time $t = 0$ to time $t = t_S$, $\sum_{t=0}^{t_S} d_j^t$, minus all demands made by customer *j* that have been served before time $t = t_S$, $\sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{t=0}^{t_S-1} d_j^t \bar{x}_{ij}^{mt}$. If $D_j^t$ is equal to 0, then $v_i^t$ is 0, otherwise $v_i^t$ is 1.

Similarly, constraint (3.25) states that a vehicle serving customer *i*, must continue to either the depot (*j*=0) or another customer ($j \in N, j \neq 0$).

- A demand constraint (constraint (3.26)) is also added. This constraint states that the total demand of all customers visited by the same vehicle must be less than or equal to the capacity of the vehicle.

$$\sum_{i=0}^{N}\left(d_i^{t_S}\left(\sum_{j=0}^{N}\sum_{t=t_S}^{T} x_{ij}^{mt}\right)\right) \leq Q \quad \forall m \in M \tag{3.26}$$

## 3.4.3. Routing Constraints

- If node *j* is visited after visiting node *i*, then the departure time, *t*, from node *j* is equal to or greater than the departure time from node *i* plus the travel time from node *i* to node *j* at time *t*. This is described by constraint (3.27).

$$\begin{aligned}&\sum_{k=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(t \times \bar{x}_{jk}{}^{mt}\right)+\sum_{t=t_S}^{T}\left(t \times x_{jk}{}^{mt}\right)\right) \geq \\ &\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(\left(t+\bar{C}_{ij}^t\right) \times \bar{x}_{ij}{}^{mt}\right)+\sum_{t=t_S}^{T}\left(\left(t+\bar{C}_{ij}^t\right) \times x_{ij}{}^{mt}\right)\right) \quad \forall i > 0, j > 0\end{aligned} \tag{3.27}$$

- A vehicle, visiting node *i*, that leaves node *p* and a vehicle visiting node *p*, that leaves to node *j*, is the same vehicle. This constraint (3.28) is a route continuity constraint.

$$\sum_{i=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ip}{}^{mt}+\sum_{t=t_S}^{T}x_{ip}{}^{mt}\right)-\sum_{j=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{pj}{}^{mt}+\sum_{t=t_S}^{T}x_{pj}{}^{mt}\right)=0\quad\forall m\in M,\ p\in N,\ p\neq 0 \tag{3.28}$$

- If node *j* is visited after visiting node *i*, then the departure time, *t*, from node *j* is equal to or greater than the departure time from node *i* plus the travel time from node *i* to node *j* at time *t*. This is described by constraint (3.29).

$$\begin{aligned}&\sum_{k=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(t\times\overline{x}_{jk}{}^{mt}\right)+\sum_{t=t_S}^{T}\left(t\times x_{jk}{}^{mt}\right)\right)\geq\\&\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(\left(t+\overline{C}_{ij}^{t}\right)\times\overline{x}_{ij}{}^{mt}\right)+\sum_{t=t_S}^{T}\left(\left(t+\overline{C}_{ij}^{t}\right)\times x_{ij}{}^{mt}\right)\right)\quad\forall i>0,\ j>0\end{aligned} \tag{3.29}$$

## 3.5. The Real-Time Multi-Objective VRP Mix LP Model

The travel time cost function, $C_{ij}^{t}$, is stochastic in nature, meaning that it may vary from one day to another. Therefore, the cost function, $C_{ij}^{t}$, is associated with a mean, $\mathbb{E}\left(C_{ij}^{t}\right)$, which describes the average travel time from customer *i* to customer *j* at time *t* and a standard deviation, $\sigma\left(C_{ij}^{t}\right)$, which shows how much variation there is from the mean. As an estimation of $C_{ij}^{t}$ the mean $\mathbb{E}\left(C_{ij}^{t}\right)$ can be used. However, for a route based on this estimation, the total travel time of the route will not reflect the possibility of arriving at a customer earlier or later than expected, and the changes in travel time it may cause. For that reason, a different estimation of the stochastic cost function, $C_{ij}^{t}$, is suggested. Let $\epsilon\left(t\right)$ be an impact factor, which defines how much the value of $C_{ij}^{t}$ is affected by possible changes in travel time (compared to the mean) in previous and future time

intervals, and is defined as $\epsilon(t) = t \cdot \max_{t' \le t} \left| \frac{\sigma(c_{ij}^{t'})}{\mathbb{E}(c_{ij}^{t'})} \right|$. The estimation of the stochastic cost function, $\hat{C}_{ij}^{t}$, is defined as $\hat{C}_{ij}^{t} = \frac{1}{2\epsilon(t)+1} \sum_{\hat{t}=t-\lfloor \epsilon(t) \rfloor}^{t+\lfloor \epsilon(t) \rfloor} \mathbb{E}(C_{ij}^{\hat{t}})$. In this definition $\hat{C}_{ij}^{t}$ equals to an average of expected values of $C_{ij}^{t}$, thereby taking into consideration the possibility of being early or late. Based on this definition, the objective functions are:

Based on the notation of $\hat{C}_{ij}^{t}$, the formulation, objectives and constraints presented above, the real-time multi-objective VRP can be defined as the following mixed integer linear programming module.

The objective of the mixed integer linear programming are:

$$\begin{aligned} \min \quad Z = & \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{m=1}^{M} \sum_{t=0}^{t_S-1} \left[ \max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + \overline{ST}_j + \overline{WT}_j \right] \bar{x}_{ij}^{mt} + \\ & + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{m=1}^{M} \sum_{t=t_S}^{T} \left[ \max\left(\hat{C}_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j \right] x_{ij}^{mt} \end{aligned} \tag{3.30}$$

$$\min \quad Z = \sum_{j=1}^{N} \sum_{m=1}^{M} \sum_{t=0}^{t=t_s-1} \bar{x}_{0j}^{mt} + \sum_{j=1}^{N} \sum_{m=1}^{M} \sum_{t=t_s}^{T} x_{0j}^{mt} \tag{3.31}$$

$$\min \quad Z = \sum_{i=0}^{n} \sigma_i S_i \left( \sum_{j=0}^{N} \sum_{m=1}^{M} \left( \sum_{t=0}^{t_S-1} \left( \left( t + \bar{C}_{ji}^{t} \right) x'^{mt}_{ji} \right) + \sum_{t=t_S}^{T} \left( \left( t + \hat{C}_{ij}^{t} \right) x_{ji}^{mt} \right) \right) \right) \tag{3.32}$$

and

$$\min\{StdDev\} = \min \quad Z = \left\{ \sqrt{\frac{\sum_{m=1}^{M} w_m^2}{M} - \left( \frac{\sum_{m=1}^{M} w_m}{M} \right)^2} \right\} \tag{3.33}$$

where

$$w_m = \sum_{j=0}^{N} \sum_{m=1}^{M} \left( \sum_{t=0}^{t_S-1} \left( \bar{C}_{ji}^{t} x'^{mt}_{ji} \right) + \sum_{t=t_S}^{T} \left( \hat{C}_{ij}^{t} x_{ji}^{mt} \right) \right) \tag{3.34}$$

$$Min\ \ Z = Max\left(t : x_{i0}^{mt} = 1 \quad \forall i \in N, m \in M, t \in T\right) \tag{3.35}$$

and the constraints are:

$$x_{ii}^{mt} = 1 \quad \forall i \in N,\ m \in M \tag{3.36}$$

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt} + \sum_{t=t_S}^{T} x_{0j}^{mt}\right) \le 1 \quad \forall m \in M \tag{3.37}$$

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}{}^{mt} + \sum_{t=t_S}^{T} x_{0j}{}^{mt}\right) = \sum_{i=1}^{N}\sum_{t=t_S}^{T} x_{i0}{}^{mt} \quad \forall m \in M \tag{3.38}$$

$$\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt} + \sum_{t=t_S}^{T} x_{ij}{}^{mt}\right) = v_j^{t_S} + \sum_{m=1}^{M}\sum_{t=t_S^m}^{t_S-1} v_j^t \overline{x}_{ij}{}^{mt} \quad \forall i \in N,\ i \ne j \tag{3.39}$$

$$\sum_{i=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt} + \sum_{t=t_S}^{T} x_{ij}{}^{mt}\right) = v_i^{t_S} + \sum_{m=1}^{M}\sum_{t=t_S^m}^{t_S-1} v_i^t \overline{x}_{ij}{}^{mt} \quad \forall j \in N,\ i \ne j \tag{3.40}$$

$$\sum_{i=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ip}{}^{mt} + \sum_{t=t_S}^{T} x_{ip}{}^{mt}\right) - \sum_{j=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{pj}{}^{mt} + \sum_{t=t_S}^{T} x_{pj}{}^{mt}\right) = 0 \quad \forall m \in M,\ p \in N,\ p \ne 0 \tag{3.41}$$

$$\begin{aligned}&\sum_{k=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(t \times \overline{x}_{jk}{}^{mt}\right) + \sum_{t=t_S}^{T}\left(t \times x_{jk}{}^{mt}\right)\right) \ge \\ &\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(\left(t + \overline{C}_{ij}^t\right) \times \overline{x}_{ij}{}^{mt}\right) + \sum_{t=t_S}^{T}\left(\left(t + \overline{C}_{ij}^t\right) \times x_{ij}{}^{mt}\right)\right) \quad \forall i > 0,\ j > 0\end{aligned} \tag{3.42}$$

$$\sum_{i=0}^{N}\left(d_i^{t_S}\left(\sum_{j=0}^{N}\sum_{t=t_S}^{T} x_{ij}^{mt}\right)\right) \le D_m \quad \forall m \in M \tag{3.43}$$

$$P\left(\left(\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{C}_{ij}^t \overline{x}_{ij}^{mt} + \sum_{t=t_S}^{T} c_{ij}^t x_{ij}^{mt}\right)\right) \le c^*\right) \ge \alpha \tag{3.44}$$

$$x_{ij}^{mt} \in \{0,1\} \quad \forall m \in V,\ i, j \in N,\ t \in T \tag{3.45}$$

Constraint (3.44) is a chance constraint stating that we are looking for a set of routes, that for a given probability, $\alpha$, the traveling time will not be higher than $c^*$. A method for the determination of the value of $c^*$ is described next in this chapter.

## 3.6. Summary

This chapter describes the formulation of the real-time multi-objective vehicle routing problem stated in chapter 1. The problem is formulated as a mixed integer linear programming problem on a network.

Several assumptions and limitations are considered, such as a system with dynamic conditions (real-time variation in travel times and real-time service requests), all demands have specified service times and service time intervals, soft time windows for service around the desired service time are considered, and more.

Five objectives functions are described and formulated:

1. Minimizing the total travel time
2. Minimizing number of vehicles - in the real world, since the fixed cost of using additional vehicles is higher than the routing operation costs, we can reduce the total cost by minimizing the number of vehicles in service.
3. Maximizing customers' satisfaction – In VRPTW with soft time windows, time windows may sometimes be violated. However, there exist certain bounds on the violation that a customer can endure. Each customer can be assigned with a customer's satisfaction function, which can describe his/her satisfaction vs. the deviation from his time window.
4. Maximizing the balance of the tours - This objective function seeks to balance the work between vehicles by minimizing the difference between the work of each vehicle and the work of the vehicle with the lowest work level.
5. Minimizing the arrival time of the last vehicle - Each vehicle starts its route and ends its route at the depot. While the start time of each vehicle is known, the end time is unknown and is subject to changes, mainly because of the stochastic nature

of the travel time. By minimizing the arrival time of the last vehicle, we guarantee two things: (1) Maximum availability of vehicles for unscheduled deliveries and (2) that there are no 'too long' routes.

The various constraints applied to this problem are also described and formulated as well:

1. Vehicle cannot drive from one node to itself.
2. All vehicles start their routes at the depot.
3. All vehicles end their routes at the depot.
4. All customers require a visit, are visited exactly once, while all other customers are not visited.
5. The total demand of all customers visited by the same vehicle must be less than or equal to the capacity of the vehicle.
6. If node *j* is visited after visiting node *i*, then the departure time, *t*, from node *j* is equal to or greater than the departure time from node *i* plus the travel time from node *i* to node *j* at time *t*.
7. A vehicle, visiting node *i*, that leaves for node *p* and a vehicle visiting node *p*, that leaves for node *j*, is the same vehicle.
8. If node *j* is visited after visiting node *i*, then the departure time, *t*, from node *j* is equal to or greater than the departure time from node *i* plus the travel time from node *i* to node *j* at time *t*.

# 4. Coping with Dynamic VRP

In many real-life applications relevant data changes during the execution of transportation processes and schedules have to be updated dynamically. Thanks to recent advances in information and communication technologies, vehicle fleets can now be managed in real-time. When jointly used, devices like geographic information systems (GIS), global positioning systems (GPS), traffic flow sensors and cellular phones are able to provide real-time data, such as current vehicle locations, new customer requests, and periodic estimates of road travel times. If suitably processed, this large amount of data can be used to reduce the cost and improve the service level of a modern company. To this end, revised routes have to be timely generated as soon as new events occur (Ghiani et al., 2003).

In this context, Dynamic Vehicle Routing Problems (DVRPs), also known as on-line or real-time Vehicle Routing Problems, are becoming increasingly important (Hanshar & Ombuki-Berman, 2007; Housroum, Hsu, Dupas & Goncalves, 2006; Montemanni, Gambardella, Rizzoli & Donati, 2005; Psaraftis, 1995). It is possible to define several dynamic features which introduce dynamism into the classical VRP: roads between two customers could be blocked off, customers could modify their orders, the travel time for some routes could be increased due to bad weather conditions, etc. This implies that Dynamic VRPs constitute in fact a set of different problems, which are of crucial importance in today's industry, accounting for a significant portion of many distribution and transportation systems.

The main goal of this chapter is to present the problem of DVRP and methods from the literature for its resolution.

## 4.1. Dynamic vs. Static Planning

This section discusses the main differences between dynamic and static vehicle routing. Although some of the issues discussed apply to dynamic and static planning, their impact on dynamic planning is often much more severe. A comprehensive discussion of dynamic vehicle routing can be found in (Psaraftis, 1988) and (Psaraftis, 1995). Psaraftis gives the following definition of a dynamic problem: a problem is dynamic if information on the

problem is made known to the decision maker or is updated concurrently with the determination of the solution. By contrast, if all inputs are received before the determination of the solution and do not change thereafter, the problem is termed static.

### 4.1.1. Evolution of information

Obviously, the major difference between dynamic and static problems is the evolution of information. In static problems information is assumed to be known for the entire duration of the transportation process. In dynamic problems, however, some input is not known at the time of planning, and some input is not known with certainty. For example, traffic conditions change, the number of vehicles available may change due to vehicle break-down, new transportation requests become known, or attributes of transportation requests may change.

Most of the literature on dynamic vehicle routing only considers one type of uncertainty: the arrival of a new transportation request. Few works consider problems with several sources of uncertainty. Among them are the works by Carvalho and Powell (2000) and Fleischmann et al. (2004), who consider the arrival of new transportation requests and uncertain travel times.

Another issue in dynamic planning is the reliability of future information. Long-term information is more likely to change than short-term information. For example, long-term estimations of travel times can only be based on historical data. In the short-term, however, actual traffic conditions can be used to estimate travel times. Those short-term forecasts allow a much better estimation of what is going to happen and thus help in improving punctuality.

### 4.1.2. Rolling horizon

In static planning, schedules are generated for a certain finite planning horizon. The duration of the planning process is bounded by the time between data collection and the start of transportation processes. Given the input data, a static planning system must be capable of calculating high quality solutions in the time available for optimization.

In dynamic planning, the planning horizon may not be bounded or even known. In fact, a typical dynamic planning scenario is that of an open-ended process, going on for an indefinite period of time. Usually, near-term events are more important than long-term

events. The consideration of requirements which have to be satisfied long into the future would not be very wise, because such future information may change anyway. In a typical rolling horizon framework, only information relevant to planning decisions within a horizon of a certain length is considered. As time unfolds, parts of the tentative schedule are applied and new information may enter the planning horizon.

### 4.1.3. Impreciseness of model representation

Real-life vehicle routing problems usually cannot be precisely represented by an analytical model which is required for computer-based decision support. Even if the analytical model is of high quality, discrepancies between model representation and real-life problems arise as a result of the sheer cost of getting information into the computer. Telematics systems can be used to improve the timely availability of information regarding the actual transportation processes. Electronic Data Interchange (EDI) can be used to integrate information systems of shippers, e.g. to obtain all relevant data regarding transportation requests and customer locations. Despite the improved possibilities of getting data into the model, the information is generally not only incomplete but also imprecise. A shipper, for example, may ask that a shipment be picked up in the morning before noon, when his dock is not as busy. In the model such restrictions are usually treated as time window constraints. The computer system has no way of interpreting whether such a request for early pickup is a hard constraint or whether the shipper was only trying to express a preference.

The impreciseness of the model representation results in two fundamental problems: (1) some solutions which are feasible according to the model may not be feasible in reality and vice versa, (2) a solution with high quality in the model may not have the same high quality in reality.

Although these problems occur in static as well as in dynamic planning, the impact is quite different. In static planning there is usually more time for the collection of data, resulting in a more accurate representation of the real-life problem. Furthermore, there is more time to manually verify and validate a solution recommended by the planning system.

### 4.1.4. Interactivity

Due to the impreciseness of any model representation and the fact that a significant amount of relevant information is not available to the computer, but only to the dispatchers who are in direct contact with drivers and shippers, model recommendations cannot always be fully implemented. According to Carvalho and Powell (2000), several motor carriers report that the average usage of model recommendations is below 60%, and good performance is considered around 70%.

In order to deal with this issue, Kopfer and Schonberger (2002) have presented a framework for interactive problem solving. The concept is founded on posting a problem to a planning method in order to let it generate a solution. The returned solution usually does not satisfy all real-life requirements. Therefore, dispatchers may add, modify, or remove certain constraints in the analytical model. The modified problem is again posted to the planning method, and after a solution has been found, further modifications to the model can be made. Although this approach can be used for static and dynamic problems, this iterative decision making is much harder in dynamic planning due to the lack of time.

In a similar approach, instead of having an iterative decision making process, a real-time decision support system allows dispatchers and dynamic planning systems to simultaneously modify the current solution. Dispatchers may add, modify, or remove certain constraints in the analytical model at any time. All changes made by the dispatchers are directly considered by the dynamic planning system which continuously searches for improved solutions. Whenever the dynamic planning system finds a better solution, the current solution is immediately updated and shown to the dispatchers. An optimistic locking scheme is used in order to maintain data consistency.

Humans use a form of cache memory for processing information, called working memory. This working memory must be regularly updated in order to consider changes in problem data and solution. A dynamic planning system must support dispatchers in quickly updating their working memory. Therefore, besides providing algorithmic optimization techniques, a dynamic planning system must also provide sophisticated graphical user interfaces (GUI), allowing dispatchers to quickly identify modified parts of the solution, and to efficiently verify feasibility and profitability of an automatically generated schedule.

### 4.1.5. Response time

The response time of an algorithm is the time which is needed until a newly calculated solution can be applied. Algorithms for dynamic planning must have fast response times for two reasons. First, a solution calculated for a dynamic problem can only be applied if the input data have not changed significantly during the planning process. Second, the longer it takes to calculate a new solution, the higher the probability that dispatchers may concurrently change the current solution.

In many cases dispatchers have to decide about load acceptance or rejection while the shipper is on the phone making the request. Dynamic planning systems should be capable of supporting dispatchers in the load acceptance decision while the shipper is on the phone. In other words, the system should give support within a couple of seconds in deciding whether a transportation request can be served feasibly and efficiently.

The main advantage in dynamic planning is that there is usually plenty of time for optimization. As noted by Kilby, Prosser and Shaw (1998), ten minutes spent to find a solution for a small problem may seem like a long time in the static case. In the dynamic case, however, 'one has all day', so the time might as well be used. A dynamic planning system can be used to successively improve the current solution. Obviously, fast response times cannot be achieved if new solutions are calculated from scratch every time the dynamic planning method is invoked. Therefore, a dynamic planning system should have a restart capability, i.e. the planning system should be able to continue from the current solution. Furthermore, dynamic planning methods require efficient information update mechanisms in order to efficiently consider modified input data.

### 4.1.6. Measuring performance

Measuring performance in dynamic planning is much more complicated than in static planning. A method for static planning can be evaluated by comparing the solution obtained with solutions obtained by other methods. In dynamic planning, however, the decisions made at one point in time determine the options and alternatives at a later point in time. It is possible to compare alternative decisions at one point in time, but then carrying the effects of those decisions forward in time creates a problem.

In dynamic planning with rolling horizons, any evaluation of operating efficiencies must address the issue of time period evaluation. The state of the system at the end of the evaluation period may have a dramatic impact on future performance. A dynamic planning method may perform quite well over a day or a week, but can have poor long-term performance, e.g. if all vehicles are sent to very distant regions where there is little hope of picking up a new load.

As mentioned above, dynamic planning methods must be interactive, enabling dispatchers to verify model recommendations. In many cases dispatchers will agree with model recommendations. However, differing problem knowledge and solution methods of computer vs. human dispatchers may result in contradictory decisions. Under the time constraints in dynamic planning, it is often very difficult or even impossible to find out why the recommendation is being made. Is the discrepancy a result of "higher reasoning" or a simple data error ? Typically, dispatchers will follow their own intuition and not the model recommendation. As noted by Powell W.B., Marar, Gelfand and Bowers (2002), a dynamic planning system is often considered successful if dispatchers agree with model recommendations, but dispatchers often do not follow model recommendations. Obviously, the question arises that if dispatchers do not comply with model recommendations, are the solutions produced any good at all ?

## 4.2. DVRP Interests

There are several important problems that must be solved in real-time. In (Gendreau & Potvin, 1998), (Larsen, 2000) and (Ghiani et al., 2003), the authors list a number of real-life applications that motivate the research in the domain of dynamic vehicle routing problems.

- *Supply and distribution companies*: In seller-managed systems, distribution companies estimate customer inventory levels in such a way as to replenish them before stock depletion. Hence, demands are known beforehand in principle and all customers are static. However, because demand is uncertain, some customers might run out of their stock and have to be serviced urgently.
- *Courier Services*: This refers to the international express mail services that must respond to customer requests in real-time. The load is collected at different customer

locations and has to be delivered at another location. The package to be delivered is brought back to a remote terminal for further processing and shipping. The deliveries form a static routing problem, since recipients are known by the driver. However, most pickup requests are dynamic because neither the driver nor the planner knows where the pickups are going to take place.

- *Rescue and repair service companies*: Repair services usually involve a utility firm (broken car rescue, electricity, gas, water and sewer, etc.) that responds to customer requests for maintenance or repair of its facilities.
- *Dial-a-ride systems*: Dial-a-ride systems are mostly found in demand-responsive transportation systems aimed at servicing small communities or passengers with specific requirements (elderly, disabled). These problems are of the many-to-many when any node can serve as a source or destination for any commodity or service. Customers can book a trip one day in advance (static customers) or make a request at short notice (dynamic customers).
- *Emergency services*: These includes the police, firefighting and ambulance services. By definition, the problem is purely dynamic, since all customers are unknown beforehand and arrive in real-time. In most situations, routes are not formed because the requests are usually served before a new request appears. The problem then is to assign the best vehicle (for instance the nearest) to the new request. Solving methods are based on location analysis for deciding where to dispatch the emergency vehicles or to escape the downtown traffic jam.
- *Taxicab services*: Managing taxicabs is still another example of a real-life dynamic routing problem. In most taxicab systems the percentage of dynamic customers is very high, i.e., only a very few customers are known by the planner before the taxicab leaves the central station at the beginning of its working day.

## 4.3. Related Works

In this section, we present a classification and an overview on the state-of-the-art of dynamic vehicle routing problems. Naturally, this chapter cannot cover all aspects of vehicle routing problems with dynamic or stochastic elements. The goal of this chapter is rather to provide a brief introduction to the literature on these subjects. Also, this chapter

is meant to provide the reader with an overview of the methodological approaches to these problems.

Different surveys have been proposed on DVRPs (Bianchi & Bianchi-idsia, 2000; Ghiani et al., 2003; Moretti Branchini, Amaral Armentano & Løkketangen, 2009).

Psaraftis (1988, 1995) was among the first to study dynamic versions of the VRP. Psaraftis defines that a vehicle routing problem is dynamic when some inputs to the problem are revealed during the execution of the algorithm. Demand information is not known when vehicles are assigned, and demand information is revealed on-line. Problem solution evolves as inputs are revealed to the algorithm and to the decision maker. Possible information attributes might include evolution of information (static/dynamic), quality of information (known-deterministic/forecast/probabilistic/unknown), availability of information (local/global), and processing of information (centralized/decentralized).

Powell W. B. et al. (1995) concentrate on stochastic programming based models, but also provide an excellent survey on various dynamic vehicle routing problems. For example, the dynamic traffic assignment problem consists of finding the optimal routing of some goods from origin to their destination through a network of links which could have time-dependent capacities. The authors also discuss how to evaluate the solutions, since it is an important issue that distinguishes static from dynamic models. They note that in static models finding an appropriate objective function is fairly easy and that the objective function is usually a good measure for evaluating the solution. However, for dynamic models the objective function used to find the solution over a rolling horizon often has little to do with the measures developed to evaluate the overall quality of a solution.

Bertsimas DJ and Simchi-Levi (1996) provide a survey of deterministic and static as well as dynamic and stochastic vehicle routing problems for which they examine the robustness and the asymptotic behavior of the known algorithms. Bertsimas and Simchi-Levi argue that analytical analysis of the vehicle routing problem offers new insights into the algorithmic structure, making performance analysis of classical algorithms possible and leading to a better understanding of models that integrate vehicle routing with other issues like inventory control. The authors conclude that a-priori optimization is an attractive policy if intensive computational power is not present. Furthermore, they point out that dealing with stochasticity in the VRP provides insights that can be useful when

constructing practical algorithms for the VRP within a dynamic and stochastic environment.

Gendreau and Potvin (1998) note that the work on local area vehicle routing and dispatching still leaves a number of questions to be answered. In particular, research on demand forecasting used for constructing routes with look-ahead is needed in the future. Furthermore, the authors point out that it is relevant to consider several sources of uncertainty like cancellation of requests and service delays rather than just to focus on uncertainty in the time-space occurrence of service requests. Gendreau and Potvin also note that the issue of diversion deserves more attention. Due to the large amount of online information it has become possible to redirect the vehicle while on-route to the new customer. Finally, the authors advocate further research on parallel implementations and worst-case analysis, in order to be able to assess the loss in not having full information available at the time of planning.

The following is a classification of DVRPs according to the degree of knowledge that we have on the input data of the problem and quality of the available information. A dynamic problem can be either deterministic or stochastic. DVRP is deterministic if all data related to the customers are known when the customer demands arrive; otherwise it is stochastic. Both of these classes can be subject to different factors such as service time window, traffic jam, road maintenance, weather changes, breakdown of vehicles and so on. These factors often change the speed of vehicles and the travel time for arriving at the depot.

1. **Deterministic**: In a deterministic case, all the data related to the inputs are known. For instance, when a new customer demand appears, customer location and the quantity of his demand are known. Different types of deterministic DVRP can be found in the literature as:
   a. **Dynamic Capacitated Vehicle Routing Problem (DCVRP)**: An important number of works exist on this variant (Gendreau et al., 1999; Kilby et al., 1998; Montemanni et al., 2005) which represents the conventional definition of the problem, and where the existence of all customers and their localizations are deterministic, but their order can arrive at any time. The objective is to find a set

of routes with the lowest traveled distance, and with respect to vehicle capacity limit.

The Dynamic Traveling Repairman Problem (DTRP) (Bertsimas D.J. & Van Ryzin, 1991, 1993) belongs to this class of problems. It is described as a problem in which demands arrive according to Poisson process in a Euclidean service region, and their locations are distributed throughout the service region. The goal is to minimize the expected time that the demand spends in the system (i.e. the average time a customer must wait before his/her request is completed), as opposed to the expected distance that the vehicle travels. The service times of requests are not known to the dispatcher, until the service at the respective customers is completed.

Where all demands are dynamic in DTRP, i.e. all customers are immediate request customers. Larsen, Madsen and Solomon (2002) define the Partially Dynamic Traveling Repairman Problem (PDTRP) that is a variant of this problem involving both advance and immediate request customers. Furthermore, the problem seeks to optimize different objective functions. The dispatcher is more interested in minimizing the distance traveled by the repairman than in minimizing the overall system time.

b. **Dynamic Vehicle Routing Problem with Time Windows (DVRPTW)**: It is one of the most well-studied variants of DVRP (Alvarenga, de Abreu Silva & Mateus, 2005; Fabri & Recht, 2006; Housroum et al., 2006; Larsen, Madsen & Solomon, 2004; Mitrović-Minić, Krishnamurti & Laporte, 2004; Oliveira, de Souza & Silva, 2008; Wang J., Tong & Li, 2007). Besides the possibility of requiring services in real time, the time window associated with each customer *i* follows a specific interval time $[a_i, b_i]$, that must be satisfied. Larsen et al. (2002) proposed on-line policies for the Partially Dynamic Traveling Salesman Problem with Time Windows (PDTSPTW) that could be considered as an instance of DVRPTW with a single vehicle. The objective is to minimize lateness at customer locations. A simple policy of requiring the vehicle to wait at the current customer location until it can service another customer without being early. Other policies may

suggest repositioning the vehicle at a location different from that of the current customer, based on prior information on future requests.

c. **Dynamic Vehicle Routing Problem with time-dependent Travel Times (DVRPTT)**: Described in (Haghani & Jung, 2005), it assumes that the travel time from customer *i* to customer *j* is variable across time. This variation could occur due to the type of the road, weather and traffic conditions that may strongly influence the speed of vehicles and hence travel times.

d. **Dynamic Pickup and Delivery Vehicle Routing Problem (DPDVRP)**: Based on the conventional Pickup and Delivery Vehicle Routing Problem (PDVRP) (Savelsbergh M. W. P. & Sol, 1995). The problem consists of determining a set of optimal routes for a fleet of vehicles in order to serve transportation requests (Mitrović-Minić et al., 2004). The objective is to minimize total route length, i.e., the sum of the distances traveled by all the vehicles, under the following constraints: all requests must be served, each request must be served entirely by one vehicle (pairing constraint), and each pickup location has to be served before its corresponding delivery location (precedence constraint). The dynamic version arises when not all requests are known in advance. Swihart and Papastavrou (1999) have introduced a new variant of the DTRP where each service request has a pickup and a delivery location. The objective is to minimize the expected system time. The authors consider the unit-capacity case where the vehicle can carry no more than one item, as well as the case where the vehicle can carry an arbitrarily large number of items. Attanasio, Cordeau, Ghiani and Laporte (2004) present a parallel implementation of a tabu search method developed previously by Cordeau and Laporte (2003) for the Dynamic Dial-a-Ride Problem (DDARP). In the latter, requests are received throughout the day and the primary objective is to accommodate as many requests as possible according to the available fleet of vehicles. Furthermore, the routes are designed under the constraint that customers specify pick-up and drop-off requests between origins and destinations. Yang, Jaillet and Mahmassani (2004) introduce a real-time multi-vehicle truck-load pickup and delivery problem. They propose a mixed-integer programming

formulation for the off-line version of the problem and propose a new rolling horizon re-optimization strategy for a dynamic version.

2. **Stochastic**: In stochastic dynamic problems (also known as probabilistic dynamic problems) uncertain data are related to customer locations, demands or travel times and are represented by stochastic processes.
   a. **Dynamic and Stochastic Capacitated Vehicle Routing Problem (DSCVRP)**: It considers the situation where customer requests are unknown and revealed over time. In addition, customer locations and service times are random variables and are realized dynamically during plan execution. Bent and Van Hentenryck (2004) considered DVRP with stochastic customers. They proposed a multiple scenario approach that continuously generates routing plans for scenarios including known and immediate requests to maximize the number of serviced customers. The approach was adapted from Solomon benchmarks, with a varying degree of dynamism. Hvattum, Løkketangen and Laporte (2006) addressed this variant of the problem. The authors assume that both customer locations and demands may be unknown in advance. They formulate the problem as a multi-stage stochastic programming problem, and a heuristic method was developed to generate routes by exploiting the information gathered on future customer demand.
   b. **Dynamic and Stochastic Vehicle Routing Problem with Time Windows (DSVRPTW)**: Proposed by Pavone, Bisnik, Frazzoli and Isler (2009), in this problem, each service request is generated according to a stochastic process; once a service request appears, it remains active for a certain deterministic amount of time, and then expires. The objective is to minimize the number of possible vehicles and ensure that each demand is visited before its expiration. Furthermore, this problem has been considered by Bent and Van Hentenryck (2007).
   c. **Dynamic Vehicle Routing Problem with Stochastic Travel Times (DVRPSTT)**: It assumes that the problem is subject to a stochastic travel time which represents a random variable in an interval. The travel times change from one period to the next. Some works present this version of the problem as in (Potvin, Xu & Benyahia, 2006), where the travel time to the next destination is perturbed by adding a value generated with a normal probability law. This

perturbation represents any unforeseen events that may occur along the current travel journey. It is known to the dispatching system only when the vehicle arrives at its planned destination.

d. **Dynamic and Stochastic Pickup and Delivery Vehicle Routing Problem (DSPDVRP)**: In this version of the problem a stochastic process concerns the quantity of demand that the vehicle must pick up or delivery to each customer. Thus, we have vagueness in quantities to pick up or deliver at the customer's location (Xu, Goncalves & Hsu, 2008). The demand of each customer is revealed only when the vehicle reaches the customer. The distribution can be modeled by using a probabilistic law, such as a normal law, for example, or by using fuzzy logic.

## 4.4. Solution Methods

This section gives an overview of algorithms for solving vehicle routing problems, based on solution methods which can be used for rich problems in which problem data may change dynamically. The solution methods discussed in this section can be categorized into assignment methods, construction methods, improvement methods, meta-heuristics and mathematical programming based methods.

### 4.4.1. Assignment Methods

Assignment methods are methods that assign transportation requests to vehicles for immediate execution. They are used in highly dynamic problems where problem data change very fast and no foresighted planning is likely to perform well. Assignment algorithms that simultaneously assign several open orders to idle vehicles are studied by Spivey and Powell (2004) and Fleischmann et al. (2004).

### 4.4.2. Construction Methods

Construction methods gradually build tours while keeping an eye on the objective function value, but they do not contain an improvement phase (Laporte & Semet, 2002). A comprehensive survey on construction methods for the VRPTW is given by Bräysy and Gendreau (2005b). One of the best-known tour construction methods for the VRP is the Savings algorithm by Clarke and Wright (1964).

Insertion methods are methods that successively insert open transportation requests into partially constructed tours. They are well suited for dynamic planning, because they permit incorporation of a new order that considers the set of tours which are currently implemented. Insertion methods are very fast and can be used for dynamic vehicle routing problems in which there may not be enough time to employ more sophisticated methods. Furthermore, insertion methods can often be applied to problems incorporating various real-life requirements without losing efficiency. A discussion of efficient insertion methods for vehicle routing problems incorporating complicating constraints can be found in (Campbell & Savelsbergh, 2004).

Early examples of insertion methods have been proposed by Solomon (1987) for the VRPTW. Parallel insertion methods for the static VRPTW which simultaneously constructs several tours via insertions are proposed by Potvin and Rousseau (1993) and Antes and Derigs (1995). Recently Lu and Dessouky (2006) presented an insertion method for the PDPTW which not only considers the classical incremental costs, but also the cost of reducing the time window slack so that more opportunities are left for future insertions. Insertion methods for the dynamic PDP are also studied by Mitrovic-Minic, Adviser-Krishnamurti and Adviser-Laporte (2001) and Fleischmann et al. (2004).

### 4.4.3. Improvement Methods

Many solution techniques for combinatorial optimization problems are based on a simple and general idea. Let $s$ be a feasible solution of the problem considered and let $f(s)$ denote the objective function value of $s$. For each feasible solution $s$ the neighborhood of $s$ is defined by the solutions $s^*$ which can be obtained by applying an appropriately defined neighborhood operator to the solution $s$. So-called local search or neighborhood search methods explore the neighborhood of the current solution $s$ by searching for a feasible solution $s^*$ of high quality in the neighborhood of the current solution $s$. This solution may be accepted as a new current solution, and in this case, the process is iterated by considering $s^*$ as a new current solution.

In maximization (minimization) problems, a new solution $s^\star$ is typically only accepted if $f(s^*) \geq f(s)$ ( $f(s^*) \leq f(s)$ ) . If no solution $s^\star$ with $f(s^*) \geq f(s)$ ( $f(s^*) \leq f(s)$ ) exists in the neighborhood of $s$, a local optimum has been reached. A comprehensive

work on local search methods is given by Aarts and Lenstra (1997). A survey and comparison of local search methods for the VRPTW has been presented by Bräysy and Gendreau (2005a).

Improvement methods are local search methods which start with a feasible solution and gradually modify the current solution in order to improve the solution quality. The most simple improvement methods operate on a single tour and optimize the sequence in which the locations are visited. They are often based on methods developed for the TSP, e.g. 2-opt by Lin (1965) and Or-opt by Or (1976). Others consider several tours simultaneously, e.g. the operators relocate, exchange, and cross originally proposed by Salesbergh (1992) for the classical VRP. Local optima produced by an improvement method can be very far from the optimal solution, as they only accept solutions that produce an improvement in the objective function value. Thus, the outcome depends heavily on the initial solution and the neighborhood definition.

### 4.4.4. Meta-heuristics

Meta-heuristics are general solution procedures that often embed some of the standard tour construction and improvement methods, as well as techniques to escape from local optima of low quality (Cordeau, Gendreau, Laporte, Potvin & Semet, 2002). A comprehensive survey on the use of meta-heuristics for the VRPTW is given by Bräysy and Gendreau (2005b). Examples of meta-heuristics are Simulated Annealing, Genetic Algorithms, Ant Systems, Tabu Search, and Iterated Local Search (Blum & Roli, 2003).

The fundamental idea of Simulating Annealing is to allow moves resulting in solutions of worse quality in order to escape from locally optimal solutions (Kirkpatrick et al., 1983). The probability of doing such a move is decreased during the search. Although successful for many static problems, it is not clear how to effectively change this probability in dynamic problems, as input data may change during the search.

Genetic Algorithms, Ant Systems, and Tabu Search are memory-based methods classified as Adaptive Memory Programming (AMP) methods by Taillard É. D., Gambardella, Gendreau and Potvin (1998). Particularly in highly dynamic problems, AMP methods require methods to efficiently update the memory. The memory can only be used effectively if there are only minor changes to the problem data. Examples of AMP methods are the Genetic Algorithm for the dynamic PDP presented by Pankratz

(2004), an Ant Colony System for the dynamic VRP by Montemanni et al. (2003), the Tabu Search algorithm for the dynamic VRP by Gendreau et al. (1999), and the Tabu Search algorithms for the dynamic PDP by Gendreau and Potvin (1998) and Mitrovic-Minic et al. (2001).

The essence of Iterated Local Search (ILS) is to iteratively build a sequence of solutions generated by an embedded heuristic. It applies the heuristic until it finds a local optimum. Then it perturbs the solution and restarts the heuristic. This generally leads to far better solutions than if one would use repeated random trials of that heuristic (Lourenço, 2002).

Variable Neighborhood Search (VNS) can be interpreted as a specialized ILS based on the idea of systematically changing the neighborhood structure during the search (Hansen P. & Mladenović, 2003; Mladenović & Hansen, 1997). VNS systematically exploits the following observations: (1) a local optimum with respect to one neighborhood structure is not necessarily so for another; (2) a global optimum is a local optimum with respect to all possible neighborhood structures; (3) for many problems local optima with respect to one or several neighborhoods are relatively close to each other. An example of a VNS algorithm for vehicle routing problems is the algorithm for the multi-depot VRPTW presented by Polacek, Hartl, Doerner and Reimann (2004).

As noted by Ahuja, Ergun, Orlin and Punnen (2002), a critical issue in the design of a neighborhood search approach is the size of the chosen neighborhood. Large neighborhoods increase the quality of the locally optimal solutions; however, locally optimal solutions are difficult to find in very large neighborhoods. In each iteration of the Large Neighborhood Search (LNS) algorithm presented by Shaw (1997) for the VRPTW, $k$ customers are first removed from their tours and then re-inserted using a branch and bound procedure. Schrimpf, Schneider, Stamm-Wilbrandt and Dueck (2000) and Ropke and Pisinger (2006) present similar LNS algorithms using fast insertion heuristics for the re-insertion of transportation requests. The use of fast insertion heuristics is more appropriate for dynamic planning, as fast response times can be easily achieved. The LNS approach is very well suited for highly constrained vehicle routing problems and rich vehicle routing problems in which data may change dynamically.

### 4.4.5. Mathematical Programming Based Methods

Finding optimal solutions for vehicle routing problems is generally too time consuming, particularly in dynamic planning. However, mathematical programming based methods can be used in dynamic vehicle routing if the number of decision variables is reduced dramatically or sub-problems are only solved approximately.

Mathematical programming based methods for the dynamic Full-Truckload PDP have been presented by Yang et al. (2004). To guarantee robustness and time-lines of the methods, the number of transportation requests involved in each optimization was limited to a fixed upper-bound. Thus, the resulting mathematical program is significantly reduced in size and is solved using a branch-and-cut procedure.

The most popular mathematical programming based methods for rich vehicle routing problems is the Column Generation approach. Column Generation has been applied to the VRPTW by Desrochers et al. (1992), the HFVRP by Taillard É. D. (1996), the PDP by Dumas, Desrosiers and Soumis (1991), a generalized PDP by Savelsbergh M. and Sol (1998). Column Generation approaches for dynamic vehicle routing problems have been presented by Savelsbergh M. and Sol (1998) and more recently by Potvin et al. (2006).

## 4.5. Summary

In many real-life applications relevant data change during the execution of transportation processes and schedules have to be updated dynamically. Thanks to recent advances in information and communication technologies, vehicle fleets can now be managed in real-time. In this context, Dynamic Vehicle Routing Problems (DVRPs), also known as on-line or real-time Vehicle Routing Problems, are becoming increasingly important.

There are several differences between static and dynamic vehicle routing problems. Obviously, the major difference is the evolution of information. In static problems information is assumed to be known for the entire duration of the transportation process. In dynamic problems, however, some input is not known at the time of planning, and some input is not known with certainty. In static planning, schedules are generated for a certain finite planning horizon, while in dynamic planning, the planning horizon may neither be bounded, nor known.

The response time of an algorithm, which is the time which is needed until a newly calculated solution can be applied, must be fast for two reasons: (1) a solution calculated for a dynamic problem can only be applied if the input data have not changed significantly during the planning process, and (2) the longer it takes to calculate a new solution the higher the probability that dispatchers concurrently change the current solution.

Several dynamic VRPs have been reviewed, which include the Dynamic Capacitated VRP, Dynamic VRP with Time Windows, Dynamic VRP with time-dependent Travel Times, Dynamic Pickup and Delivery VRP, Dynamic and Stochastic Capacitated VRP, Dynamic and Stochastic VRP with Time Windows, Dynamic VRP with Stochastic Travel Times and Dynamic and Stochastic Pickup and Delivery VRP.

Next, the various solution methods, including heuristics and meta-heuristics methods were reviewed.

# 5. Solving Multi-Objective Optimization Problems

Real-world problems often have multiple conflicting objectives. For example, when purchasing computing equipment, we would usually like to have a high-performance system, but we also want to spend less money buying it. Obviously, in these problems, there is no single, best solution when measured on all objectives. These problems are examples of a special class of optimization problems called multi-objective optimization problems (MOPs).

In single objective problems, an optimum solution is a solution for which the criterion value is maximized (or minimized) when compared to any other alternative in the set of all feasible alternatives. In multi-objective problems, the notion of an "optimum solution" does not usually exist in the context of conflicting, multiple objectives. In general, it is called a Pareto optimal solution if no other feasible solution exists that would decrease some objectives (suppose a minimization problem) without causing a simultaneous increase in at least one other objective (Coello C. A. C., 2006).

With this definition of optimality, we usually find several trade-off solutions (called the *Pareto optimal set* or *Pareto optimal front* (POF)). In that sense, the search for an optimal solution has fundamentally changed from what we see in the case of single-objective problems. The task of solving MOPs is called *multi-objective optimization*.

However, practically speaking, users need only one solution from the set of optimal trade-off solutions. Therefore, solving MOPs can be seen as the combination of both searching and decision-making (Horn, 1997). In order to support this, there are four main approaches in the literature (Miettinen, 1999).

1. No-preference - These methods solve a problem and give a solution directly to the decision maker without using preference information.
2. Decision making after search / Posteriori – These methods find all possible solutions of the non-dominated set and use the user preference to determine the most suitable one.
3. Decision making before search / Priori – These methods incorporate the use of preference before the optimization process, and thus will result in only one solution at the end. With this approach, the bias (from the user preference) is imposed all the time.

4. Decision making during search / Interactive – These methods are a hybridization of the second and third methods, in which human decision making is periodically used to refine the obtained trade-off solutions and thus to guide the search.

In general, the second method is preferred mostly within the research community since it is less subjective than the other two.

## 5.1. Concepts and Notations

This section will define some common concepts that have been widely used in the literature. Interested readers might refer to (Coello C. A. C., Lamont & Van Veldhuizen, 2007; Deb, 2001; Ehrgott, 2005) or (Miettinen, 1999) for a more detailed description.

Mathematically, in a *k*-objective optimization problem, a vector function $f(x)$ of *k* objectives is defined as:

$$f(x) = \left( f_1(x), f_2(x), ..., f_n(x) \right) \tag{5.1}$$

in which *x* is a vector of decision variables in the *n*-dimensional space $\mathbb{R}^n$; *n* and *k* are not necessarily the same. A solution is assigned a vector *x* and therefore the corresponding objective vector, *f*. Therefore, a general MOP is defined as follows:

$$\min \ f_i(x)\Big|_{x \in D} \tag{5.2}$$

where $i = 1, 2, ..., k$ and $D \in \mathbb{R}^n$, is called the *feasible search region*. All solutions (including optimal solutions) that belong to *D* are called *feasible solutions.*

In general, when dealing with MOPs, a solution $x_1$ is said to dominate solution $x_2$ if $x_1$ is better than $x_2$ when measured on all objectives. If $x_1$ does not dominate $x_2$ and $x_2$ also does not dominate $x_1$, they are said to be non-dominated. If we use $\prec$ between $x_1$ and $x_2$ as $x_1 \prec x_2$ to denote that $x_1$ dominates $x_2$ and $\lhd$ between two scalars *a* and *b*, as $a \lhd b$ to denote that *a* is better than *b* (similarly, $a \rhd b$ to denote that *a* is worse than *b*, and $a \not\rhd b$ to denote that *a* is not worse than *b*), then the dominance concept is formally defined as follows.

**Definition 5.1**: $x_1 \prec x_2$ if the following conditions are held:

1. $f_j(x_1) \not\rhd f_j(x_2) \quad \forall j \in \{1,2,...,k\}$

2. $\exists j \in \{1,2,...,k\}$ in which $f_j(x_1) \lhd f_j(x_2)$

The concept defined in definition 5.1 is sometimes referred to as *weak dominance*. For the *strict dominance* concept, solution $x_1$ must be strictly better than $x_2$ in all objectives. However, we follow the weak dominance concept as defined in definition 5.1.

Several optimization algorithms, mainly EAs, use a population of individuals during the optimization process. At the end, we usually have a set of individuals where no single individual dominates any other in the set. This set is an approximation of the real optimal solutions for the problem.

In general, if an individual in a population is not dominated by any other individual in the population, it is called a non-dominated individual. All non-dominated individuals in a population form the non-dominated set (as formally described in definition 5.2). Note that these definitions are equivalent to that from (Deb, 2001).

**Definition 5.2:** A set *S* is said to be the non-dominated set of a population *P* if the following conditions are held:

1. $S \subseteq P$

2. $\forall s \in S, \ \nexists x \in P : x \prec s$

When the set *P* represents the entire search space, the set of non-dominated solutions *S* is called the *global Pareto optimal set*. If *P* represents a subspace, *S* will be called the *local Pareto optimal set*. There is only one global Pareto optimal set, but there could be multiple local ones. However, in general, we simply refer to the global Pareto optimal set as the Pareto optimal set. Although there are several conditions established in the literature for optimality (Ehrgott, 2005; Miettinen, 1999), for practical black-box optimization problems, these conditions generally cannot be easily verified.

Finally, we define two special objective vectors (assuming that the problem is minimization) that are related to the Pareto optimal set (Ehrgott, 2005). For the sake of simplicity, these vectors are also called "solutions."

- **Ideal solution:** This represents the lower bound of each objective in the Pareto optimal set. It can be obtained by optimizing each objective individually in the entire feasible objective space.
- **Nadir solution:** This contains all the upper bounds of each objective in the Pareto optimal set. Obtaining the Nadir solution over the Pareto optimal set is not an easy task. One of the common approaches is to estimate the Nadir point by a pay-off table based on the Ideal solution.

## 5.2. Traditional Multi-Objective Algorithms

There are many traditional methods (the term "traditional" is used to differentiate such methods from evolutionary ones), such as the method of global criterion, weighted-sum (Cohon, 2004; Miettinen, 1999), $\epsilon$-constraint (Haimes, Lasdon & Wismer, 1971), weighted metric (Miettinen, 1999) and goal programming (Steuer, 1986). This section will only summarize several approaches that represent four different categories.

### 5.2.1. No-Preference Methods

In no-preference methods, in which user preference is not considered, the decision maker receives the solution of the optimization process, which he can either accept or reject. No-preference methods are suitable in the case that the decision maker does not have specific assumptions on the solution.

As an example we consider the *global criterion* method (Miettinen, 1999; Zeleny, 1982). The global criterion method transforms MOPs into single objective optimization problems by minimizing the distance between some reference points and the feasible objective region. In the simplest form, the reference point is the ideal solution and the problem is represented as follows:

$$\min\left(\sum_{i=1}^{k}\left|f_i(x)-z_i^*\right|^p\right)^{\frac{1}{p}} \tag{5.3}$$

where $z^*$ is the ideal vector, and $k$ is the number of objectives.

From the equation, one can see that the obtained solutions depend very much on the choice of the $p$'s value. Also, in the end the method will only give one solution to the decision maker.

### 5.2.2. Posteriori Methods

In posteriori methods, the decision maker is given a set of Pareto optimal solutions and the most suitable one is selected based on the decision maker's preference. Here, the two most popular approaches, weighted sum and $\epsilon$-constraint, are summarized.

In the weighted-sum method, all the objectives are combined into a single objective by using a weight vector. The problem in equation (5.2) is now transformed as in equation (5.4).

$$\min f(x) = w_1 f_1(x) + w_2 f_2(x) + ... + w_k f_k(x) \mid x \in D \tag{5.4}$$

where $i = 1, 2, \ldots, k$ and $D \in \mathbb{R}^n$.

The weight vector is usually normalized such that $\Sigma w_i = 1$.

Although the weighted-sum method is simple and easy to use, there are two inherent problems. First, there is the difficulty of selecting the weights in order to deal with scaling problems since the objectives usually have different magnitudes. Therefore, when combining them together, it is easy to cause biases when searching for tradeoff solutions. Secondly, the performance of the method is heavily dependent on the shape of the POF. Consequently, it cannot find all the optimal solutions for problems that have a non-convex POF.

To overcome the difficulty of non-convexity, the $\epsilon$-constraint method has been introduced, where only one objective is optimized while the others are transformed as constraints. The problem in equation (5.2) is now transformed as in equation (5.5). Again, the problem is now transformed into a single objective one.

$$\min f_j(x) \mid x \in D \tag{5.5}$$

subject to $f_j(x) \leq \epsilon_i$ where $i$=1,2, …,$k$, $i \neq j$ and $D \in \mathbb{R}^n$.

In this method, the $\epsilon$ vector is determined and uses the boundary (upper bound in the case of minimization) for all objectives $i$. For a given $\epsilon$ vector, this method will find an optimal solution by optimizing objective $j$. By changing $\epsilon$, we will obtain a set of optimal solutions. Although this method alleviates the difficulty of non-convexity, it still has to face the problem of selecting appropriate values for the $\epsilon$ vector, since it can happen that for a given $\epsilon$ vector, no feasible solution exists.

### 5.2.3. Priori Methods

In priori methods, the decision maker must indicate the assumption about the preferences before the optimization process. Therefore, the issue is how to quantify the preference and incorporate it into the problem before the optimization process.

One obvious method is the weighted-sum method, described in the previous section, where the weights can be used to represent the decision maker's preference.

We also consider the lexicographic ordering and goal programming (Fishburn, 1974; Ignizio, 1983; Miettinen, 1999) as examples of priori preference methods.

When using the lexicographic method, the decision maker is asked to arrange the objective functions by their importance. The optimization process is performed individually on each objective following the order of importance, when the result of each optimization process is used as constraints for the next process.

When using goal programming, aspiration levels of the objective functions have to be specified by the decision maker. Optimizing the objective function with an aspiration level is seen as a goal to be achieved. In its simplest and general form, goal programming can be stated as follows:

$$\min \sum_{i=1}^{k} \left| f_i(x) - z_i \right|^p \tag{5.6}$$

where $z$ is the vector indicating the aspiration levels.

### 5.2.4. Interactive Methods

This section on traditional methods is concluded by looking at the class of interactive methods, which allows the decision maker to interact with the optimization algorithm. In general, the interaction can be described step-by-step as follows (Miettinen, 1999):

1. Find an initial feasible solution
2. Interact with the decision maker, and
3. Obtain a new solution (or a set of new solutions). If the new solution (or one of them) or one of the previous solutions is acceptable to the decision maker, stop. Otherwise, go to (2).

As indicated in Miettinen (1999), using the interaction between the algorithm and the decision maker, , many weaknesses of the above approaches can be alleviated. To date, there are many approaches using an interactive style, namely, GDF (Geoffrion, Dyer & Feinberg, 1972), Tchebycheff method (Steuer, 1986), Reference point method (Wierzbiki, 1980), NIMBUS (Miettinen, 1994). Recently, interactive methods have also been incorporated with MOEAs (Abbass, 2006).

## 5.3. Multi-Objective Evolutionary Algorithms

### 5.3.1. Overview

Multi-objective evolutionary algorithms (MOEAs) are stochastic optimization techniques. Similar to other optimization algorithms, MOEAs are used to find Pareto optimal solutions for a particular problem, but differ by using a population-based approach. The majority of existing MOEAs employ the concept of dominance in their courses of action (however, see VEGA (Miettinen, 1999) for an example of not using a dominance relation); therefore, the focus here is on the class of dominance-based MOEAs.

The optimization mechanism of MOEAs is quite similar to that of EAs, except for the use of the dominance relation. In more detail, at each iteration, the objective values are calculated for every individual and are then used to determine the dominance relationships within the population, in order to select potentially better solutions for the

production of the offspring population. This population might be combined with the parent population to produce the population for the next generation. Further, the existence of the objective space might give MOEAs the flexibility to apply some conventional supportive techniques such as niching.

Generally, MOEAs have to deal with two major issues (Deb, 2001): (1) How to get close to the Pareto optimal front, which is not an easy task, because converging to the POF is a stochastic process. (2) The second is how to maintain the diversity of solutions in the obtained set. These two issues have become common criteria for most current algorithmic performance comparisons (Deb, Zitzler & Thiele, 2000). A diverse set of solutions will give more options for decision makers, designers and so forth. However, working on a set of solutions instead of only one, makes the measurement of the convergence of a MOEA harder, since the closeness of one individual to the optima does not act as a measure for the entire set.

To date, many MOEAs have been developed. Generally speaking, there are several ways to classify MOEAs. However, this chapter follows the one used by Coello C. A. C. (2006), where they are classified into two broad categories: Non-elitism and Elitism.

### 5.3.2. Non-Elitism Approach

In the non-elitism approach, best solutions of current population are not preserved when the next generation, based on the individuals of the current population, is created (Deb, 2001). Instead, selected individuals from the current generation are used to exclusively generate solutions for the next generation by crossover and mutation operators as in EAs. Coello C. A. C. (2006) refers to all algorithms using this approach as instances of the *first generation* of MOEAs which implies simplicity. The only difference from conventional EAs is that they use the dominance relation when assessing solutions. Instances of this category include MOGA (Fonseca C. M. & Fleming, 1993), NPGA (Horn, Nafpliotis & Goldberg, 1994) and NSGA (Deb, 2001).

Although MOEAs are different from each other, the common steps of these algorithms can be summarized as follows:

1. Initialize a population *P*

2. Select elitist solutions from *P* to create/update an external set *FP* (For non-elitism algorithms, *FP* is empty) (optional)
3. Create mating pool from one or both of *P* and *FP*
4. Perform reproduction based on the pool to create the next generation P
5. Possibly combine *FP* into *P*
6. Go to (2) if the termination condition is not satisfied.

Note that Steps (2) and (5) are used for elitism approaches that will be summarized in the next subsection.

### 5.3.3. Elitism Approach

Elitism is a mechanism to preserve the best individuals from generation to generation. In this way, the system never loses the best individuals found during the optimization process. Elitism was used at quite an early stage of evolutionary computing (De Jong, 1975); and to date, it has been used widely with EAs. Elitism can be done by placing one or more of the best individuals directly into the population for the next generations, or by comparing the offspring individual with its parents and then the offspring will only be considered if it is better than the parent (Storn & Price, 1995).

In the domain of evolutionary multi-objective optimization, elitist MOEAs usually (but not necessarily) employ an external set (the archive) to store the non-dominated solutions after each generation. In general, when using the archive, there are two important aspects, as follows:

1. **Interaction between the archive and the main population**: This is about how we use the archive during the optimization process; for example, one such way is to combine the archive with the current population to form the population for the next generation (Zitzler, Laumanns & Thiele, 2001).
2. **Updating the archive**: This is about the methodology to build the archive; one such method is by using the neighborhood relationship between individuals using crowded dominance (Deb et al., 2002), clustering (Zitzler et al., 2001), or geographical grid (Knowles J. D. & Corne, 2000), while another method is by controlling the size of the archive through truncation when the number of non-dominated individuals are over a predefined threshold.

Obviously, the current archive might then be a part of the next generation; however, the way to integrate this archive may be different from one algorithm to another. In general, with elitism, the best individuals in each generation are always preserved, and this helps the algorithms to get closer to the POF; a proof of convergence for MOEAs using elitism can be found in (Rudolph & Agapie, 2000). Algorithms such as PAES (Knowles J. D. & Corne, 2000), SPEA2 (Zitzler et al., 2001), PDE (Abbass, Sarker & Newton, 2001), NSGA-II (Deb et al., 2002) and MOPSO (Coello CAC, Pulido & Lechuga, 2004) are typical examples of this category.

## 5.3.4. Selected MOEAs

This section will summarize several approaches in the literature.

### 5.3.4.1 Non-Dominated Sorting Genetic Algorithms Version 2: NSGA-II

NSGA-II is an elitism algorithm (Deb, 2001; Deb et al., 2002). The main feature of NSGA-II lies in its elitism-preservation operation. Note that NSGA-II does not use an explicit archive; a population is used to store both elitist and non-elitist solutions for the next generation. However, for consistency, it is still considered as an archive. Firstly, the archive size is set equal to the initial population size. The current archive is then determined based on the combination of the current population and the previous archive. To do this, NSGA-II uses dominance ranking to classify the population into a number of layers, such that the first layer is the non-dominated set in the population, the second layer is the non-dominated set in the population with the first layer removed, the third layer is the non-dominated set in the population with the first and second layers removed and so on. The archive is created based on the order of ranking layers: the best rank being selected first. If the number of individuals in the archive is smaller than the population size, the next layer will be taken into account and so forth. If adding a layer makes the number of individuals in the archive exceed the initial population size, a truncation operator is applied to that layer using *crowding distance*.

The *crowding distance D* of a solution $x$ is calculated as follows: the population is sorted according to each objective to find adjacent solutions to $x$; boundary solutions are assigned infinite values; the average of the differences between the adjacent solutions in

each objective is calculated; the truncation operator removes the individual with the smallest *crowding distance*.

$$D(x) = \sum_{m=1}^{M} \frac{F_m^{I_x^m+1} - F_m^{I_x^m-1}}{F_m^{\max} - F_m^{\min}} \tag{5.7}$$

in which $F$ is the vector of objective values, and $I_x^m$ returns the sorted index of solution $x$, according to objective $m^{\text{th}}$.

An offspring population of the same size as the initial population is then created from the archive, by using crowded tournament selection, crossover, and mutation operators. Crowded tournament selection is a traditional tournament selection method, but when two solutions have the same rank, it uses the crowding distance to break the tie.

#### 5.3.4.2 A Pareto-Frontier Differential Evolution Algorithm for MOPs: PDE

This algorithm works as follows (Abbass et al., 2001): an initial population is generated at random from a Gaussian distribution with a predefined mean and standard deviation. All dominated solutions are removed from the population. The remaining non-dominated solutions are retained for reproduction. If the number of non-dominated solutions exceeds some threshold, a distance metric relation is used to remove those parents who are very close to each other. Three parents are selected at random. A child is generated from the three parents as in conventional single-objective Differential Evolution and is placed into the population if it dominates the first selected parent; otherwise a new selection process takes place. This process continues until the population is completed. A maximum number of non-dominated solutions in each generation was set to 50. If this maximum is exceeded, the following nearest neighbor distance function is adopted:

$$D(x) = \frac{\min\|x - x_i\| + \min\|x - x_j\|}{2} \tag{5.8}$$

where $x \neq x_i \neq x_j$. That is, the nearest neighbor distance is the average Euclidean distance between the closest two points. The non-dominated solution with the smallest neighbor distance is removed from the population until the total number of non-dominated solutions is retained at 50.

#### 5.3.4.3 Strength Pareto Evolutionary Algorithm: SPEA2

SPEA2 is actually an extension of an elitism MOEA called "The Strength Pareto Evolution Algorithm" - SPEA (Zitzler & Thiele, 1999). This section only concentrates on the main points of SPEA2 (Zitzler et al., 2001). The initial population, representation and evolutionary operators are standard: uniform distribution, binary representation, binary tournament selection, single-point crossover, and bit-flip mutation. However, the distinctive feature of SPEA2 lies in the elitism-preserved operation.

An external set (archive) is created for storing primarily non-dominated solutions. It is then combined with the current population to form the next archive that is then used to create offspring for the next generation. The size of the archive is fixed. It can be set to be equal to the population size. Therefore, two special situations exist when filling solutions in the archive:.If the number of non-dominated solutions is smaller than the archive size, other dominated solutions taken from the remainder part of the population are filled in. This selection is carried out according to a fitness value, specifically defined for SPEA. That is, the individual fitness value defined for a solution *x*, is the total of the SPEA-defined strengths of solutions which dominate *x*, plus a density value.

The second situation happens when the number of non-dominated solutions is over the archive size. In this case, a truncation operator is applied. For that operator, the solution which has the smallest distance to the other solutions will be removed from the set. If solutions have the same minimum distance, the second nearest distance will be considered, and so forth. This is called the *k-th nearest distance rule*.

#### 5.3.4.4 Pareto Archived Evolutionary Strategy: PAES

This algorithm uses an evolutionary strategy for solving multi-objective problems (Knowles J. D. & Corne, 2000). Therefore, it uses the mutation operator only, and the parental solutions are mutated to generate offspring. Similar to evolutionary strategies, it

also has different versions such as (1+1), (1+λ), or (μ, λ). The unique property of PAES is the way it uses and maintains elitism. We consider the case (1+1) as an example.

If the newly generated offspring dominates the parent, it replaces its parent. Conversely, if the parent dominates the offspring, it is discarded and new offspring will be generated. However, if both of them are non-dominated, there is a further mechanism to compare them (note that PAES also has an archive to store the non-dominated solutions over time). To do this, the offspring will be compared against all of the non-dominated solutions found so far in the archive. There will be several possible cases as follows:

- **Offspring is dominated by a member of the archive**: It is discarded and the parent is mutated again.
- **Offspring dominates some members of the archive**: These members are deleted from the archive and the offspring is included into the archive. It also will be a parent in the next generation.
- **Offspring is non-dominated with all members of the archive**: Offspring will be considered to be included into the archive depending on the current size of the archive. Note that the parent is also a non-dominated solution and belongs to the archive. Therefore, it is necessary to calculate the density in the areas of both solutions in order to decide which one will be the parent of the next generation. For this, a hyper-grid is built in the area of the objective occupied by the archive, where all solutions in the archive will belong to different hyper-cells of the grid depending on their locations. Thus, the offspring is selected if its cell is less crowded than that of the parent.

To keep the size of the archive always below its limit, PAES also uses a density measure. The solution associated with the highest-density cell will be replaced by the newcomer (the offspring).

#### 5.3.4.5 Multi-Objective Particle Swarm Optimizer: MOPSO

This is an MOEA which incorporates Pareto dominance into a particle swarm optimization algorithm in order to allow the PSO algorithm to handle problems with several objective functions (Coello CAC et al., 2004). In PSO, a population of solutions (particles) are used without either crossover or mutation operators. Each solution is

assigned a velocity and uses this velocity to make a move in the search space. The determination of the velocity of a particle is dependent on both the best position the particle has achieved (the local best) and the best position the population has found so far (the global best). Applying PSO to multi-objective optimization relies very much on how to define the local and global best positions.

MOPSO keeps tracking the local best for every solution over time. In order to find the global best position for each solution, MOPSO uses an external archive (secondary repository) of particles to store all non-dominated particles. Each particle will be assigned to a selected one in the archive (as the global best). The selection of a particle in the archive is dependent on the density of the areas surrounding the particle. Further, the archive is updated continuously and its size is controlled by using the grid technique proposed in PAES where a hyper-grid is built in the area of the objective occupied by the archive, and all solutions in the archive will belong to different hyper-cells of the grid depending on their locations.

### 5.3.5. Performance Assessments

Performance metrics are usually used to compare algorithms in order to form an understanding of which one is better and in what aspects. However, it is hard to define a concise definition of algorithmic performance. In general, when doing comparisons, a number of criteria are employed (Zitzler, Deb & Thiele, 2000):

- Closeness of the obtained non-dominated set to the Pareto optimal front.
- A good (in most cases, uniform) distribution of solutions within the set.
- Spread of the obtained non-dominated front, that is, for each objective, a wide range of values should be covered by the non-dominated solutions.

Based on these criteria, the community of evolutionary multi-objective optimization has developed a number of performance metrics. Recently, there have been a number of works to develop platforms for performance assessments including the most popular metrics such as the PISA system (Bleuler, Laumanns, Thiele & Zitzler, 2003). This section will provide a summary of the most popular of these metrics.

#### 5.3.5.1 Metric Evaluation Closeness to the POF

The first obvious metric is the error rate, *ER*, introduced by Veldhuizen (1999). It is calculated by the percentage of solutions that are not in the POF:

$$ER = \frac{\sum_{i=1}^{N} e_i}{N} \tag{5.9}$$

where $N$ is the size of the obtained set and $e_i = 1$ if the solution $i$ is not in the POF, otherwise $e_i = 0$. The smaller the *ER*, the better the convergence to the POF. However, this metric does not work in the case when all the solutions of two compared sets are not in the POFs. In this case, a threshold is employed, such that if the distance from a solution $i$ to the POF is greater than the threshold, $e_i = 1$, otherwise $e_i = 0$.

The second metric is the generation distance, *GD*, which is the average distance from the set of solutions found by evolution to the POF (Veldhuizen, 1999)

$$GD = \frac{\sqrt{\sum_{i=1}^{N} d_i^{\,2}}}{N} \tag{5.10}$$

where $d_i$ is the Euclidean distance (in objective space) from solution $i$ to the nearest solution in the POF. If there is a large fluctuation in the distance values, it is also necessary to calculate the variance of the metric. Finally, the objective values should be normalized before calculating the distance.

#### 5.3.5.2 Metric Evaluating Diversity among Obtained Non-Dominated Solutions

The spread metric is also animport performance comparison. One of its instances is introduced by Schott (1995), called the *spacing* method.

$$S = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\bar{d} - d_i\right)^2}}{\bar{d}} \tag{5.11}$$

where

$$d_i = \min_{j=1,\ldots,N} \sum_{m=1}^{M} \left| f_m^i - f_m^j \right| \tag{5.12}$$

and $f_m$ is the $m^{th}$ objective function. $N$ is the population size and $M$ is the number of objectives. The interpretation of this metric is that the smaller the value of $S$, the better the distribution in the set. For some problems, this metric might be correlated with the number of obtained solutions. In general, this metric focuses on the distribution of the Pareto optimal set, not the extent of the spread.

Deb et al. (2002) proposed another method to alleviate the problem of the above spacing method. The spread of a set of non-dominated solutions is calculated as follows:

$$\Delta = \frac{\sum_{i=1}^{M} d_i^e + \sum_{i=1}^{N} \left| d_i - \bar{d} \right|}{\sum_{i=1}^{M} d_i^e + N\bar{d}} \tag{5.13}$$

where $d_i$ can be any distance measure between neighboring solutions and $\bar{d}$ is the mean value of these distances. $d_i^e$ is the distance between extreme solutions of the obtained non-dominated set and the true Pareto optimal set. $\Delta$ ranges from 0 to 1. If it is close to 1, the spread is bad.

#### 5.3.5.3 Metric Evaluation: both Closeness and Diversity

All the metrics discussed in the previous section focus on a single criterion only. This section summarizes two metrics that take into account both closeness and diversity. The first one is the hyper-volume ratio (Zitzler & Thiele, 1999), one of the most widely accepted by the research community of MOEAs. To calculate the hyper-volume, an area of objective space covered by the obtained POF is measured, called the hyper-area.

Calculating the hyper-volume is a time consuming process, although recently several attempts have been made to speed up this process (While, Bradstreet, Barone & Hingston, 2005; While, Hingston, Barone & Huband, 2006). In general, for two sets of solutions, whichever has the greater value of hyper-volume will be the best. However, when using hyper-volume it is sometimes difficult to understand the quality of the obtained POF in comparison with the true POF.

As recommended by Coello C. A. C. (2006) and Veldhuizen (1999), it is considered better to use the hyper-volume ratio (HR) that is measured by the ratio between the hyper-volumes of hyper-areas covered by the obtained POF and the true POF, called $H_1$ and $H_2$ respectively. *HR* is calculated as in equation (5.14). For this metric, the greater the value of *HR*, the better the convergence the algorithm provides.

$$HR = \frac{H_1}{H_2} \tag{5.14}$$

There are some questions on how to determine the reference point for the calculation of the hyper-volume. For example, it can be the origin (Veldhuizen, 1999). However, generally it is dependent on the area of the objective space that is visited by all comparing algorithms. In this revised version, as suggested elsewhere (Deb, 2001), the reference point is the one associated with all the worst values of objectives found by all the algorithms under investigation.

The second metric uses a statistical comparison method. It was first introduced by Fonseca C. and Fleming (1996). For experiments of MOEAs which generate a large set of solutions, this metric is often the most suitable, as their data can easily be assessed by statistical methods. Knowles J. D. and Corne (2000) modified this metric and instead of drawing parallel lines, all lines originate from the origin. The basic idea is as follows: suppose that two algorithms ($A_1$, $A_2$) result in two non-dominated sets: $P_1$ and $P_2$. The lines that join the solutions in $P_1$ and $P_2$ are called attainment surfaces. The comparison is carried out in the objective space. In order to do the comparison, a number of lines are drawn from the origin (assuming a minimization problem), such that they intersect with the surfaces. The comparison is then individually done for each sampling line to determine which one outperforms the other. Each intersection line will then yield a

number of intersection points. In this case, statistical tests are necessary to determine the percentage an algorithm outperformed the other in each section. For both of these methods, the final results are two numbers that show the percentage of the space where each algorithm outperforms the other.

### 5.3.6. Statistical Testing

Since MOEAs (and EAs in general) are stochastic, we cannot rely on the results obtained from only one run tested on a particular problem. Therefore, it is necessary that every algorithm involved in the comparison be tested on the problem for a number of independent runs (equivalent to using different random seeds). In general, all algorithms were usually tested for a number of runs. By applying the aforementioned metrics (except the one using attainment surfaces), at the end, a set of numerical values was obtained for each algorithm. All comparisons will be done on these sets. From the statistical point of view, there are a number of concepts that can be used to compare the sets, including the mean, standard deviation, and median. However, the confidence on using these concepts in comparison is questionable. In general, the final decision on the performance of algorithms will be made after completing statistical testing.

## 5.4. Summary

Real-world problems often have multiple conflicting objectives. In single objective problems, an optimum solution is a solution for which the criterion value is maximized (or minimized) when compared to any other alternative in the set of all feasible alternatives. In multi-objective problems, the notion of an “optimum solution” does not usually exist in the context of conflicting, multiple objectives. In general, it is called a Pareto optimal solution if no other feasible solution exists which would decrease some objectives (suppose a minimization problem) without causing a simultaneous increase in at least one other objective.

Several traditional multi-objective optimization algorithms have been reviewed, such as the global criterion method, which transforms MOPs into single objective optimization problems by minimizing the distance between some reference points and the feasible objective region, or the weighted-sum method, in which all the objectives are combined into a single objective by using a weight vector.

Multi-objective Evolutionary algorithms have also been reviewed. Some of the evolutionary algorithms discussed use a non-elitism approach, such as MOGA, NPGA and NSGA. In a non-elitism approach the best solutions of the current population are not preserved when the next generation is created. Other evolutionary algorithms do use an elitisim approach, such as the NSGA2, SPEA2, PAES and others.

Finally, performance assessment methods have been discussed as well as statistical testing.

# 6. Evolutionary Algorithms for Solving Real-Time Multi-Objective Vehicle Routing Problems

## 6.1. Evolutionary Algorithms

Evolutionary Algorithms belong to the Evolutionary Computation field of study concerned with computational methods inspired by the process and mechanisms of biological evolution. The process of evolution by means of natural selection (descent with modification) was proposed by Darwin to account for the variety of life and its suitability (adaptive fit) for its environment. The mechanisms of evolution describe how evolution actually takes place through the modification and propagation of genetic material (proteins). Evolutionary Algorithms are concerned with investigating computational systems that resemble simplified versions of the processes and mechanisms of evolution, toward achieving the effects of these processes and mechanisms, namely the development of adaptive systems. Additional subject areas that fall within the realm of Evolutionary Computation are algorithms that seek to exploit the properties from the related fields of Population Genetics, Population Ecology, Coevolutionary Biology, and Developmental Biology.

Evolutionary Algorithms share properties of adaptation through an iterative process that accumulates and amplifies beneficial variation through trial and error. Candidate solutions represent members of a virtual population striving to survive in an environment defined by a problem specific objective function. In each case, the evolutionary process refines the adaptive fit of the population of candidate solutions in the environment, typically using surrogates for the mechanisms of evolution such as genetic recombination and mutation.

There are many excellent texts on the theory of evolution, although Darwin's original source can be an interesting and surprisingly enjoyable read (Darwin, 1859). Huxley's book defined the modern synthesis in evolutionary biology that combined Darwin's natural selection with Mendel's genetic mechanisms (Huxley, 1942), although any good textbook on evolution will suffice. Popular science books on evolution are an easy place to start, such as Dawkins' "The Selfish Gene" that presents a gene-centric perspective on

evolution (Dawkins, 2006), and Dennett's "Darwin's Dangerous Idea" that considers the algorithmic properties of the process (Dennett, 1996).

## 6.1.1. Genetic Algorithms

### 6.1.1.1 Introduction

Genetic Algorithms (Mitchell, 1996; Sivanandam & Deepa, 2007) are a family of computational models inspired by evolution. These algorithms encode a potential solution to a specific problem on a simple chromosome-like data structure and apply recombination operators to these structures in order to preserve critical information.

An implementation of a genetic algorithm begins with a population of (typically random) chromosomes. One then evaluates these structures and allocated reproductive opportunities in such a way that these chromosomes which represent a better solution to the target problem are given more chances to 'reproduce' than those chromosomes which are poorer solutions. The 'goodness' of a solution is typically defined with respect to the current population.

### 6.1.1.2 Biological Background

Genetic Algorithms (GA) search by simulating evolution, starting from an initial set of solutions, and generating successive "generations" of solutions. Genetic Algorithms are inspired by the way living things evolved into more successful organisms in nature. The main idea is survival of the fittest, a.k.a. natural selection.

A chromosome is a long, complicated thread of DNA (deoxyribonucleic acid). Hereditary factors that determine particular traits of an individual are strung along the length of these chromosomes, like beads on a necklace. Each trait is coded by some combination of nucleotides (A (Adenine), C (Cytosine), T (Thymine) and G (Guanine)). Like an alphabet in a language, meaningful combinations of the nucleotides produce specific instructions to the cell.

Changes occur during reproduction. The chromosomes from the parents exchange information randomly by a process called crossover. Therefore, the offsprings exhibit some traits of the father and some traits of the mother. A rarer process called mutation also changes some traits. Sometimes an error may occur during copying of chromosomes

(mitosis). As an example, the parent cell may have A-C-G-C-T but an accident may occur and changes the new cell to A-C-T-C-T. Usually this results in a nonsensical sequence of nonsensical and the cell does not survive. But over millions of years, sometimes the accidental mistake produces a meaningful sequence of nonsensical, thus producing a better species.

In nature, the individual that has better survival traits will survive for a longer period of time. This in turn provides it a better chance to produce offspring with its genetic material. Therefore, after a long period of time, the entire population will consist of lots of genes from the superior individuals and less from the inferior individuals. In a sense, the fittest survived and the unfit died out. This force of nature is called natural selection.

#### 6.1.1.3 Genetic Algorithms

The major steps of genetic algorithms are the generation of a population of solutions, finding the objective function and fitness function and the application of genetic operators. These aspects are described next in this section. The working principle of a traditional GA is as follows:

1. Set $Population = \varnothing$.
2. Add *PopulationSize* randomly created feasible individuals to *Population*.
3. While stop condition is not met do
    a. Evaluate the fitness value of each individual in *Population*.
    b. Set $NewPopulation = \varnothing$.
    c. While the size of *NewPopulation* is less than *PopulationSize* do
        i. Select *Parent1* and *Parent2* from *Population* based on the fitness values of each individual.
        ii. Apply crossover operation, with probability $p_C$, on *Parent1* and *Parent2* to create *Child1* and *Child2*.
        iii. Apply mutation operation, with probability $p_M$, on *Child1*.
        iv. Apply mutation operation, with probability $p_M$, on *Child*2.
        v. Add *Child1* and *Child2* to *NewPopulation*.
    d. Replace *Population* with NewPopulation.

An important characteristic of genetic algorithm is the coding of variables that describethe problem. The most common coding method is to transform the variables to a binary string or vector; GAs perform best when solution vectors are binary. If the problem has more than one variable, a multi-variable coding is constructed by concatenating as many single variables coding as the number of variables in the problem. Genetic Algorithm processes a number of solutions simultaneously. Hence, in the first step a population having *P* individuals is generated by pseudo-random generators whose individuals represent a feasible solution. This is a representation of a solution vector in a solution space and is called initial solution. This ensures that the search is robust and unbiased, as it starts from a wide range of points in the solution space.

In the next step, individual members of the population are evaluated to find the objective function value. In this step, the exterior penalty function method is utilized to transform a constrained optimization problem to an unconstrained one. This is exclusively problem specific. In the third step, the objective function is mapped into a fitness function that computes a fitness value for each member of the population. This is followed by the application of GA operators.

There are three main operators: reproduction, crossover and mutation to create a new population. The purpose of these operators is to create new solutions by selection, combination or alteration of the current solutions that have shown to be good temporary solutions. The new population is further evaluated and tested until termination. If the termination criterion is not met, the population is iteratively operated by the above three operators and evaluated. This procedure is continued until the termination criterion is met. One cycle of these operations and the subsequent evaluation procedure is known as a generation in GAs terminology.

Reproduction (or selection) is an operator that makes more copies of better solutions in a new population. Reproduction is usually the first operator applied on a population. Reproduction selects good solutions in a population and forms a mating pool. This is one of the reasons that the reproduction operation is sometimes known as the selection operator. Thus, in the reproduction operation the process of natural selection causes those individuals that encode successful structures to produce copies more frequently. To sustain the generation of a new population, the reproduction of the individuals in the

current population is necessary. For better individuals, these should come from the fittest individuals of the previous population. There are a number of reproduction operators in GA literature, but the essential idea in all of them is that the above average solutions are picked from the current population, and their multiple copies are inserted in the mating pool in a probabilistic manner.

The commonly-used reproduction operator is the proportionate reproduction operator (Roulette-Wheel selection), where a solution is selected for the mating pool with a probability proportional to its fitness. Thus, the $i^{th}$ solution in the population is selected with a probability proportional to *Fi*. Since the population size is usually kept fixed in a simple GA, the sum of the probability of each solution being selected for the mating pools must be one. Therefore, the probability for selecting the $i^{th}$ string is

$$p_i = \frac{F_i}{\sum_{j=1}^{n} F_j} \tag{6.1}$$

where *n* is the population size.

A crossover operator is used to recombine two solutions to get a better solution. In a crossover operation, the recombination process creates different individuals in the successive generations by combining material from two individuals from the previous generation. In reproduction, good solutions in a population are probabilistically assigned a larger number of copies and a mating pool is formed. It is important to note that no new solutions are formed in the reproduction phase. In the crossover operator, new solutions are created by exchanging information among solutions of the mating pool.

The two solutions participating in the crossover operation are known as parent solutions, and the resulting solutions are known as children solutions. Children solutions produced by the crossover may or may not be better than the parent solutions, but this is not a matter of serious concern, because if good solutions are created by crossover, there will be more copies of them in the next mating pool generated by crossover. It is clear from this discussion that the effect of crossover may be detrimental or beneficial. Thus, in order to preserve some of the good solutions that are already present in the mating pool, all solutions in the mating pool are not used in crossover. When a crossover probability,

defined here as $p_C$ is used, only $100p_C$ per cent solutions in the population are used in the crossover operation and $100(1-p_C)$ per cent of the population remains as they are in the current population.

Many crossover operators exist in the GA literature. One site crossover and two site crossover are the most common ones adopted. As noted before, solutions are usually encoded using a string of binary digits. In most crossover operators, two strings (solutions) are picked from the mating pool at random and some portion of the strings are exchanged between the strings.

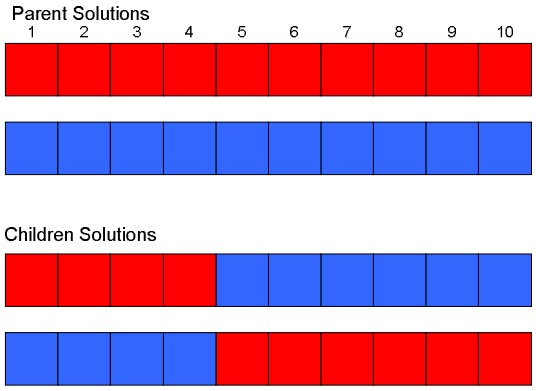



**Figure 6.1 - One site crossover**

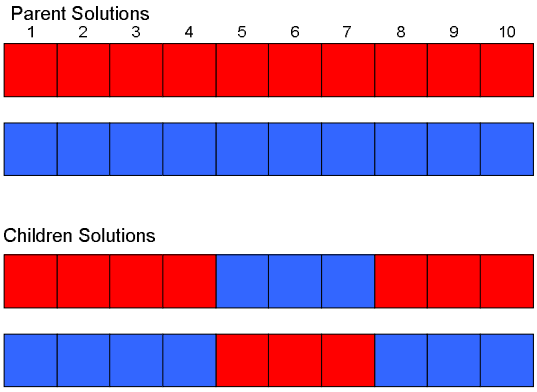



**Figure 6.2 – Two site crossover operation**

In the one site crossover, a crossover site is selected randomly. The portions on the right of the selected site of these two strings are exchanged to form a new pair of strings. The

new strings are thus a combination of the old strings (Figure 6.1). Two site crossover is a variation of the one site crossover, except that two crossover sites are chosen and the bits between the sites are exchanged as shown in Figure 6.2.

One site crossover is more suitable when string length is small, while two site crossover is suitable for large strings. Hence, the present work adopts a two site crossover. The underlying objective of crossover is to exchange information between strings to get a string that is possibly better than the parents.

Mutation adds new information in a random way to the genetic search process and ultimately helps to avoid getting trapped at local optima. It is an operator that introduces diversity in the population whenever the population tends to become homogeneous due to repeated use of reproduction and crossover operators. Mutation may cause the chromosomes of individuals to be different from those of their parent individuals.

In a sense, mutation is the process of randomly disturbing genetic information. They operate at the bit level; when the bits are being copied from the current string to the new string, there is a probability that each bit may become mutated. This probability is usually quite a small value, referred to as mutation probability $p_M$. A coin toss mechanism is employed; if a random number between zero and one is less than the mutation probability, then the bit is inverted, so that zero becomes one and one becomes zero. This helps in introducing a bit of diversity to the population by scattering the occasional points. This random scattering could result in a better optima, or even modify a part of the genetic code that would be beneficial in later operations. On the other hand, it might produce a weak individual that will never be selected for further operations.

These three operators are simple and straightforward. The reproduction operator selects good solutions and the crossover operator recombines them, to create better solutions. The mutation operator alters a solution locally expecting a better solution. Even though none of these claims are guaranteed and/or tested while creating a solution, it is expected that if bad solutions are created they will be eliminated by the reproduction operator in the next generation and if good solutions are created, they will be increasingly emphasized.

Application of these operators on the current population creates a new population. This new population is used to generate subsequent populations and so on, yielding solutions

that are closer to the optimum solution. The values of the objective function of the individuals of the new population are again determined by decoding the strings. These values express the fitness of the solutions of the new generations. This completes one cycle of a genetic algorithm called a generation. In each generation if the solution is improved, it is stored as the best solution. This is repeated until convergence.

Problems with multiple objectives arise in a natural fashion in most disciplines, and their solution has long been a challenge to researchers. Despite the considerable variety of techniques developed in Operations Research (OR) and other disciplines to tackle these problems, the complexities of their solution calls for alternative approaches.

The use of evolutionary algorithms (EAs) to solve problems of this nature has been motivated mainly because of the population-based nature of EAs which allows the generation of several elements of the Pareto optimal set in a single run (Coello C. A. C. et al., 2007). Additionally, the complexity of some multi-objective optimization problems (MOPs) (e.g., very large search spaces, uncertainty, noise, disjoint Pareto curves, etc.) may prevent use (or application) of traditional OR MOP-solution techniques.

In this study, two multi-objective Genetic algorithms are used, VEGA and SPEA2.

#### 6.1.1.4 VEGA

The Vector Evaluated Genetic Algorithm (VEGA proposed by David Schaffer (Schaffer J. D., 1985; Schaffer & Grefenstette, 1985), is normally considered the first implementation of a multi-objective evolutionary algorithm (MOEA). The vector is by definition the vector of *k* objective functions of the MOP. The VEGA approach is an example of a criterion or objective selection technique where a fraction of each succeeding population is selected based on separate objective performance. The specific objectives for each fraction are randomly selected at each generation. VEGA tends to converge to solutions close to local optima with regard to each individual objective.

The VEGA concept is that, for a problem with *NumObj* objectives, *NumObj* sub-populations of size *PopSize*/*NumObj* each would be generated (assuming a total population size of *PopSize*). Each sub-population uses only one of the *NumObj* objective functions for fitness assignment. The proportionate selection operator is used to generate the mating pool. These sub-populations are then shuffled together to obtain a new

population of size *PopSize*, on which the GA would apply the crossover and mutation operators in the usual way. Shuffling is done prior to sub-population partitioning in order to reduce positional population bias. This process is illustrated in Algorithm 1. The complexity of VEGA is clearly the same as the single-objective GA.

1. Set $Population = \varnothing$.
2. Add *PopSize* randomly created feasible individuals to *Population*.
3. While stop condition is not met do
   a. For each individual $i \in Population$, evaluate $f_{ik}$, which is the fitness value of individual *i* in regard to objective function *k*, for all $k \in NumObj$, where *NumObj* is the number of objective functions.
   b. Set $MatingPool = \varnothing$.
   c. While the size of *MatingPool* is less than *PopSize* do
      i. Set *k*=1.
      ii. Select $PopSize/NumObj$ individuals from *Population*, based on the fitness value of each individual calculated for objective function *k*, $f_{ik}$, and add them to *MatingPool*.
      iii. Increase *k* by 1.
   d. Shuffle the *MatingPool*.
   e. Set $NewPopulation = \varnothing$.
   f. While the size of *NewPopulation* is less than *PopSize* do
      i. Select *Parent1* and *Parent2* from *MatingPool*.
      ii. Apply crossover operation, with probability $p_C$, on *Parent1* and *Parent2* to create *Child1* and *Child2*.
      iii. Apply mutation operation, with probability $p_M$, on *Child1*.
      iv. Apply mutation operation, with probability $p_M$, on *Child*2.
      v. Add *Child1* and *Child2* to *NewPopulation*.
   g. Replace *Population* with *NewPopulation*.
4. The result of the algorithm is the set of all non-dominated solutions in *Population*.

**Algorithm 1 - VEGA algorithm**

Schaffer realized that the solutions generated by VEGA were non-dominated in a local sense, because their non-dominance was limited to the current population. And while a locally dominated individual is also globally dominated, the converse is not necessarily true (Schaffer J. D., 1985). An individual that is not dominated in one generation may become dominated by an individual who emerges in a later generation. Also, Schaffer noted a problem that in genetics is known as "speciation" (i.e., one could have the evolution of "species" within the population which excel on different aspects of performance). This problem arises because this technique selects individuals that excel in one dimension of performance, without considering other dimensions. The potential danger is that one could have individuals with what Schaffer called "middling" performance in all dimensions, which could be very useful for compromise solutions, but that would not survive under this selection scheme, since they are not at the extreme for any dimension of performance (i.e., they do not produce the best value for any objective function, but only moderately good values for all of them). Speciation is undesirable because it is opposed to our goal of finding a compromise solution. Schaffer suggested some heuristics to deal with this problem. For example, one could use a heuristic selection preference approach for non-dominated individuals in each generation, to protect the "middling" chromosomes. Also, crossbreeding among the "species" could be encouraged by adding some mate selection heuristics instead of using the random mate selection of the traditional GA (i.e., the use of mating restrictions). In accordance with the discussion, VEGA uses a localized criterion for ranking as depicted in Figure 6.3.

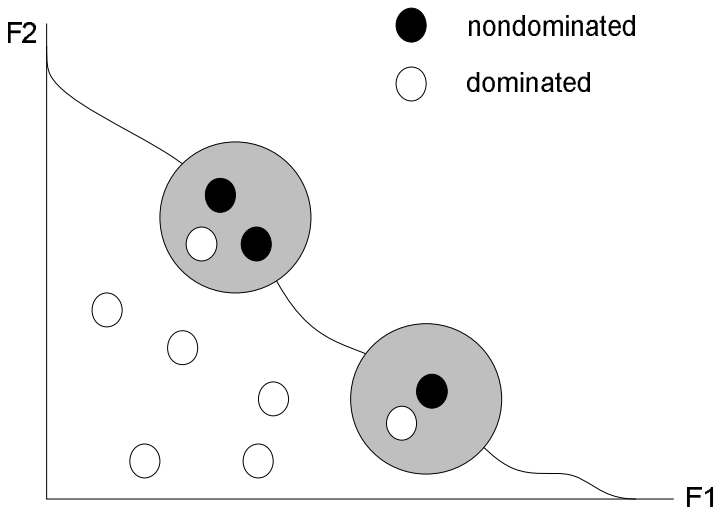



**Figure 6.3 - VEGA's criterion-based ranking mechanism**

Norris and Crossley (1998) and Crossley, Cook, Fanjoy and Venkayya (1999) believe this technique reduces the diversity of any given $PF_{current}(t)$. They implemented elitist selection to ensure $PF_{known}(t)$ endpoints (or in other words, $PF_{known}(t)$'s extrema) survive between generations. Otherwise, the MOEA converges to a single design rather than maintaining a number of alternatives. In other attempts to preserve diversity in $PF_{current}(t)$ they also employ a VEGA variant. Here, "*k*"-branch tournaments (where *k* is the number of MOP objectives) allow each solution to compete once in each of *k* tournaments, where each set of tournaments selects $\frac{1}{k}^{th}$ of the next population (Khuri, Bäck & Heitkötter, 1994).

*Criticism of criterion selection techniques* - VEGA is very simple and easy to implement, since only the selection mechanism of a traditional GA has to be modified. One of its main advantages is that despite its simplicity, this sort of approach can generate several solutions in one run of the MOEA. However, note that the shuffling and merging of all the sub-populations that VEGA performs corresponds to averaging the fitness components associated with each of the objectives (Knowles J. & Corne, 2002). Since Schaffer uses proportional fitness assignment (Goldberg, 1989), these fitness components are in turn proportional to the objectives themselves (Fonseca C.M. & Fleming, 1995). Therefore, the resulting expected fitness corresponds to a linear

combination of the objectives where the weights depend on the distribution of the population at each generation as shown by Richardson, Palmer, Liepins and Hilliard (1989). This means that VEGA has the same problems as the aggregating approaches previously discussed (i.e., it is not able to generate concave portions of the Pareto front). Nevertheless, VEGA has been found useful in other domains such as constraint-handling, where its biased behavior can be of great help (Coello C., Aguirre & Buckles, 2000; Coello Coello & Aguirre, 2002; Surry, Radcliffe & Boyd, 1995). Note that these algorithmic developments were in part based upon consideration of the computational hardware performance at the time. Other variations and extensions of the VEGA concept included the Vector Optimized Evolution Strategy (VOES) by Kursawe (1991). His approach was based on an evolution strategy along with a fitness evaluation process similar to VEGA. It also employed a diploid chromosome scheme with preservation of non-dominated solutions using an elitist approach. The WBGA (weight-based genetic algorithm) proposed by Hajela and Lin (1992) is related to VEGA's sampling approach, but it uses a set of weights (each individual is assigned a vector containing such weights). These vectors remain diverse across the population through niching and appropriately selected sub-populations that are evaluated for different objectives in a way analogous to VEGA. Again, this MOEA is simple, but the use of weighted vectors has the same disadvantages as the independent sampling approach.

**An Improved VEGA Algorithm**

Elitism guarantees that the best solutions found in each iteration are passed on to the next iteration and not lost. The original VEGA algorithm does not use elitism. Conventionally, elitism is achieved by simply copying the solutions directly into the new generation; Next, an extended version of the VEGA algorithm, which uses elitism, is presented. In this version of the algorithm, the set of non-dominated chromosomes is passed on to the next generation.

In order to describe how the elitism, or the preservation of high performance solutions, is done in the enhanced VEGA algorithm, the concepts of dominated and non-dominated solution have to be defined first. In single objective optimization problems, the "best" solution is defined in terms of an "optimum solution" for which the objective function

value is optimized when compared to any other alternative in the set of all feasible alternatives. In multi-objective optimization problems, however, the notion of an "optimum solution" does not usually exist, since the optimum of each criterion does not usually point to the same alternative. The optimal solution in a multi-objective optimization problem is usually equivalent to choosing the best compromise solution. In the absence of an optimal solution, the concepts of dominated and non-dominated solutions become relevant.

A feasible solution, $x_1$, dominates another feasible solution, $x_2$, if $x_1$ is at least as good as $x_2$ with respect to all objective functions and is better than $x_2$ with respect to at least one objective function. A *non-dominated solution* is a feasible solution that is not dominated by any other feasible solution. Hence the solution of a multi-objective problem is a set of non-dominated feasible solutions.

Using the definition above, the set of high performance solutions can be defined as the set of non-dominated solutions obtained in all iterations of the algorithm. This set of non-dominated solutions, denoted as *E*, can be obtained if, in each iteration, any newly obtained solution is added to the set *E* if it is not dominated by any solution already in *E*. Moreover, if a newly obtained solution should be added to the set *E*, then any solution already in *E* that is dominated by the newly obtained solution is removed from *E*. After the last iteration, the result of the algorithm is the set *E*, which is the set of non-dominated solutions obtained in all of the algorithm's iterations.

The process of the improved VEGA algorithm is illustrated in Algorithm 2.

1. Set $Population = \varnothing$.
2. Set $E = \varnothing$.
3. Add *PopSize* randomly created feasible individuals to *Population*.
4. Add all dominated solution in *Population* into *E*.
5. While stop condition is not met do
    a. For each individual $i \in Population$, evaluate $f_{ik}$, which is the fitness value of individual *i* in regard to objective function *k*, for all $k \in NumObj$, where *NumObj* is the number of objective functions.
    b. Set $MatingPool = \varnothing$.

c. While the size of *MatingPool* is less than *PopSize* do
   i. Set *k*=1.
   ii. Select $^{PopSize}/_{NumObj}$ individuals from *Population*, based on the fitness value of each individual calculated for objective function *k*, $f_{ik}$, and add them to *MatingPool*.
   iii. Increase *k* by 1.
d. Shuffle the *MatingPool*.
e. Set $NewPopulation = \varnothing$.
f. While the size of *NewPopulation* is less than *PopSize* do
   i. Select *Parent1* and *Parent2* from *MatingPool*.
   ii. Apply crossover operation, with probability $p_C$, on *Parent1* and *Parent2* to create *Child1* and *Child2*.
   iii. Apply mutation operation, with probability $p_M$, on *Child1*.
   iv. Apply mutation operation, with probability $p_M$, on *Child*2.
   v. Add *Child1* and *Child2* to *NewPopulation*.
g. Replace *Population* with *NewPopulation*.
h. Set $\overline{E} = \varnothing$.
i. Add all non dominated solution in $Population \cup E$ into $\overline{E}$.
j. Replace $E$ with $\overline{E}$.

6. The result of the algorithm is the set of all non-dominated solution *E*.

**Algorithm 2 - Improved VEGA algorithm**

#### 6.1.1.5 Strength Pareto Evolutionary Algorithm: SPEA2

The Strength Pareto Evolutionary Algorithm (SPEA) was introduced by Zitzler and Thiele (1999). This approach was conceived as a way of integrating different MOEAs. SPEA uses an external archive containing non-dominated solutions previously found (the so-called external non-dominated set). At each generation, non-dominated individuals are copied to the external non-dominated set. For each individual in this external set, a

strength value is computed. This strength is similar to the ranking value of MOGA (Fonseca C. M. & Fleming, 1993), since it is proportional to the number of solutions where a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external non-dominated solutions that dominate it. The fitness assignment process of SPEA considers both closeness to the true Pareto front and even distribution of solutions at the same time. Thus, instead of using niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external non-dominated set. In fact, since the external non-dominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, the authors decided to adopt a technique that prunes the contents of the external non-dominated set so that its size remains below a certain threshold. The approach adopted for this sake was a clustering technique called average linkage method (Morse, 1980).

A revised version of SPEA, called SPEA2, has three main differences with respect to its predecessor (Zitzler et al., 2001): (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals which it dominates; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

The following pseudo code describes how SPEA2 works (Zitzler et al., 2001).

1. Set $P_0 = \varnothing$, where $P_0$ denotes the population set at generation 0.
2. Set $\overline{P}_0 = \varnothing$, where $\overline{P}$ denotes an archive (external set).
3. Set $t$=0.
4. Add *PopSize* randomly created feasible individuals to $P_0$.
5. For each individual $i \in \left(P_t \cup \overline{P}_t\right)$, evaluate its fitness value.
6. Set $\overline{P}_{t+1} = \varnothing$.

7. Copy all non-dominated individuals from $P_t \cup \bar{P}_t$ to $\bar{P}_{t+1}$. If $\left|\bar{P}_{t+1}\right| > ArchiveSize$ then reduce $\bar{P}_{t+1}$ by means of the truncation operator, otherwise if $\left|\bar{P}_{t+1}\right| < ArchiveSize$ then fill $\bar{P}_{t+1}$ with dominated individuals in $P_t \cup \bar{P}_t$
8. If stopping condition is met then the result of the algorithm is the set of all non-dominated individuals in $\bar{P}_{t+1}$. Stop.
9. Perform binary tournament selection with replacement on $\bar{P}_{t+1}$ in order to fill the mating pool.
10. Apply recombination and mutation operators to the mating pool and set $P_{t+1}$ to the resulting population.
11. Set $t=t+1$
12. Go to Step 5.

**Algorithm 3 – SPEA Algorithm**

In step 5 of the SPEA2 pseudo code, the fitness value of each individual in $P_t \cup \bar{P}_t$ is evaluated. To avoid the situation where individuals dominated by the same archive members have identical fitness values, with SPEA2 for each individual both dominating and dominated solutions are taken into account. In detail, each individual *i* in the archive $\bar{P}_t$ and the population $P_t$ is assigned a strength value $S(i)$, representing the number of solutions it dominates:

$$S(i) = \left|\left\{ j : j \in P_t + \bar{P}_t \wedge i \succ j \right\}\right| \tag{6.2}$$

where $|\cdot|$ denotes the cardinality of a set, + stands for multi-set union and the symbol $\succ$ corresponds to the Pareto dominance relation. On the basis of the *S* values, the raw fitness $R(i)$ of an individual *i* is calculated:

$$R(i) = \sum_{j \in P_t \cup \bar{P}_t,\, j \succ i} S(j) \tag{6.3}$$

In other words, the raw fitness is determined by the strengths of its dominators in both archive and population, as opposed to SPEA where only archive members are considered in this context. It is important to note that fitness is to be minimized here, i.e., $R(i) = 0$ corresponds to a non-dominated individual, while a high $R(i)$ value means that $i$ is dominated by many individuals (which in turn dominate many individuals).

Although the raw fitness assignment provides a sort of niching mechanism based on the concept of Pareto dominance, it may fail when most individuals do not dominate each other. Therefore, additional density information is incorporated to discriminate between individuals having identical raw fitness values. The density estimation technique used in SPEA2 is an adaptation of the $k^{\text{th}}$ nearest neighbor method (Silverman, 1986), where the density at any point is a (decreasing) function of the distance to the $k^{\text{th}}$ nearest data point. Here, we simply take the inverse of the distance to the $k^{\text{th}}$ nearest neighbor as the density estimate. To be more precise, for each individual $i$ the distances (in objective space) to all individuals $j$ in archive and population are calculated and stored in a list. After sorting the list in increasing order, the $k^{\text{th}}$ element gives the distance sought, denoted as $\sigma_i^k$. As a common setting, we use $k$ equal to the square root of the sample size (Silverman, 1986), thus, $k = \sqrt{N + \bar{N}}$ . Afterwards, the density $D(i)$ corresponding to $i$ is defined by

$$d(i) = \frac{1}{\sigma_i^k + 2} \tag{6.4}$$

In the denominator, two is added to ensure that its value is greater than zero and that $D(i) < 1$. Finally, adding $D(i)$ to the raw fitness value $R(i)$ of an individual $i$ yields its fitness $F(i)$:

$$F(i) = R(i) + D(i) \tag{6.5}$$

The archive update operation (Step 7 in the algorithm's pseudo code) in SPEA2 differs from the one in SPEA in two respects: (1) the number of individuals contained in the archive is constant over time, and (2) the truncation method prevents boundary solutions from being removed.

During environmental selection, the first step is to copy all non-dominated individuals, i.e., those which have a fitness lower than one, from archive and population to the archive of the next generation:

$$\overline{P}_{t+1} = \left\{ i : i \in P_t \cup \overline{P}_t \wedge F\left(i\right) < 1 \right\} \tag{6.6}$$

If the non-dominated front fits exactly into the archive ($\left|\overline{P}_{t+1}\right| = \overline{N}$) the environmental selection step is completed. Otherwise, there can be two situations: Either the archive is too small ($\left|\overline{P}_{t+1}\right| < \overline{N}$) or too large ($\left|\overline{P}_{t+1}\right| > \overline{N}$). In the first case, the best $\overline{N} - \left|\overline{P}_{t+1}\right|$ dominated individuals in the previous archive and population are copied to the new archive. This can be implemented by sorting the multi-set $P_t \cup \overline{P}_t$ according to the fitness values and copy the first $\overline{N} - \left|\overline{P}_{t+1}\right|$ individuals *i* with $F\left(i\right) \geq 0$ from the resulting ordered list to $\overline{P}_{t+1}$. In the second case, when the size of the current non-dominated (multi)set exceeds $\overline{N}$, an archive truncation procedure is invoked which iteratively removes individuals from $\overline{P}_{t+1}$ until j $\left|\overline{P}_{t+1}\right| = \overline{N}$. Here, at each iteration that individual *i* is chosen for removal for which $i \leq_d j$ for all $j \in \overline{P}_{t+1}$ with

$$i \leq_d j \quad :\Leftrightarrow \quad \begin{array}{l} \forall 0 < k < \left|\overline{P}_{t+1}\right| : \sigma_i^k = \sigma_j^k \vee \\ \exists 0 < k < \left|\overline{P}_{t+1}\right| : \left[ \left( \forall 0 < l < k : \sigma_i^l = \sigma_j^l \right) \wedge \sigma_i^k < \sigma_j^k \right] \end{array} \tag{6.7}$$

where $\sigma_i^k$ denotes the distance of *i* to its $k^{\text{th}}$ nearest neighbor in $\overline{P}_{t+1}$. In other words, the individual which has the minimum distance to another individual is chosen at each stage;

if there are several individuals with minimum distance, the tie is broken by considering the second smallest distances and so forth.

## 6.1.2. Artificial Bee Colony

The artificial bee colony (ABC) algorithm proposed by Karaboga Dervis (2005) and later modified by Karaboga D. and Akay (2011) is a new evolutionary meta-heuristic technique inspired by the intelligent behavior of natural honey bees in their search for nectar.

### 6.1.2.1 Biological Background

A bee colony can be thought of as a swarm whose individual agents are bees. Each bee at the low-level component works through a swarm at the global level of component to form a system. Thus, the system’s global behavior is determined from its individual's local behavior, where the different interactions and coordination among individuals leads to an organized teamwork system. This system is characterized by nteracting collective behavior through labor division, distributed simultaneous task performance, specialized individuals, and self- organization.

The exchange of information among bees leads to the formation of a tuned collective knowledge. A colony of honey bees consists of a queen, many drones (males) and thousands of workers (non-reproductive females). The queen's job is to lay eggs and to start new colonies. The sole function of the drones is to mate with the queen and during the fall they are ejected from the colony. The worker bees build honeycomb, and the young bees clean the colony, feed the queen and drones, guard the colony, and collect food.

As nectar is the bees' energy source, two kinds of worker bees are responsible for food. These are scout bees and forager bees. A bee does many things in its life history, and does not become a scout/work bee until late in its life.

While scout bees carry out the exploration process of the search space, forager bees control the exploitation process. However, exploration and exploitation processes must be carried out together by the colony’s explorers and the colony’s exploiters. As the increase

in the number of scouts encourages the exploration process, the increase of foragers encourages the exploitation process.

Studying the foraging behavior leads to optimal foraging theory that directs activities towards achieving goals. This theory states that organisms forage in such a way as to maximize their intake energy per unit of time. In other words, the swarm of bees behaves in such a way as to find and capture the food containing the most energy while expending the least possible amount of time in real variables. There are two forms of scenarios for any bee in the forging process, either scout or forager. The following subsections present these two scenarios:

**The Behavior of Scouts Scenario**

Scouts fly around and search for food. When they find a source of nectar or pollen, they fly back to the colony and start dancing to communicate with other bees on a particular region in the comb.

Hence the behavior of the scout scenario is summarized according to the following activities:

The scout flies from its colony searching for food sources in a random way. Once it finishes a full trip, it returns back to its colony. When a scout arrives at the colony, it goes inside and announces its presence by the wing vibrations. This means that it has a message to communicate.

If it has found a nearby source of nectar or pollen, it performs a circular dance. The nearby bees follow it through this circular dance and smell it for the identity of the flowers. They listen to the intensity of the wing vibrations to indicate the value of the food source.

If the source is very close, no directions are given. Alternatively, if the flower source is far away, careful directions must be given.

The abstract convention that the scout makes is that the up position on the comb is the position of the sun. Because bees can see polarized light, they can tell sun position without actually seeing the sun. The scout dances in a precise angle from the vertical. This equals to the horizontal angle of the sun with reference to the colony exit with the location of the food source.

Next, the scout bee must tell the other bees how far away the flower source is. This is done by waggling the abdomen from side to side. The slower the waggling, the further away is the distance of the food flower.

Thus, the dance of scouts points to the direction, distance, and quality of food source. Since various groups of scouting bees compete with each other, the decision is finally made in favor of the best domicile.

**The Behavior of Foragers Scenario**

The reaction of the forager bees to this show concludes in the following steps:

The bees in the colony closely follow the scout to learn their directions, and also learn the odor of the flower on the scout bee, so they can find the flower when they arrive at the source location.

Because the sun is moving in the sky, the bees should use an accurate clock sense to adjust for the changing sun position with reference to the food source and the colony exit.

Even more remarkable, if a trained bee is removed from the colony to another location where the flower is not visible, but the colony is, the bee does not return to the colony to get its bearing, but reads sun position, triangulates, and flies directly to the flower. Subsequently, the forager bees take a load of nectar from the source and return to the colony and unload the nectar to the store of food.

Foraging requires energy and the honeybee's evaluation as to where, what, and how long to forage are all related to the economics of energy consumption and the net gain of food to the colony.

Generally bees fly only as far as necessary to secure an acceptable food source from which there is a net-gain. Therefore, these are the factors that influence foraging behavior and determine profitability. The net rate of energy intake is defined as the energy gained while foraging minus the energy spent on foraging, divided by time spent foraging.

#### 6.1.2.2 Artificial Bee Colony Algorithm

In the ABC algorithm, the colony of artificial bees consists of three groups of bees: (1) employed bees - bees that are currently exploiting a food source; (2) onlookers - bees that

are waiting in the hive for the employed bees to share information about the food sources; and (3) scouts - bees that are searching for new food sources in the neighborhood of the hive.

The ABC algorithm is an iterative algorithm. It starts by assigning each employed bee to a randomly generated solution (known as a food source). Next, in each iteration, each employed bee, using a neighborhood operator, finds a new food source near its assigned food source. The nectar amount (defined as a fitness function) of the new food source is then evaluated. If the amount of nectar in the new food source is higher than the amount of nectar in the old one, then the older source is replaced by the newer one. Next, the nectar information of the food sources is shared with the onlookers (real bees do this by dancing in the dance area inside the hive). The onlooker chooses a food source according to the probability proportional to the quality of that food source. Roulette wheel selection is the usual method. Therefore, good food sources, as opposed to bad ones, attract more onlooker bees. Subsequently, using a neighborhood operator, each onlooker finds a food source near its selected food source and calculates its nectar amount. Then, for each old food source, the best food source among all the food sources near the old one is determined. The employed bee associated with the old food source is assigned to the best food source and abandons the old one if the best food source is better than the old food source. A food source is also abandoned by an employed bee if the quality of the food source has not improved in the course of a predetermined and limited number of successive iterations. The employed bees then become scouts and randomly search for new food source. After a scout finds a new food source, it becomes an employed bee again. After each employed bee is assigned to a food source, another iteration of the ABC algorithm begins. The iterative process is repeated until a stopping condition is met.

Szeto, Wu and Ho (2011) describe the steps of the ABC algorithm as follows:

1. Randomly generate a set *S* of *i* solutions as initial food sources, where *i* is the number of employed bees. Assign an employed bee to each food source.
2. Evaluate the nectar amount (fitness), $f(s_i)$, of each food source, for each objective function *j*.
3. Repeat until a stopping condition is met:

a. For each food source $s_i \in S$, apply the neighborhood operator to generate a neighbor food source, $\overline{s_i}$. If the fitness of the neighbor food source is better than that of the original food source, i.e., $f(s_i) > f(\overline{s_i})$ for maximization problems, then replace the original food source with this neighbor food source.
b. Set $G_0 = \varnothing, G_1 = \varnothing, ... G_i = \varnothing,$ where *i* is the number of employed bees.
c. For each onlooker, use the fitness-based roulette wheel selection method to select a food source, $s_i$. Apply a neighborhood operator to $s_i$ to find a neighbor food source, say $\overline{s_i}$. Add $\overline{s_i}$ to $G_i$, i.e. $G_i = G_i \cup \{\overline{s_i}\}$.
d. For each food source $s_i \in S$, if $G_i \neq \varnothing$ then let $\overline{s}$ be the source food with best fitness value in $G_i$. If the fitness of $\overline{s}$ is better than that of $s_i$, then replace $s_i$ with $\overline{s}$.
e. Replace any food sources $s_i \in S$ whose fitness has not been improved for *limit* iterations with randomly generated solutions.

4. Output the best food source (solution) found (meaning the set *E*).

**Algorithm 4 - ABC Algorithm**

## Vector Evaluated Artificial Bee Colony Algorithm

Since ABC algorithms share common characteristics with GAs, simple modifications made to the basic GAs can be adopted and applied to ABC algorithms in order to solve multi-objective Real-Time VRPs. The vector evaluated genetic algorithm (VEGA) proposed by Schaffer J. (1985) is an example of such modification that can easily be applied to ABC algorithms.

Assuming *NumObj* represents the number of objective functions and *NumEmpBees* represents the number of employed bees, and based on the structure of the ABC algorithm, the vector evaluated technique and the use of elitism, which is the process of preserving previous high performance solutions from one generation to the next, the new combined VE-ABC algorithm that we propose is defined as follows:

1. Set $E = \varnothing$.
2. Randomly generate a set *S* of *i* solutions as initial food sources, where *i* is the number of employed bees. Assign an employed bee to each food source.
3. Evaluate the nectar amount (fitness), $f_j(s_i)$, of each food source, for each objective function *j*.
4. For each $s \in S$, if *s* is a non-dominated solution add *s* to *E*.
5. Repeat until a stopping condition is met
   a. For each food source $s_i \in S$, apply the neighborhood operator to generate a neighbor food source, $\overline{s}_i$. If the fitness of the neighbor food source is better than that of the original food source, based on objective function *j*, i.e. $f_j(s_i) > f_j(\overline{s}_i)$ for maximization problems, when $j = \left\lfloor \frac{i}{\left\lfloor \frac{NumEmpBees}{NumObj} \right\rfloor} \right\rfloor \bmod NumObj$, then replace the original food source with this neighbor food source.
   b. Set $G_0 = \varnothing, G_1 = \varnothing, \ldots G_i = \varnothing,$ where *i* is the number of employed bees.
   c. For each onlooker, use the fitness-based roulette wheel selection method to select a food source, $s_i$, using objective function *j*, where $j = \left\lfloor \frac{i}{\left\lfloor \frac{NumEmpBees}{NumObj} \right\rfloor} \right\rfloor \bmod NumObj$. Apply a neighborhood operator to $s_i$ to find a neighbor food source, say $\overline{s}_i$. Add $\overline{s}_i$ to $G_i$, i.e. $G_i = G_i \cup \{\overline{s}_i\}$.
   d. For each food source $s_i \in S$, if $G_i \neq \varnothing$ then let $\overline{s}$ be the source food with best fitness value in $G_i$, when the fitness is evaluated regarding objective *j*, when $j = \left\lfloor \frac{i}{\left\lfloor \frac{NumEmpBees}{NumObj} \right\rfloor} \right\rfloor$. If the fitness of $\overline{s}$ is better than that of $s_i$, then replace $s_i$ with $\overline{s}$.

e. For each $s \in S$, if *s* is not dominated by any solution $e \in E$, add *s* to *E* and check each solution $e \in E$. If *e* is dominated by *s*, remove *e* from *E*.

f. Replace any food sources $s_i \in S$ whose fitness has not been improved for *limit* iterations with randomly generated solutions.

2. Output the best food source (solution) found (meaning the set *E*).

## 6.2. Representation and Genetic Operations

### 6.2.1. Representation

The first step in designing an Evolutionary Algorithm (EA) for a particular problem is to devise a suitable representation scheme. This is very important because the rest of the algorithm depends on this representation. Traditionally, solutions are represented by a simple binary string. This simple representation is not appropriate for the VRP. During the last few years several representations have been considered in connection with the VRP. A very popular, if not the most popular, representation method for the VRPs (or TSPs) that are solved by EAs is permutation representation. The permutation representation is easy to understand and to represent a simple TSP tour for a single vehicle. The problem we want to solve in this research is a real-time multi-vehicle problem, and therefore, the permutation representation method needs some modifications.

A candidate solution to an instance of the VRP must specify the number of vehicles required, the partition of the demands through all these vehicles, the delivery order for each route as well as waiting time at each customer. Let a node object define an object that has two properties, customer number and waiting time at customer. A solution to the multi-objective real-time VRPs can be encoded using an array of node objects, and based on the permutation representation. A solution contains several routes, each one of them composed by an ordered subset of the costumers. All demands belonging to the problem being solved must be present in one of the routes (Pereira, Tavares, Machado & Costa, 2002; Szeto et al., 2011). As an example, consider a VRP in which 10 customers have to be served from a central depot. A possible solution to the problem is presented in Figure 6.4.

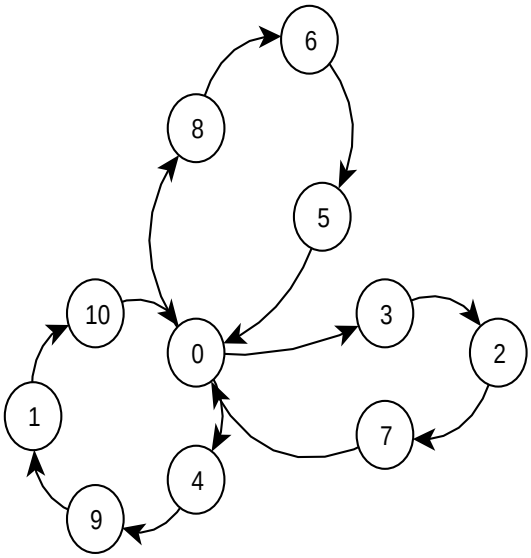



**Figure 6.4 - A possible solution to VRP with 10 customers**

As stated earlier, the solution presented in Figure 6.4 can be encoded using an array of node objects. The solution from

Figure 6.5 represents the encoding of the solution presented in Figure 6.4.

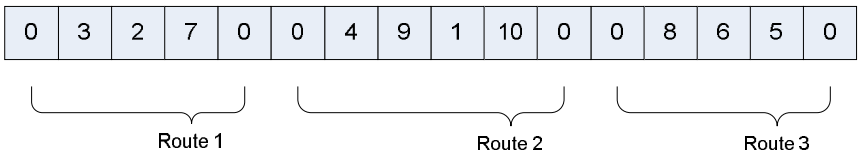



**Figure 6.5 – Representation of a solution to VRP with 10 customers**

As seen in the above example, ten customers have to be served from a central depot. This is done using three vehicles, each assigned to a different route. The first route starts at the depot, and visits customers 3, 2 and 7 in that same order. The first route ends at the depot. The second route starts at the depot, and visits customers 4, 9, 1 and 10 in that same order and ends at the depot. The third and last route starts at the depot, visits customers 8, 6 and 6 in that same order and ends at the depot. Since all routes end at the depot, it is possible to define a solution using an array of integers in the following way:

Let *S*[*i*] denote the node object stored at index *i* of the node objects array, and let *S*[*i*].*CustNum* denote the value of the customer number property belonging to node object *S*[*i*].

*S*[1].*CustNum*, the value of the customer number proper that belongs to the first node object of the array, points to the start location of the first route (it also contains information about the waiting time at the same location, *S*[1].*WaitTime* (not shown in

Figure 6.5)). A start location with value of zero means that the route starts at the depot. The value of *S*[2].*CustNum* is the first customer of the first route. *S*[3].*CustNum* is the second customer in the route and so on. The route continues until it reaches an index, *i*, for which *S*[*i*].*CustNum* is zero. Since all routes end at the depot, this means that after visiting some customers, the route ends at the depot. The value of *S*[*i*+1].*CustNum* is the start location of the next route, which is described in the same way.

In real-time problems, routes have to reflect the status of vehicles which are not always located at the depot. If, at calculation time, a vehicle is positioned at a customer, then there should be a corresponding route in the array, which starts with the same customer. In the same way, if, at calculation time, a vehicle is driving from customer one to customer two, then there should be a corresponding route in the array, which starts with customer one, and whose second location is customer two.

## 6.2.2. Genetic Operations

Crossover and mutation are the genetic operators used in the general GAs. In ABCs only neighborhood operators, which are equivalent to GA's mutation operators, are used. Solutions used in a specific problem have their own characteristics, and some particular crossover operators are needed. We use crossover and mutation for the real-time multi-objective VRP. All offspring created after three genetic operators are tested for feasibility. If the offspring do not satisfy feasibility, they need the repairing steps and then they are included in the sampling space.

### 6.2.2.1 Crossover

Based on the idea that the exchange of information between good chromosomes will generate even better offspring, the crossover operator combines information, or sub-

solutions, from two solutions to create a better solution. As mentioned earlier, children solutions, produced by the crossover, may or may not be better than the parents solutions, but this is not a matter of serious concern, because if good solutions are created by crossover, there will be more copies of them in the next mating pool generated by crossover. Since, from the above, the effect of crossover may be detrimental or beneficial, in order to preserve some of the good solutions that are already present in the mating pool, none of the solutions in the mating pool are used in crossover. When a crossover probability, defined here as $p_C$ is used, only $100\,p_C$ per cent solutions in the population are used in the crossover operation and $100(1-p_C)$ per cent of the population remains as they are in the current population.

Many crossover operators exist in the GA literature. One site crossover and two site crossovers are the most common ones adopted.

In a one site crossover, two parent solutions, $Parent_1$ and $Parent_2$, are picked from the mating pool. Assuming that the solutions are encoded using a bit-string of size $N$, a single crossover point, $C$, is randomly selected. Two new children solutions are not built based on the parents solutions in the following ways: Bits 1 to $C$, of $Child_1$ bit-string encoding, are equal to bits 1 to $C$ of $Parent_1$ bit-string encoding. Bits $C$+1 to $N$, of $Child_1$ bit-string encoding, are equal to bits $C$+1 to $N$ of $Parent_2$ bit-string encoding. Similarly, bits 1 to $C$, of $Child_2$ bit-string encoding, are equal to bits 1 to $C$ of $Parent_2$ bit-string encoding and bits $C$+1 to $N$, are equal to bits $C$+1 to $N$ of $Parent_1$ bit-string encoding.

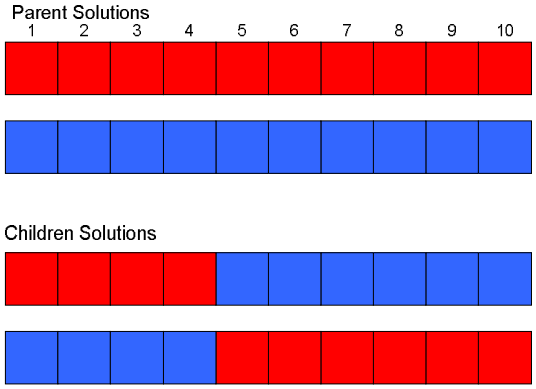



**Figure 6.6 - One site crossover**

In a two sites crossover, two parent solutions, $Parent_1$ and $Parent_2$, are picked from the mating pool. Two crossover points¸ $C_1$ and $C_2$, are randomly selected. Two new children solutions are not built based on the parents solutions in the following ways: Bits 1 to $C_1$, of $Child_1$ bit-string encoding, are equal to bits 1 to $C_1$ of $Parent_1$ bit-string encoding. Bits $C_1$+1 to $C_2$, of $Child_1$ bit-string encoding, are equal to bits $C_1$+1 to $C_2$ of $Parent_2$ bit-string encoding. Bits $C_2$+1 to $N$, of $Child_1$ bit-string encoding, are equal to bits $C_2$+1 to $N$ of $Parent_1$ bit-string encoding. Similarly, bits 1 to $C_1$, of $Child_2$ bit-string encoding, are equal to bits 1 to $C_1$ of $Parent_2$ bit-string encoding, bits $C_1$+1 to $C_2$, of $Child_2$ bit-string encoding, are equal to bits $C_1$+1 to $C_2$ of $Parent_1$ bit-string encoding and bits $C_2$+1 to $N$, are equal to bits $C_2$+1 to $N$ of $Parent_2$ bit-string encoding.

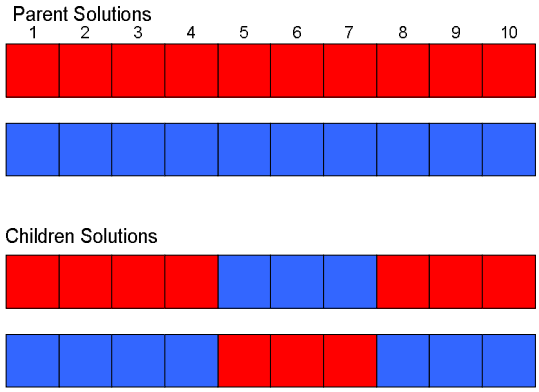



**Figure 6.7 – Two site crossover operation**

The crossover operator used in our approach does not promote a mutual exchange of genetic material between two parents. Instead, when submitted to this kind of operation, one individual receives a fragment of genetic material (more precisely, a sub-route) from another parent and inserts it in one of its own routes. The donor is not modified. The geographical location of the costumers is used to determine the position where the sub-route is inserted. The following algorithm clarifies how crossover is applied to the individuals of the selected set:

1. Get a sub-sequent from 2$^{nd}$ chromosome (sub-sequent must begin with start-route and end with end-route)

2. remove all customers found in the sub-sequent from the first chromosome
3. insert the sub-sequent at the end of the 1st chromosome

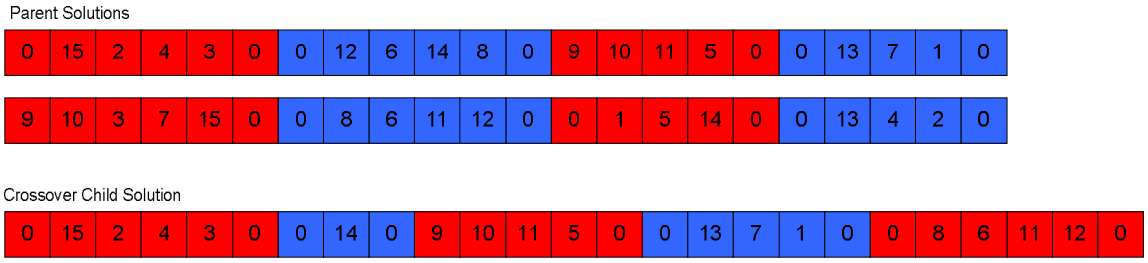



**Figure 6.8 – Crossover operation**

The example from Figure 6.8 helps to illustrate how crossover acts. In the example two parent solutions, $Parent_1$ and $Parent_2$, were chosen. In the example, both $Parent_1$ and $Parent_2$ represent solutions with four routes. The first route of $Parent_1$ is 0-15-2-4-3-0, the second route is 0-12-16-14-8-0, the third route is 9-10-11-5-0 and the fourth and last route is 0-13-7-1-0. Similarly, the first route of $Parent_2$ is 9-10-3-7-15-0, the second route is 0-8-6-11-12-0, the third route is 0-1-5-14-0 and the fourth and last route is 0-13-4-2-0. In order to create the child solution (the result of the crossover operation), we first make a copy of $Parent_1$. Second, a route is randomly selected from $Parent_2$. While in the example, one route, 0-8-6-11-12-0, was selected, the crossover allows for more than one route to be selected, as long as they are sequential routes. The next step is to remove all customers found in the selected route(s) from the copy of $Parent_1$. This operation may result in empty routes in the copy of $Parent_1$, so a cleanup procedure has to be applied on it. Next, a route in the copy of $Parent_1$ is randomly selected, and the selected route(s) from $Parent_2$ are inserted before or after it. In the example, the randomly selected route from the copy of $Parent_1$ is the last route, and the select route from $Parent_2$ is inserted after it, meaning, added to the end of the solution. The result of the last operation is the result of the crossover operation, the child route.

Since each parent solution represents a feasible solution, applying the described crossover operation results in a new feasible solution.

#### 6.2.2.2 Mutation

Mutation adds new information in a random way to a given solution and ultimately helps to avoid getting trapped at local optima. It is an operator that introduces diversity in the population whenever the population tends to become homogeneous due to repeated use of reproduction and crossover operators.

In the multi-objective real-time VRPs, a mutation operation applied to a given solution can result in one of the following: (1) Change the order of customers in a route encoded by the solution. (2) Reduce the number of routes encoded by the solution, by merging two routes together; or (3) Increase the number of routes encoded by the solution by splitting a single route into two routes.

##### 6.2.2.2.1 Merge Routes operation

The merge operation, takes two routes from the solution and merges them into one new route. This operation is used to reduce the number of vehicles used by the solution.

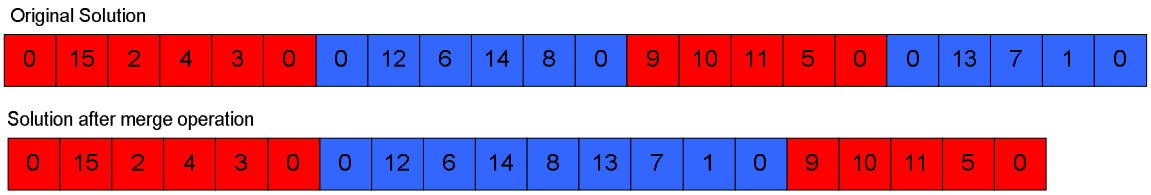



**Figure 6.9 - An example of merge route operation**

A candidate solution to an instance of the VRP is encoded using an array of node objects (node object is an object that has two properties, customer number and waiting time at customer), and based on the permutation representation.

A route is a sequence of node objects in the array of node objects. The first node in the route corresponds to the first location of the route (which can be either the depot, denoted as 0, or a customer). The last node of the route must be a depot, since all routes end at the depot.

In the example illustrated in Figure 6.9, two routes were randomly selected, 0-12-6-14-8-0 and 0-13-7-1-0. Next, the two routes are merged into a single route, 0-12-6-14-8-13-7-1-0. This is done by removing the end location of the first route and the start location of

the second route, which in both cases is the depot, and concatenating the second route to the first route. Next, the second route is removed from the solution's array of node objects, and the first route in the solution's array of objects is replaced with the new merged route.

The merge route operation can be applied to two routes, only if the first location of the second route is the depot, and there is no truck driving from the deport to the second location of the route. Otherwise, the solution will not reflect real-life information.

#### 6.2.2.2.2 Swap operation

A candidate solution to an instance of the VRP, is encoded using an array of node objects (node object is an object that has two properties, customer number and waiting time at customer), and based on the permutation representation. The swap operation randomly selects two node objects from the nodes objects array, and swaps them. The swap operation is used to change the order of customers in a route. It can also decrease the number of routes in the solution, as illustrated later in this chapter.

An example of the swap operation is illustrated in Figure 6.10.

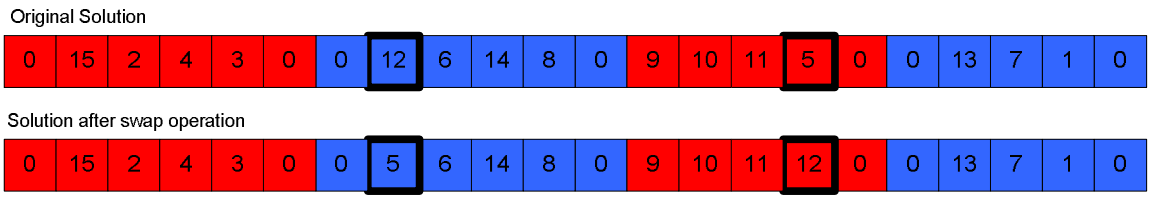



**Figure 6.10 - An example of a swap operation**

The solution shown in Figure 6.10 contains four routes. Two of the routes are 0-12-6-14-8-0 and 9-10-11-5-0. Two node objects are randomly selected, the first node object corresponds to customer 12, and the seconds corresponds to customer 5. As stated before, the swap operation interchanges the two randomly selected node objects, so the result is two new routes, 0-5-6-14-8-0 and 9-10-11-12-0.

An example of the swap operation is illustrated in Figure 6.10.

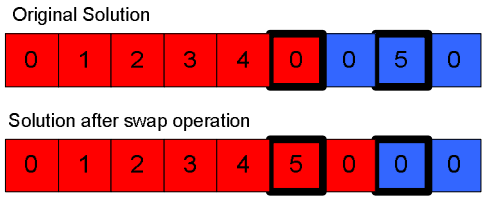



**Figure 6.11 - An example of a swap operation that decreases the number of routes**

The solution shown in Figure 6.11 contains two routes, 0-1-2-3-4-0 and 0-5-0. Two node objects are randomly selected, the first node object corresponds to the last location in the first route (the depot), and the seconds corresponds to customer 5. After swapping the two randomly selected node objects, the result is two new routes, 0-1-2-3-4-5-0 and 0-0. The second route, 0-0, is an empty route, since there are no customers to visit in the route. Therefore, this route should be removed from the solution, and we are left with only one route, instead of two.

If by swapping two node objects, the new solution does not reflect real-life information (a vehicle driving from customer *i* to customer *j* is no longer present in the solution), the swapping is not allowed.

#### 6.2.2.2.3 Split Routes operation

The split operation, takes one route from the solution and splits it into two new routes. This operation is used to increase the number of vehicles used by the solution.

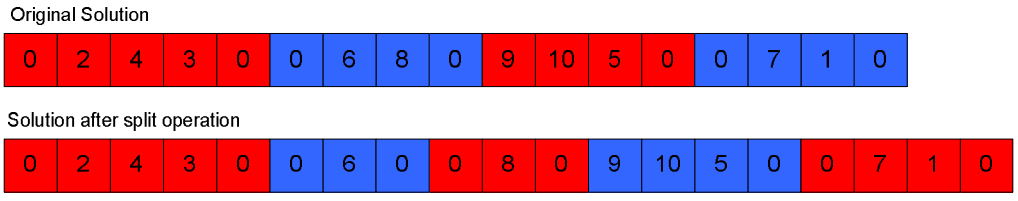



**Figure 6.12 - An example of split route operation**

Since a candidate solution to an instance of the VRP is encoded using an array of node objects (node object is an object that has two properties, customer number and waiting time at customer), splitting a route into two routes is a simple operation – randomly select

a node object from the node objects array, and insert two nodes that represent depots after it.

In the example illustrated in Figure 6.12, there are four routes, 0-2-4-3-0, 0-6-8-0, 9-10-5-0 and 0-7-1-0. A node object has to be randomly selected; in the example the selected node object corresponds to customer 6. Two node objects, each represents a depot, are inserted after the selected node object. The result is two near routes, 0-6-0 and 0-8-0.

If the selected node object represents the depot, the result of the split operation may be an empty route, which will have to be removed from the solution.

Mutation may cause a vehicle to exceed its capacity. When this happens, and to guarantee that the interpretation yields a valid solution, we split the route that exceeds capacity into several ones. An example illustrates this adjustment: assume that the original route 0-1-2-3-4-5-6-0 causes the vehicle to exceed its capacity at node 4. When this situation occurs, the itinerary is divided in two sections: 0-1-2-3-0 and 0-4-5-6-0, and a new vehicle is added to the solution. If necessary, further divisions can be made in the second section. Notice that these changes only occur at the interpretation level, and therefore the information codified in the chromosome is not altered.

#### 6.2.2.2.4 Change Wait Time operation

The last operation described is the change wait time operation. The change wait time operation changes the waiting time for a randomly chosen customer, to a random value in the range of 0 to *T*, where *T* is a predefined value. This operation is much simpler than the previously described operations.

A candidate solution to an instance of the VRP must specify, among others, waiting times at each customer. Node object is an object that has two properties, customer number and waiting time at customer. A solution to the multi-objective real-time VRPs is encoded using an array of node objects, and based on the permutation representation.

The change wait time operation is therefore a simple operation. Randomly select a node object, *N*, from the node objects array. For the randomly selected node object *N*, change the waiting time property to a random value in the range of 0 to *T*.

When this operation is applied on a solution, it does not change its feasibility.

## 6.3. Summary

Evolutionary Algorithms belong to the Evolutionary Computation field of study concerned with computational methods inspired by the process and mechanisms of biological evolution. Evolutionary Algorithms share properties of adaptation through an iterative process that accumulates and amplifies beneficial variation through trial and error. Candidate solutions represent members of a virtual population striving to survive in an environment defined by a problem specific objective function. In each case, the evolutionary process refines the adaptive fit of the population of candidate solutions in the environment, typically using surrogates for the mechanisms of evolution such as genetic recombination and mutation.

In this chapter three evolutionary algorithms for solving the real-time multi objective vehicle routing problem were presented. The first two algorithms are genetic algorithms. These algorithms encode a potential solution to a specific problem on a simple chromosome-like data structure and apply recombination operators to these structures in order to preserve critical information.

The first genetic algorithm is an improved version of the vector evaluated genetic algorithm (VEGA). The VEGA concept is that, for a problem with *NumObj* objectives, *NumObj* sub-populations of size *PopSize*/*NumObj* each would be generated (assuming a total population size of *PopSize*). Each sub-population uses only one of the *NumObj* objective functions for fitness assignment. The proportionate selection operator is used to generate the mating pool. These sub-populations are then shuffled together to obtain a new population of size *PopSize*, on which the GA would apply the crossover and mutation operators in the usual way. In each generation the set of not-dominated solutions is added to the optimal solutions set, from which non-dominated solutions are removed.

The second genetic algorithm is the SPEA2 algorithm. SPEA2 is actually an extension of an elitism MOEA called “The Strength Pareto Evolution Algorithm” – SPEA.The distinctive feature of SPEA2 lies in the elitism-preserved operation. An external set (archive) is created for storing primarily non-dominated solutions. It is then combined with the current population to form the next archive that is then used to create offspring for the next generation. The size of the archive is fixed. It can be set to be equal to the population

size. Therefore, two special situations exist when filling solutions in the archive. If the number of non-dominated solutions is smaller than the archive size, other dominated solutions taken from the remainder part of the population are filled in. This selection is carried out according to a fitness value, specifically defined for SPEA. In other words, the individual fitness value defined for a solution *x*, is the total of the SPEA-defined strengths of solutions which dominate *x*, plus a density value.

The second situation happens when the number of non-dominated solutions is over the archive size. In this case, a truncation operator is applied. For that operator, the solution which has the smallest distance to the other solutions will be removed from the set. If solutions have the same minimum distance, the second nearest distance will be considered, and so forth. This is called the *k-th nearest distance rule*.

The third revolutionary algorithm is a combination of the vector evaluated technique and artifical bee colony algorithm. In the ABC algorithm, the colony of artificial bees consists of three groups of bees: (1) employed bees - bees that are currently exploiting a food source; (2) onlookers - bees that are waiting in the hive for the employed bees to share information about the food sources; and (3) scouts - bees that are searching for new food sources in the neighborhood of the hive. The ABC algorithm starts by assigning each employed bee to a randomly generated solution. Next, in each iteration, each employed bee, using a neighborhood operator, finds a new food source near its assigned food source. The nectar amount of the new food source is then evaluated. If the amount of nectar in the new food source is higher than the amount of nectar in the old one, then the older source is replaced by the newer one. Next, the nectar information of the food sources is shared with the onlookers. The onlooker chooses a food source according to the probability proportional to the quality of that food source. Roulette wheel selection is the usual method. Therefore, good food sources, as opposed to bad ones, attract more onlooker bees. Subsequently, using a neighborhood operator, each onlooker finds a food source near its selected food source and calculates its nectar amount. Then, for each old food source, the best food source among all the food sources near the old one is determined. The employed bee associated with the old food source is assigned to the best food source and abandons the old one if the best food source is better than the old food source. A food source is also abandoned by an employed bee if the quality of the food source has not improved in the course of a predetermined and limited number of

successive iterations. The employed bees then become scouts and randomly search for new food source. After a scout finds a new food source, it again becomes an employed bee. After each employed bee is assigned to a food source, another iteration of the ABC algorithm begins. The iterative process is repeated until a stopping condition is met.

Next, solutions representation was described. A candidate solution to an instance of the VRP must specify the number of vehicles required, the partition of the demands through all these vehicles, the delivery order for each route as well as waiting time at each customer. Let a node object define an object that has two properties, customer number and waiting time at customer. A solution to the multi-objective real-time VRPs can be encoded using an array of node objects, and based on the permutation representation. A solution contains several routes, each one of them composed by an ordered subset of the costumers. All demands belonging to the problem being solved must be present in one of the routes.

Methods such as crossover and mutations, which are needed for diversity purposes, were also described. Crossover and mutation are the genetic operators used in the general GAs. In ABCs only neighborhood operators, which are equivalent to GA's mutation operators, are used. Solutions used in a specific problem have their own characteristics, and some particular crossover operators are needed.

# 7. Fitness Functions and Algorithm Convergence

A fitness function is a particular type of objective function that is used to summarize, as a single figure of merit, how close a given design solution is to achieving the set aims.

In the field of evolutionary algorithms, at each iteration, the idea is to delete the *n* worst design solutions, and to breed *n* new ones out of the best design solutions. Therefore, each design solution needs to be awarded a figure of merit to indicate how close it came to meeting the overall specifications, which is generated by applying the fitness function to the test, or simulation, results obtained from that solution.

Evolutionary algorithms work mainly due to the effort involved in designing a workable fitness function. Even though it is no longer the human designer, but the computer, that comes up with the final design, it is the human designer who has to design the fitness function. If this is designed wrongly, the algorithm will either converge to an inappropriate solution, or will have difficulty converging at all.

Furthermore, the fitness function must not only correlate closely with the designer's goal; it must also be computed quickly. Speed of execution is very important, as a typical evolutionary algorithm must be iterated many times in order to produce a usable result for a non-trivial problem.

In some cases, fitness approximation may be appropriate, especially if (1) the fitness computation time of a single solution is extremely high, (2) a precise model for fitness computation is missing or (3) the fitness function is uncertain or noisy.

Two main classes of fitness functions exist: one where the fitness function does not change, as in optimizing a fixed function or testing with a fixed set of test cases; and one where the fitness function is mutable, as in niche differentiation or co-evolving the set of test cases.

In both the improved VEGA algorithm and the VE-ABC algorithm, in each generation, a number of sub-populations are generated by performing proportional selection according to each objective function in turn. Thus, for a problem with $q$ objectives, $q$ sub-populations of size $N/q$ each would be generated, assuming a population size of $N$. This means that for each objective function, a corresponding fitness function has to be designed and calculated for all proposed solutions.

In the case of the SPEA2 algorithm, in order to avoid the situation that individuals dominated by the same archive members have identical fitness values, each individual is assigned a strength value $S(i)$, representing the number of solutions it dominates. A solution dominates another solution if for all objective functions, the first solution is better than the second solution. Therefore, in order to compute $S(i)$, for each individual, the values of all objective functions need to be computed.

Five objective functions are addressed in this study; four of which rely on travel time: (1) minimizing the total travel time; (2) minimizing the difference of travel times among the routes of the solution, (3) Minimizing the total dissatisfaction of all customers and (4) Minimizing the arrival time of the latest vehicle.

Due to the stochastic nature of travel time, in order to get an accurate value, or accurate fitness functions for the previously mentioned objectives, simulation has to be used.

Simulation works by traveling paths. Each path is traveled $w$ times, when $w$ is pre-determined by the user. The traveling times are stored in a sorted array. The returned traveling time, $C$, returned by the simulation is defined as the traveling time stored in entry $w \cdot \alpha$ of the array. Assuming that $\alpha$=0.95, this means that in 95% of all cases, the actual traveling time will be shorter than $C$. A higher value of $w$ will usually increase the accuracy of the result obtained from the simulation.

Usually, a high value of $w$ results in accurate results of the simulation; however, it dramatically increases the running time of the algorithm. For example, in this study, values of $w$=1 and $w$=1000 were used. Using the improved VEGA algorithm, several problems were solved. The average running time when $w$=1 was about 20 minutes, and when $w$=1000, about 8 hours. In this study, it will be shown that $w$=1 (fitness approximation) can be used without affecting the algorithm performance (meaning that for different values of $w$ the algorithm converges to the same results).

### 7.1.1. Convergence

To validate this approach, a methodology normally adopted in the evolutionary multi-objective optimization literature was used.

Using performance measures (or metrics) allows the assessment (in a quantitative way) of an algorithm's performance. For multi objective optimization problems, measures tend

to focus on the objective domain as to the accuracy of the results. For this comparative study, the two following metrics were implemented:

**Two Set Coverage** (SC): This metric was proposed by Zitzler et al. (2000), and it can be termed as *relative coverage comparison of two sets*. Consider $X'$ and $X''$ as two competing sets of phenotype decision vectors. SC is defined as the mapping of the order pair $(X', X'')$ to the interval $[0,1]$, which reflects the percentage of individuals in one set ($X''$) dominated by the individuals of the other set ($X'$). The mathematical definition of this metric is shown in equation (7.1):

$$SC(X', X'') = \frac{\left|a'' \in X''; \forall a' \in X' : a' \succ a''\right|}{\left|X''\right|} \tag{7.1}$$

This definition implies that $SC$=1 when all points in $X'$ dominate or are equal to all points in $X''$. $SC$=0 implies the opposite. In general, $SC(X', X'')$ and $SC(X'', X')$ both have to be considered due to set intersections not being empty. Of course, this metric can be used for both spaces (objective function or decision variable space), but in this case, it was applied to objective function space. It should be noted that knowledge of the $PF_{true}$ is not required for this metric. This important property is the main reason for choosing this metric.

**Error Ratio** (ER): This metric was proposed by Veldhuizen (1999) to indicate the percentage of solutions in the known Pareto front, $PF_{known}$, that are not members of the true Pareto front, $PF_{true}$. In order to use this metric, it is essential that the researcher know the $PF_{true}$. The mathematical representation of this metric is shown in equation (7.2):

$$ER = \frac{\sum_{i=0}^{n} e_i}{n} \tag{7.2}$$

where $n$ is the number of vectors in $PF_{known}$ and $e_i$ is a 0 when the $i$ vector is an element of $PF_{true}$ or 1 if $i$ is not an element. It should then be clear that $ER$=0 indicates an ideal

behavior, meaning that the $PF_{known}$ is the same as $PF_{true}$; but when $ER$=1 indicates that none of the points in $PF_{known}$ are in $PF_{true}$.

However, since $PF_{true}$ is usually not known, a slightly different definition of the error ratio metric is presented. Given a set of non-dominated solutions, *ND*, (obtained from the last iteration of the algorithm) and a known Pareto front, $PF_{known}$, the error ratio metric is defined as the percentage of vectors in *ND*, that are not members of $PF_{known}$. Using the formulation presented in (7.2), the new *ER* can be calculated, where *n* is the number of vectors in *ND* and $e_i$ is a 0 when the *i* vector is an element of $PF_{known}$ or 1 if *i* is not an element.

An evolutionary algorithm usually starts with a randomly generated first generation. However, if the first generation is smartly generated, for example, by using results obtained from a heuristic algorithm, then the genetic algorithm will converge to the optimal solution much faster, and the result, assuming that the same parameters, such as the number of generations, are kept, will be more accurate.

As stated before, the fitness evaluation procedure uses simulation, which works by traveling paths. Each path is traveled *w* times, when a high value of *w* usually results in accurate results of the simulation, and therefore, a more accurate fitness value is obtained. It will be shown that *w*=1 can be used without affecting the algorithm performance (meaning that for different values of *w* the algorithm converges to the same results). In order to show that, 30 test problems were randomly generated, 10 with 50 customers, another 10 with 100 customers, and the last 10 problems with 150 customers. In all test problems, the number of time intervals is 24, and in each time interval the speed is within the range of 80-120 KM/H. Each problem was solved 4 times using the improved VEGA algorithm, twice with *w*=1 ("approximated" fitness evaluation) and twice with *w*=1000 ("exact" fitness evaluation), while using only three objective functions: (1) minimizing the total travel time; (2) minimizing the number of vehicles and (3) minimizing the difference of travel times among the routes of the solution. The VEGA algorithm was chosen for test due to its simplicity. The VEGA algorithm is very similar to the original GA, and therefore, there are no additional calculations and operations that may affect the process of the algorithm. Only three objective functions were chosen, again, to reduce the calculations of the algorithm, that may affect the convergence of the algorithm.

### 7.1.2. Metrics Comparison Results

In this section, the results of the metrics comparison, using paired-samples t-tests are reported. Throughout this section *X*' refers to a solution obtained when *w*=1 and *X*'' to a solution obtained when *w*=1000. In evolutionary algorithms, since the way the first generation was generated may change the algorithm's results, each comparison was conducted twice, once using a randomly generated first population and the second using a Savings based first population.

Eight paired-samples t-tests were conducted to compare the results of the two set coverage metric (*SC*(*X*',*X*'') vs. *SC*(*X*'',*X*')). The results are listed in Table 7.1.

The results show that for problems with 50 and 100 customers, when the first generation was randomly generated or Savings based, there is no significant difference in the scores for *SC*(*X*',*X*'') and *SC*(*X*'',*X*'). However, for problems with 150 customers, there is a significant difference in the scores for *SC*(*X*',*X*'') and *SC*(*X*'',*X*'). This indicates that on average, 59% of the non-dominated solutions, when the first generation was randomly generated, and 29% of the non-dominated solutions, when the first generation was Savings based, obtained from the last iteration of the genetic algorithm, when *w*=1, are dominated by the non-dominated solutions obtained when *w*=1000. In addition, 26% of the non-dominated solutions, when the first generation was randomly generated, and 54% of the non-dominated solutions, when the first generation was Savings based, obtained when *w*=1000, are dominated by the non-dominated solutions obtained when *w*=1. From the above, it can be concluded that for problems with 150, the results obtained when *w*=1000 are better than the results obtained when *w*=1 when using a randomly generated first generation. However, if the first generation is Savings based, then the results obtained when *w*=1 are better than the results obtained when *w*=1000. Two paired-samples t-tests, in which all groups of problems are combined into a single sample, show that there is no significant difference in the scores for *SC*(*X*',*X*'') and *SC*(*X*'',*X*').

| | *SC*(*X*',*X*'') | | *SC*(*X*'',*X*') | | | | |
|---|---|---|---|---|---|---|---|
| Problem Size | M | SD | M | SD | t | df | Sig. |
| Randomly generated fist population | | | | | | | |
| 50 | 0.479 | 0.411 | 0.493 | 0.459 | -0.112 | 39 | 0.911 |
| 100 | 0.349 | 0.447 | 0.397 | 0.447 | -0.382 | 39 | 0.705 |
| 150 | 0.59 | 0.458 | 0.258 | 0.411 | 2.654 | 39 | 0.011 |
| All | 0.411 | 0.424 | 0.416 | 0.411 | -0.064 | 119 | 0.949 |
| Savings based first population | | | | | | | |
| 50 | 0.535 | 0.41 | 0.452 | 0.376 | 0.690 | 39 | 0.494 |
| 100 | 0.408 | 0.435 | 0.254 | 0.365 | 1.427 | 39 | 0.162 |
| 150 | 0.29 | 0.399 | 0.541 | 0.446 | -2.199 | 39 | 0.034 |
| All | 0.473 | 0.447 | 0.389 | 0.446 | 1.235 | 119 | 0.219 |

**Table 7.1 – A comparison of SC(X',X'') and SC(X'',X') using paired-samples t-tests**

As with the results of the two set coverage metric, paired-samples t-tests were used to check if there are any differences in the results of the error ratio metric for $w$=1 and for $w$=1000. The results are listed in Table 7.2.

The results show that for problems with 50 customers, when the first generation was randomly generated or Savings based, there is no significant difference in the scores for *ER*(*X*') and *ER*(*X*''). This is also the case for problems with 100 customers, when the first generation was randomly generated and for problems with 150 customers, when the first generation is Savings based.

However, for problems with 150 customers, when the first generation was randomly generated and for problems with 100 customers, when the first generation is Savings based, there is a significant difference in the scores for *ER*(*X*') and *ER*(*X*''). This means that for problems with 150 customers, when the first generation was randomly generated, on average 55% of the non-dominated solutions do not belong to $PF_{known}$ when $w$=1, whereas when $w$=1000, 80% of the non-dominated solutions do not belong to $PF_{known}$. Similarly, for problems with 100 customers, when the first generation is Savings based, on average 49% of the non-dominated solutions do not belong to $PF_{known}$ when $w$=1 but when $w$=1000, 73% of the non-dominated solutions do not belong to $PF_{known}$.

This means that for problems with 150 customers, when the first generation was randomly generated and for problems with 100 customers, when the first generation was Savings based, the chance for a non-dominated solution that belongs to $PF_{known}$ is twice as high when $w$=1 than when $w$=1000.

The results of the paired-samples t-tests, in which all groups of problems are combined into a single sample, show that there is no significant difference in the scores for *ER*(*X'*) and *ER*(*X''*).

| | *SC*(*X'*,*X''*) | | *SC*(*X''*,*X'*) | | | | |
|---|---|---|---|---|---|---|---|
| Problem Size | M | SD | M | SD | t | df | Sig. |
| Randomly generated fist population | | | | | | | |
| 50 | 0.852 | 0.248 | 0.759 | 0.308 | 1.402 | 39 | 0.169 |
| 100 | 0.734 | 0.383 | 0.589 | 0.458 | 1.416 | 39 | 0.165 |
| 150 | 0.552 | 0.481 | 0.803 | 0.353 | -2.535 | 39 | 0.015 |
| All | 0.712 | 0.399 | 0.717 | 0.389 | -0.088 | 119 | 0.93 |
| Savings based first population | | | | | | | |
| 50 | 0.699 | 0.307 | 0.74 | 0.306 | -0.532 | 39 | 0.598 |
| 100 | 0.496 | 0.404 | 0.731 | 0.342 | -2.592 | 39 | 0.013 |
| 150 | 0.695 | 0.343 | 0.585 | 0.427 | 1.229 | 39 | 0.227 |
| All | 0.63 | 0.364 | 0.686 | 0.366 | -1.092 | 119 | 0.277 |

**Table 7.2 - A comparison of ER(X') and ER(X'') using paired-samples t-tests**

From the results, it can be concluded that the results obtained from the genetic algorithm, whether using $w$=1 or $w$=1000, are the same, regardless of the problem size and the method of the generation of the first population.

### 7.1.3. TOPSIS Comparison

In most cases, when solving a multi-objective optimization problem, the result is a set of non-dominated solution, from which the decision maker has to choose his preferred alternative. In an automated environment, a mechanism for choosing a preferred solution from a set of non-dominated solutions needs to be implemented. A number of techniques for automating the process of choosing have been developed. Among the various methods, one can find the Max-Min method, Min-Max method, Compromise Programming, ELECTRE Method and more (Masud & Ravindran, 2008). In this paper, the TOPSIS method was used as a means for choosing a preferred alternative.

TOPSIS (technique for order preference by similarity to ideal solution) was originally proposed by Hwang and Yoon (1981) for the MCSP. TOPSIS operates on the principle that the preferred solution (alternative) should simultaneously be closest to the ideal solution and farthest from the negative-ideal solution. TOPSIS does not require the specification of a value (utility) function, but it assumes the existence of monotonically increasing value (utility) function for each (benefit) criterion. The method uses an index

that combines the closeness of an alternative to the positive-ideal solution with its remoteness from the negative-ideal solution. The alternative maximizing this index value is the preferred alternative.

In the previous section, it has been shown that a set of non-dominated solutions obtained when $w$=1 is as good as a set of non-dominated solutions obtained when $w$=1000. However, this does not mean that the same results exist in both sets, and therefore, it is not guaranteed that the TOPSIS method selects similar results from both sets. In this section, a comparison of the results of TOPSIS method applied on the solution sets obtained from the 30 test cases is presented.

A solution is a set of three results, each for every objective function. The analysis begins with correlation analysis. Correlation analysis is used to check whether or not there is a correlation between the three values of a result. As with the metrics comparison, each comparison is conducted twice, once using a randomly generated first population, and the second using a Savings based first population.

Eight Pearson product-moment correlation coefficients were computed to assess the relationship between the results of the first objective function and the second objective function. The results are listed in Table 7.3.

| Problem Size | $w$=1 | | $w$=1000 | |
|---|---|---|---|---|
| | | Obj. 2 | | Obj. 2 |
| Randomly generated first population | | | | |
| 50 | Obj. 1 | $r$=0.94, $n$=40, $p$=0 | Obj. 1 | $r$=0.954, $n$=40, $p$=0 |
| 100 | Obj. 1 | $r$=0.973, $n$=40, $p$=0 | Obj. 1 | $r$=0.986, $n$=40, $p$=0 |
| 150 | Obj. 1 | r=0.909, $n$=40, $p$=0 | Obj. 1 | $r$=0.944, $n$=40, $p$=0 |
| All | Obj. 1 | r=0.81, $n$=120, $p$=0 | Obj. 1 | $r$=0.785, $n$=120, $p$=0 |
| Savings based first population | | | | |
| 50 | Obj. 1 | $r$=0.835, $n$=40, $p$=0 | Obj. 1 | $r$=0.814, $n$=40, $p$=0 |
| 100 | Obj. 1 | $r$=0.82, $n$=40, $p$=0 | Obj. 1 | $r$=0.865, $n$=40, $p$=0 |
| 150 | Obj. 1 | r=0.281, $n$=40, $p$=0.079 | Obj. 1 | $r$=0.174, $n$=40, $p$=0.282 |
| All | Obj. 1 | r=0.716, $n$=120, $p$=0 | Obj. 1 | r=0.716, $n$=120, $p$=0 |

**Table 7.3 - Pearson product-moment correlation coefficients between the first and second objectives, for $w$=1 and $w$=1000**

For problems with 50, 100 and 150 customers, whether the first generation was randomly created or Savings based, a positive correlation between the two variables was found for solutions obtained when $w$=1 and for solutions obtained when $w$=1000. Since a strong positive correlation exists between the two variables, a correlation analysis was

performed, using the results of all problems as a single result. The results show a positive correlation when using *w*=1 and when using *w*=1000.

A second set of eight Pearson product-moment correlation coefficients was computed to assess the relationship between the results of the first objective function and the third objective function. All tests, whether the first generation was randomly created or Savings based, show a negative correlation. However, since the third objective minimizes the difference of travel times among the routes of the solution, and is defined by means of standard deviation, and since the maximum value obtained for this objective is 0.05 when *w*=1 and 0.009 when *w*=1000, it can be assumed that the value of the third objective is always 0, and therefore, can be ignored in the analysis.

Since it has been shown that a correlation exists between the first and second objectives, and that the third objective can be ignored, since it can be treated as zero, a paired t-test can be used to compare the results obtained by using the TOPSIS method.

Eight paired-samples t-tests were conducted to compare the results obtained by using the TOPSIS method when *w*=1 and when *w*=1000. The results are listed in Table 7.4.

All paired-samples t-tests show that there is no significant difference in the scores for *w*=1 and for *w*=1000.

| | *w*=1 | | *w*=1000 | | | | |
|---|---|---|---|---|---|---|---|
| Problem Size | M | SD | M | SD | t | df | Sig. |
| Randomly generated fist population | | | | | | | |
| 50 | 31.2 | 10.6 | 30.9 | 10.2 | 0.429 | 39 | 0.67 |
| 100 | 72.1 | 28.8 | 70.9 | 26.3 | 0.899 | 39 | 0.374 |
| 150 | 76.3 | 15.8 | 79.1 | 14.2 | -1.967 | 39 | 0.056 |
| All | 59.8 | 28.2 | 60.3 | 27.8 | -0.676 | 119 | 0.5 |
| Savings based first population | | | | | | | |
| 50 | 24.9 | 12.3 | 24.8 | 11.9 | 0.02 | 39 | 0.984 |
| 100 | 52.7 | 31.7 | 51.9 | 30.6 | 0.244 | 39 | 0.808 |
| 150 | 42.2 | 27.0 | 40.6 | 26.4 | 0.34 | 39 | 0.736 |
| All | 39.9 | 27.4 | 39.1 | 26.6 | 0.417 | 119 | 0.678 |

**Table 7.4 - A comparison of TOPSIS results for *w*=1 and *w*=1000 using paired-samples t-tests**

## 7.2. Travel time characteristics

The previous analysis shows that it is possible to increase the running time of the algorithm by using an "approximated" fitness function, without influencing the accuracy of the algorithm. The analysis was done using 30 randomly generated test problems, with

50, 100 and 150 customers, all having 24 time intervals, when for each time interval the travel speed ranges from 80-120 KM/H, with empiric probability. However, travel time is more likely to be lognormally distributed because (1) the positive skew shape (i.e., right skewed) is more suitable for travel time description; that is, a higher probability exists for long travel time than for short travel time, and (2) the range [0, ∞) of the distribution is more natural than a truncated normal distribution (because negative travel times are impossible) (Hadas & Ceder, 2008).

For that reason, a second set of tests was conducted, this time using Solomon's instances. Since Solomon's instances were designed for TWVRP, a simple modification had to be done. Solomon's instances provide the location of each customer, assuming that the travel speed is constant. Since this is not the case in this problem, time intervals were added (24 of them) and for each time interval a lognormal random travel time function was assigned for which σ=0.03 and μ=4.1 (theses values may change slightly between time intervals) and therefore the average traveling speed is 60 KM/H. In order to decrease running time, Solomon's instances were solved using the improved VEGA algorithm, using 500 generations and population size of 200, once when *w*=1 and next when *w*=100. The results of the test are presented in Table 7.5.

| | | *w*=1 | | *w*=100 | | | | |
|---|---|---|---|---|---|---|---|---|
| Problem Size | Problem Type | M | SD | M | SD | t | df | Sig. |
| Randomly generated first population | | | | | | | | |
| 25 | C1 | 3.35 | 0.03 | 3.34 | 0.04 | 0.641 | 35 | 0.526 |
| | C2 | 2.47 | 0.09 | 2.45 | 0.1 | 0.561 | 31 | 0.579 |
| | R1 | 4.14 | 0.86 | 3.63 | 0.22 | 3.912 | 47 | 0 |
| | R2 | 3.15 | 0.28 | 3.15 | 0.22 | -0.059 | 43 | 0.953 |
| | RC1 | 3.96 | 0.24 | 3.84 | 0.21 | 2.492 | 31 | 0.018 |
| | RC2 | 2.57 | 0.13 | 2.59 | 0.15 | -0.69 | 31 | 0.495 |
| 50 | C1 | 6.6 | 0.22 | 6.61 | 0.44 | -0.133 | 35 | 0.895 |
| | C2 | 5.4 | 0.55 | 4.94 | 0.1 | 4.616 | 31 | 0 |
| | R1 | 11.83 | 1.21 | 11.4 | 1.45 | 1.466 | 47 | 0.149 |
| | R2 | 11.63 | 0.67 | 11.54 | 0.94 | 0.538 | 43 | 0.593 |
| | RC1 | 9.46 | 0.69 | 9.27 | 0.78 | 0.931 | 31 | 0.395 |
| | RC2 | 8.6 | 0.72 | 8.46 | 0.41 | 1.027 | 31 | 0.312 |
| 100 | C1 | 16.44 | 1.01 | 16.35 | 0.94 | 0.467 | 35 | 0.643 |
| | C2 | 14.4 | 1.00 | 13.85 | 0.94 | 2.292 | 31 | 0.029 |
| | R1 | 38.94 | 2.38 | 37.43 | 1.85 | 3.462 | 47 | 0.001 |
| | R2 | 35.74 | 3.01 | 32.37 | 2.71 | 5.224 | 43 | 0 |
| | RC1 | 32.88 | 2.45 | 30.99 | 0.29 | 3.514 | 31 | 0.001 |

| | | *w*=1 | | *w*=100 | | | | |
|---|---|---|---|---|---|---|---|---|
| Problem Size | Problem Type | M | SD | M | SD | t | df | Sig. |
| | RC2 | 28.41 | 3.29 | 24.76 | 1.96 | 5.604 | 31 | 0 |
| Savings based first population | | | | | | | | |
| 25 | C1 | 3.38 | 0.07 | 3.37 | 0.07 | 0.453 | 35 | 0.653 |
| | C2 | 2.38 | 0.09 | 2.35 | 0.08 | 1.544 | 31 | 0.133 |
| | R1 | 4.36 | 1.07 | 3.66 | 0.28 | 4.413 | 47 | 0 |
| | R2 | 3.32 | 0.3 | 3.23 | 0.31 | 1.427 | 43 | 0.161 |
| | RC1 | 4.11 | 0.26 | 4 | 0.19 | 1.971 | 31 | 0.058 |
| | RC2 | 2.51 | 0.1 | 2.54 | 0.1 | -1.275 | 31 | 0.212 |
| 50 | C1 | 6.4 | 0.08 | 6.34 | 0.01 | 4.128 | 35 | 0 |
| | C2 | 5.34 | 0.85 | 4.96 | 0.26 | 2.42 | 31 | 0.022 |
| | R1 | 12.56 | 1.26 | 11.21 | 0.8 | 6.353 | 47 | 0 |
| | R2 | 11.91 | 0.6 | 11.27 | 0.63 | 5.16 | 43 | 0 |
| | RC1 | 9.17 | 0.63 | 8.89 | 0.22 | 2.412 | 31 | 0.022 |
| | RC2 | 8.61 | 0.67 | 8.42 | 0.53 | 1.258 | 31 | 0.218 |
| 100 | C1 | 14.92 | 0.63 | 14.53 | 0.08 | 4.253 | 35 | 0 |
| | C2 | 12.33 | 0.57 | 11.45 | 0.54 | 6.486 | 31 | 0 |
| | R1 | 38.88 | 2.42 | 37.81 | 1.78 | 2.466 | 47 | 0.017 |
| | R2 | 35.76 | 2.19 | 32.16 | 2.27 | 7.837 | 43 | 0 |
| | RC1 | 29.82 | 2.17 | 28.54 | 1.34 | 2.894 | 31 | 0.007 |
| | RC2 | 28.21 | 4.07 | 24.22 | 1.74 | 4.836 | 31 | 0 |

**Table 7.5 - A comparison of TOPSIS results for *w*=1 and *w*=100 using paired-samples t-tests**

As seen from the results, for problems with 25 and 50 customers, when using a randomly generated first generation, and for problems with 25 customers when using a Savings based first generation, there is no difference in the TOPSIS results when using *w*=1 and *w*=100. However, for problems with 100 customers, when using a randomly generated first generation, and for problems with 50 and 100 customers when using a Savings based first generation, a better solution is obtained when *w*=100 compared to the solution obtained when *w*=1.

As stated before, in order to decrease running time, Solomon's instances were solved using the improved VEGA algorithm, using 500 generations and population size of 200. It is known that the number of generations used by an evolutionary algorithm may affect its results. Generally, a high number of generations gives the algorithm more chance to converge towards the optimal solution than a low number of generations. However, in real-time applications, the number of generations is bounded by the time given to the algorithm to come up with a solution. Therefore, the algorithm was tested again, this time with a stopping condition of 30 minutes running time, instead of the 500 generations. Results for problems with 100 customers are reported in Table 7.6.

| | | *w*=1 | | *w*=100 | | | | |
|---|---|---|---|---|---|---|---|---|
| Problem Size | Problem Type | M | SD | M | SD | t | df | Sig. |
| Randomly generated first generation | | | | | | | | |
| 100 | C1 | 14.85 | 0.67 | 16.43 | 0.81 | -8.883 | 35 | 0 |
| 100 | C2 | 12.64 | 1.58 | 14.19 | 0.74 | -4.814 | 31 | 0 |
| 100 | R1 | 33.67 | 2.08 | 39.61 | 1.69 | -14.488 | 47 | 0 |
| 100 | R2 | 32.77 | 4.57 | 38.62 | 2.94 | -7.073 | 43 | 0 |
| 100 | RC1 | 30.14 | 2.23 | 33.22 | 1.07 | -7.905 | 31 | 0 |
| 100 | RC2 | 27.13 | 4.42 | 28.14 | 2.49 | -0.979 | 31 | 0.335 |
| Savings based first generation | | | | | | | | |
| 100 | C1 | 14.56 | 0.4 | 14.5 | 0.45 | -0.411 | 35 | 0.684 |
| 100 | C2 | 11.72 | 1.31 | 12.15 | 2.03 | 0.985 | 31 | 0.332 |
| 100 | R1 | 34.44 | 2.56 | 40.09 | 1.53 | -13.098 | 47 | 0 |
| 100 | R2 | 31.25 | 2.79 | 39.58 | 2.31 | -16.071 | 43 | 0 |
| 100 | RC1 | 27.29 | 1.12 | 30.16 | 2.11 | -6.568 | 31 | 0 |
| 100 | RC2 | 24.92 | 4.21 | 26.27 | 1.4 | -1.841 | 31 | 0.075 |

**Table 7.6 - A comparison of TOPSIS results for *w*=1 and *w*=100 using paired-samples t-tests**

As it can be seen from Table 7.6, when using a randomly generated first generation, the results obtained by the algorithm when *w*=1 were better than the results obtained when *w*=100, except for RC2, in which no significant differences were found between the results. When using a Savings based first generation, the results obtained by the algorithm when *w*=1 were better than the results obtained when *w*=100, for problems R1, R2 and RC1, while for problems C1, C2 and RC2, no significant differences were found between the results.

Furthermore, an analysis of the convergence of the algorithm shows, that when *w*=100, the best solution is reached after almost 30 minutes of running, while the same solution is found much earlier when *w*=1. To illustrate these finding, the analysis of problems C101, C201, R101, R201, RC101 and RC201 is given for both randomly generated and Savings based first generation.

For problem C101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 16.52, objective two equals 10 and objective three equals 0, after 422 generations (see Figure 7.1). Since the algorithm, when *w*=100, was able to generate 486 generations in 30 minutes, this means that the algorithm's best solution was reached after 26 minutes and 10 seconds. For the same problem, C101, when *w*=1, the algorithm reached the best solution after 326 generations out of 7916 generations that were generated during 30 minutes, namely, after one minute and ten seconds. Moreover,

when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 19.95, objective two equals 10 and objective three equals 0, after 7394 generations, i.e., after 28 minutes and one second.

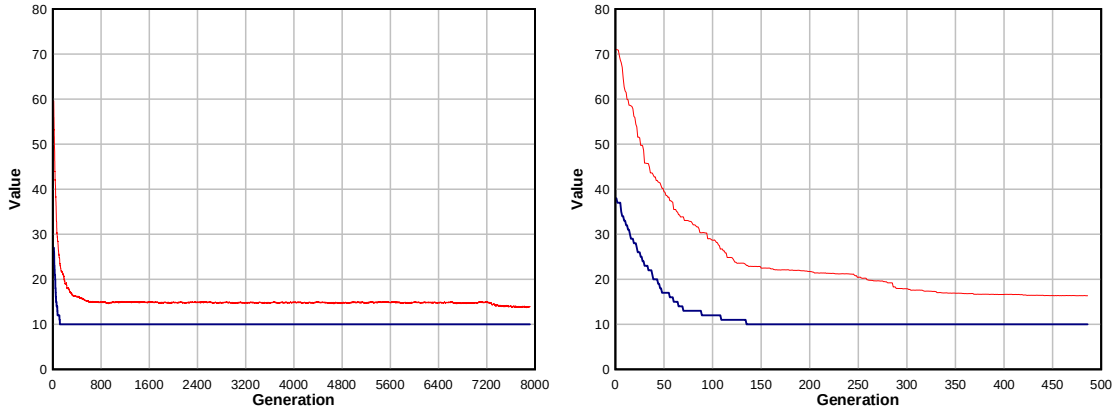



**Figure 7.1 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem C101 during the first 30 minutes**

For problem C201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 13.23, objective two equals 3 and objective three equals 0, after 422 generations (see Figure 7.2). Since the algorithm, when *w*=100, was able to generate 464 generations in 30 minutes, this means that the algorithm's best solution was reached after 27 minutes and 17 seconds. For the same problem, C201, when *w*=1, the algorithm reached the best solution after 733 generations out of 7397 generations that were generated during 30 minutes, i.e., after 9 minutes and ten seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 10.54, objective two equals 3 and objective three equals 0, after 1799 generations, i.e., after 22 minutes and 30 seconds.

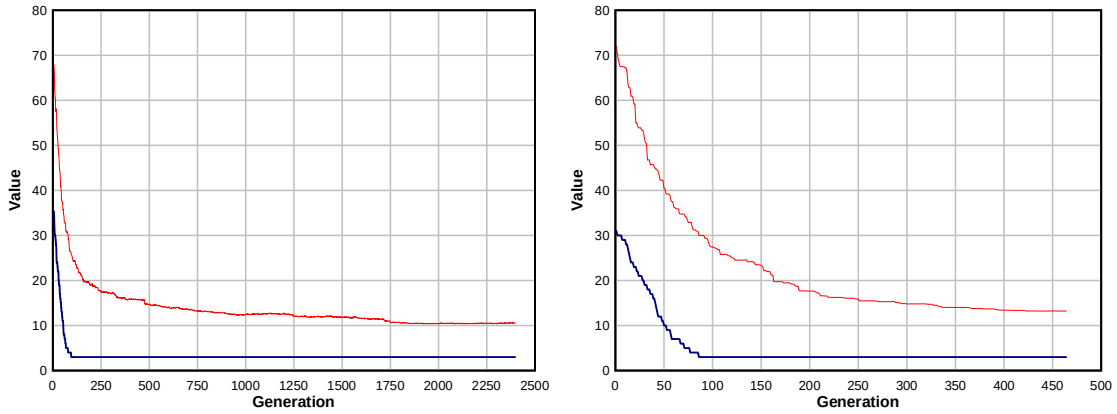



**Figure 7.2 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem C201 during the first 30 minutes**

For problem R101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 38.72, objective two equals 8 and objective three equals 0, after 345 generations (see Figure 7.3). Since the algorithm, when *w*=100, was able to generate 365 generations in 30 minutes, this means that the algorithm's best solution was reached after 28 minutes and 21 seconds. For the same problem, R101, when *w*=1, the algorithm reached the best solution after 309 generations out of 1692 generations that were generated during 30 minutes, i.e., after 5 minutes and 28 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 32.07, objective two equals 8 and objective three equals 0, after 1052 generations, i.e., after 18 minutes and 39 seconds.

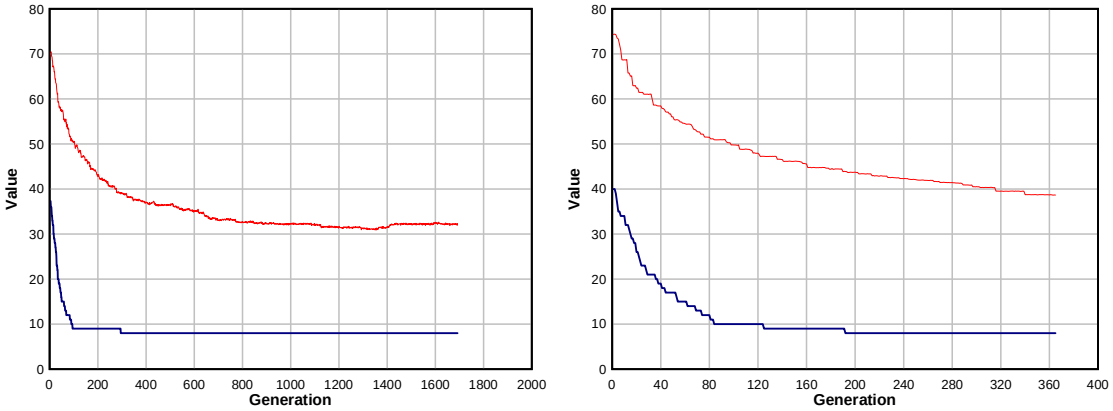



**Figure 7.3 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem R101 during the first 30 minutes**

For problem R201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 39.15, objective two equals 2 and objective three equals 0.00047, after 207 generations (see Figure 7.4). Since the algorithm, when *w*=100, was able to generate 215 generations in 30 minutes, this means that the algorithm's best solution was reached after 28 minutes and 53 seconds. For the same problem, R201, when *w*=1, the algorithm reached the best solution after 154 generations out of 1246 generation that were generated during 30 minutes, i.e., after 3 minutes and 42 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 32.91, objective two equals 2 and objective three equals 0.00008, after 785 generations, or after 18 minutes and 54 seconds.

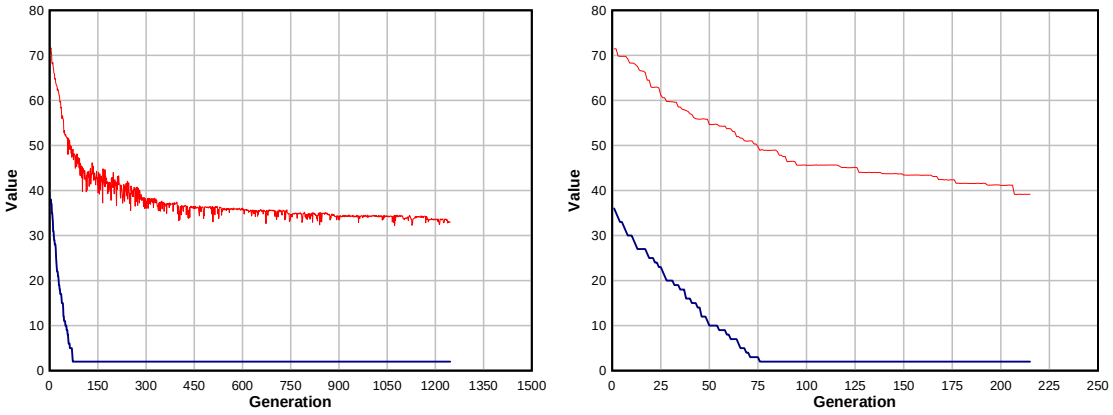



**Figure 7.4 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem R201 during the first 30 minutes**

For problem RC101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 30.53, objective two equals 10 and objective three equals 0, after 321 generations (see Figure 7.5). Since the algorithm, when *w*=100, was able to generate 327 generations in 30 minutes, this means that the algorithm's best solution was reached after 29 minutes and 26 seconds. For the same problem, RC101, when *w*=1, the algorithm reached the best solution after 432 generations out of 806 generation that were generated during 30 minutes, i.e., after 16 minutes and 4 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 28.52, objective two equals 10 and objective three equals 0, after 704 generations, or after 26 minutes and 12 seconds.

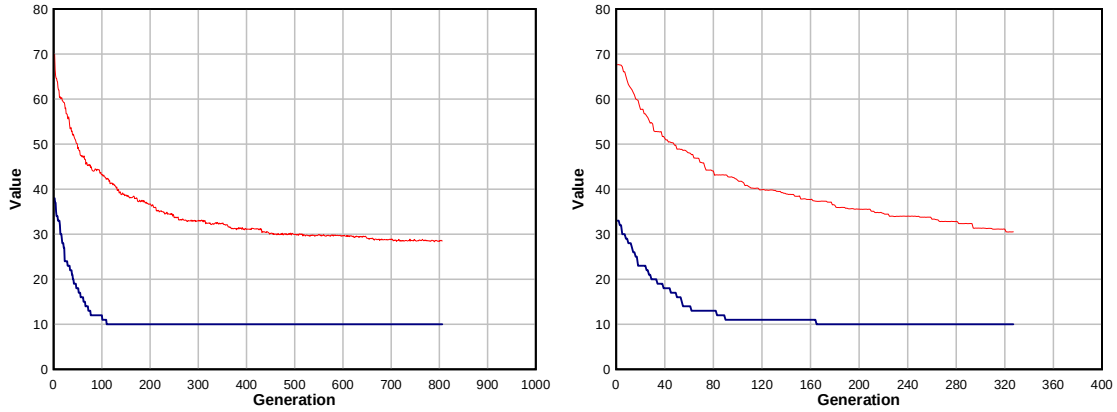



**Figure 7.5 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC101 during the first 30 minutes**

For problem RC201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 26.99, objective two equals 2 and objective three equals 0.00001, after 307 generations (see Figure 7.6). Since the algorithm, when *w*=100, was able to generate 323 generations in 30 minutes, this means that the algorithm's best solution was reached after 28 minutes and 30 seconds. For the same problem, RC201, when *w*=1, the algorithm reached the best solution after 425 generations out of 979 generation that were generated during 30 minutes, i.e., after 13 minutes and one second. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 24.22, objective two equals 2 and objective three equals 0, after 967 generations, or after 29 minutes and 37 seconds.

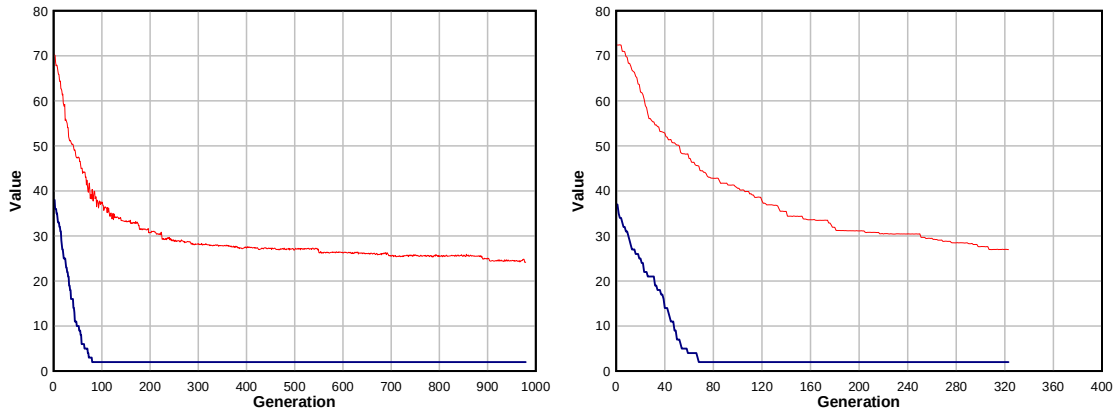



**Figure 7.6 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC201 during the first 30 minutes**

For problem C101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 13.92, objective two equals 10 and objective three equals 0, after one generation (see Figure 7.7). Since the algorithm, when *w*=100, was able to generate 513 generations in 30 minutes, this means that the algorithm's best solution was reached after 0 minutes and 0 seconds. For the same problem, C101, when *w*=1, the algorithm reached the best solution after 1 generation out of 7747 generations that were generated during 30 minutes, after 0 minutes and 0 seconds.

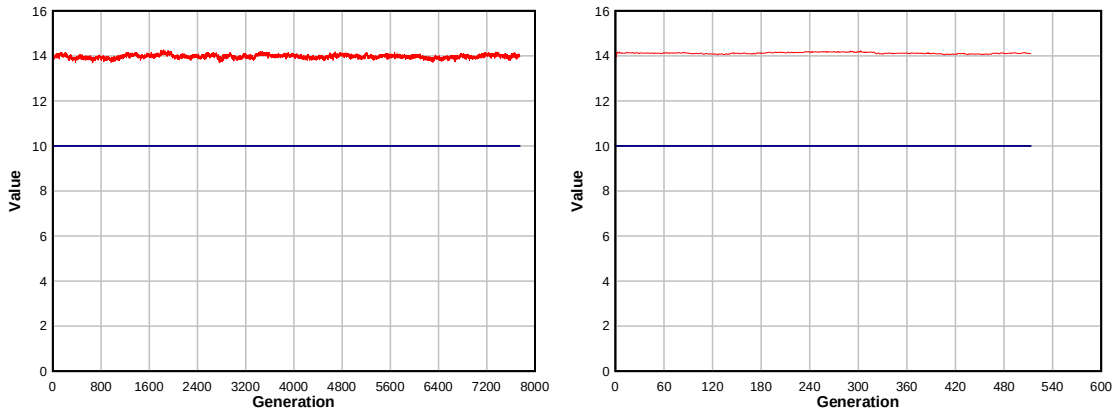



**Figure 7.7 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem C101 during the first 30 minutes**

For problem C201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 10.89, objective two equals 3 and objective three equals 0, after 494 generations (see Figure 7.8). Since the algorithm, when *w*=100, was able to generate 500 generations in 30 minutes, this means that the algorithm's best solution was reached after 29 minutes and 38 seconds. For the same problem, C201, when *w*=1, the algorithm didn't reach the best solution after 2569 generations that were generated during 30 minutes.

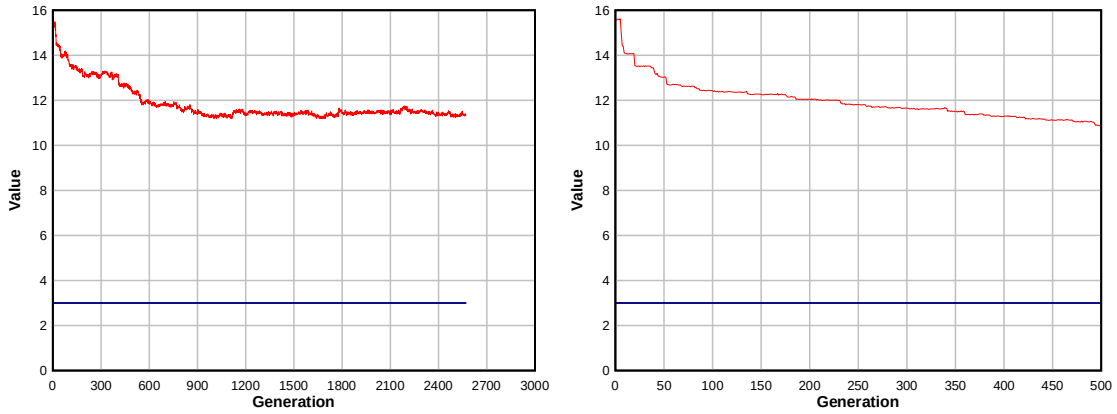



**Figure 7.8 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem C201 during the first 30 minutes**

For problem R101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 38.07, objective two equals 8 and objective three equals 0, after 371 generations (see Figure 7.9). Since the algorithm, when *w*=100, was able to generate 380 generations in 30 minutes, this means that the algorithm's best solution was reached after 29 minutes and 17 seconds. For the same problem, R101, when *w*=1, the algorithm reached the best solution after 362 generations out of 1812 generation that were generated during 30 minutes, or after 5 minutes and 59 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 30.04, objective two equals 8 and objective three equals 0, after 1706 generations, i.e., after 28 minutes and 14 seconds.

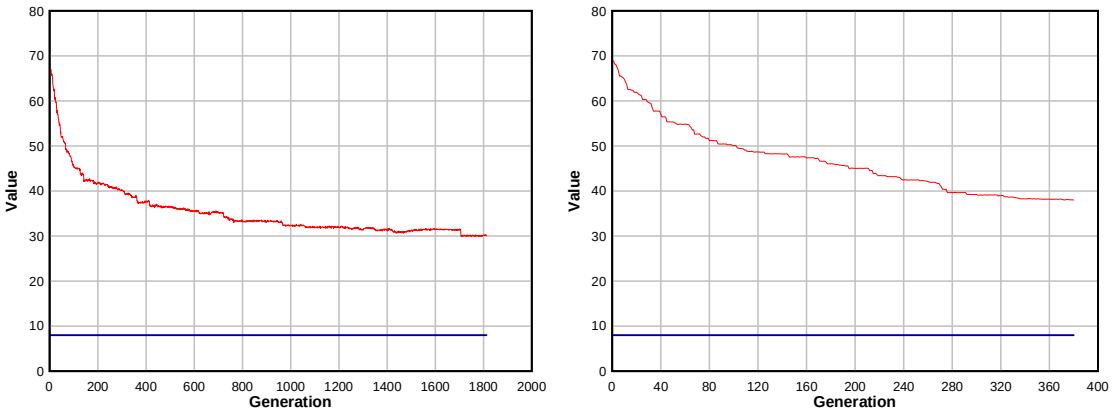



**Figure 7.9 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem R101 during the first 30 minutes**

For problem R201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 37.9, objective two equals 2 and objective three equals 0.00007, after 175 generations (see Figure 7.10). Since the algorithm, when *w*=100, was able to generate 177 generations in 30 minutes, this means that the algorithm's best solution was reached after 29 minutes and 14 seconds. For the same problem, R201, when *w*=1, the algorithm reached the best solution after 162 generations out of 1308 generation that were generated during 30 minutes, or after 3 minutes and 51 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 33.61, objective two equals 2 and objective three equals 0.00009, after 874 generations, or after 20 minutes and two seconds.

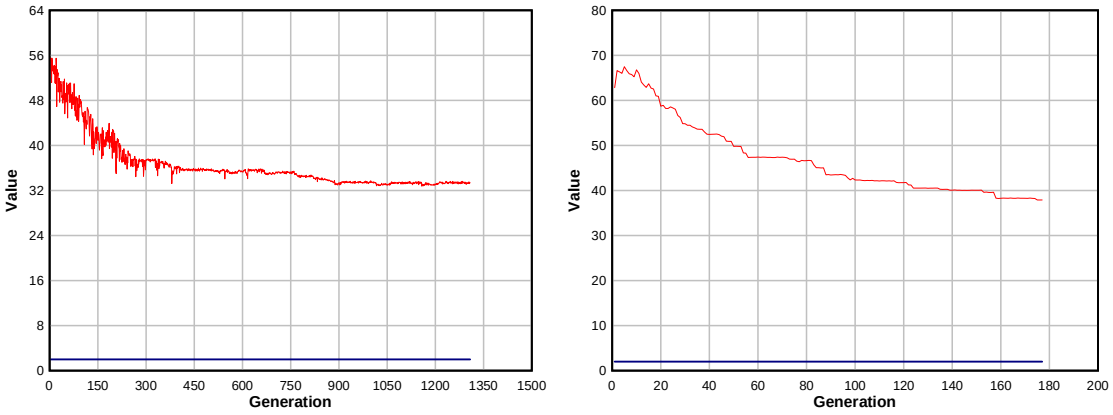



**Figure 7.10 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem R201 during the first 30 minutes**

For problem RC101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 28.47, objective two equals 9 and objective three equals 0, after 358 generations (see Figure 7.11). Since the algorithm, when *w*=100, was able to generate 362 generations in 30 minutes, this means that the algorithm's best solution was reached after 29 minutes and 40 seconds. For the same problem, RC201, when *w*=1, the algorithm reached the best solution after 798 generations out of 1164 generations that were generated during 30 minutes, i.e., after 20 minutes and 34 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 25.93, objective two equals 9 and objective three equals 0, after 1099 generations, or after 28 minutes and 19 seconds.

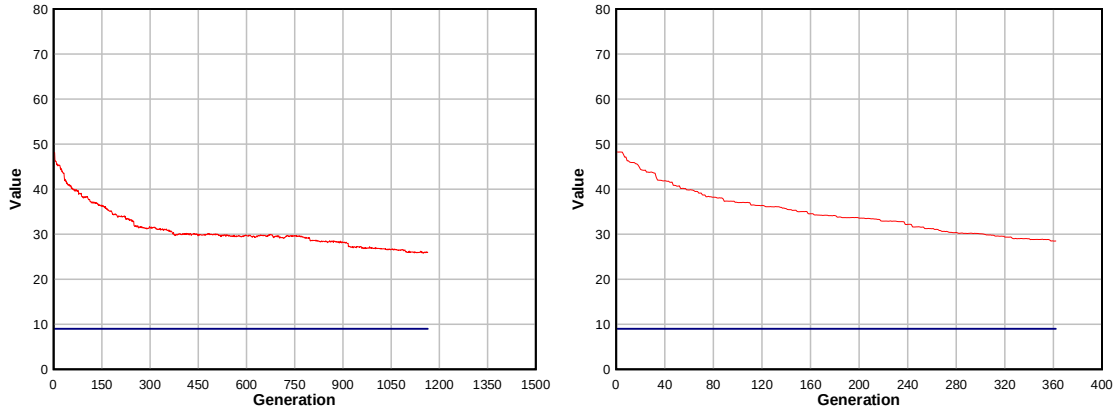



**Figure 7.11 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC101 during the first 30 minutes**

For problem RC201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 26.95, objective two equals 2 and objective three equals 0.00002, after 343 generations (see Figure 7.12). Since the algorithm, when *w*=100, was able to generate 363 generations in 30 minutes, this means that the algorithm's best solution was reached after 28 minutes and 20 seconds. For the same problem, RC201, when *w*=1, the algorithm reached the best solution after 314 generations out of 1057 generations that were generated during 30 minutes, or after 8 minutes and 54 seconds. Furthermore, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 21.69, objective two equals 2 and objective three equals 0, after 930 generations, i.e., after 26 minutes and 23 seconds.

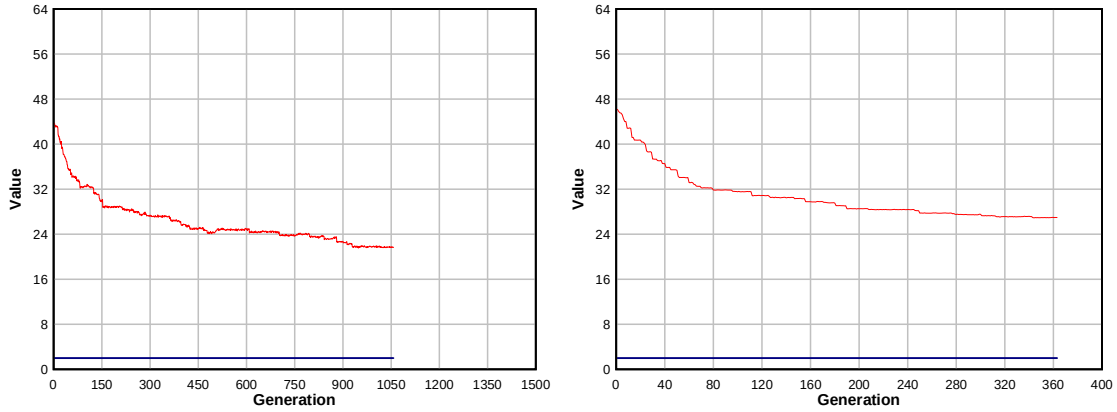



**Figure 7.12 - Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC201 during the first 30 minutes**

## 7.3. Summary

A fitness function is a particular type of objective function that is used to summarize, as a single figure of merit, how close a given design solution is to achieving the set aims.

In the field of evolutionary algorithms, at each iteration, the idea is to delete the *n* worst design solutions, and to breed *n* new ones from the best design solutions. Each design solution, therefore, needs to be awarded a figure of merit, to indicate how close it came to meeting the overall specification, and this is generated by applying the fitness function to the test, or simulation, results obtained from that solution.

In some cases, fitness approximation may be appropriate, especially if (1) the fitness computation time of a single solution is extremely high, (2) a precise model for fitness computation is missing or (3) the fitness function is uncertain or noisy.

In all three algorithms presented, the fitnesses of all five objective functions have to be calculated. Due to the stochastic nature of travel time, in order to get an accurate value, or accurate fitness functions, simulation has to be used. Simulation is a time consuming process.

It was shown that it is possible to increase the running time of the algorithm by using an "approximated" fitness function, without influencing the accuracy of the algorithm. A fast algorithm is necessary when coping with real-time problems, which is the final goal of this study.

Usually, when solving a multi-objective optimization problem, the result is a set of non-dominated solutions, from which the decision maker has to choose his preferred alternative. Since the final goal is to create an automated algorithm for solving a real-time multi-objective vehicle routing problem, the TOPSIS method, a mechanism for choosing a preferred solution from a set of non-dominated solutions, has been implemented. It was shown that there is no difference in the quality of the results obtained using the "approximated" or "accurate" methods, but this does not mean that the same results exist in both sets, and therefore it is not guaranteed that the TOPSIS method selects similar results from both sets. It was shown, by means of correlation testing and paired-samples t-tests, that the solutions selected by the TOPSIS methods are similar regardless of the method used for calculating the fitness functions.

Since travel time is more likely to be lognormally distributed, a second set of tests was done, using Solomon's instances. Using 500 generations and a population of 200 chromosomes, the result of the improved VEGA algorithm showed that for problems with a large number of chromosomes (50 and 100 customers) using $w$=100 results was a better solution than when using $w$=1, while for problems with a small number of customers (25 and 50), no significant difference was found. Since it is known that the number of generations used by a genetic algorithm may affect its results, and since in real-time applications, the number of generations is bounded by the time given to the algorithm to come up with a solution, the algorithm was tested again, this time when the stopping condition was 30 minutes of running time, instead of the 500 generations. In all cases, the result obtained by the algorithm when $w$=1 are better than the results obtained when $w$=100. Furthermore, when $w$=1, the algorithm converges to the best solution much faster than when $w$=100.

# 8. Setting Wait Time Parameter

The first objective function considered in chapter 3.3.1 is minimizing the total travel time, and is defined as

$$\begin{aligned}\min \quad Z = &\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(C'^{t}_{ij}, TW_j^S - t\right) + ST'_j + WT'_j\right]x'^{mt}_{ij} + \\ &+\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(\bar{C}^{t}_{ij}, TW_j^S - t\right) + ST_j + WT_j\right]x^{mt}_{ij}\end{aligned} \tag{8.1}$$

where the total traveling time is composed from two parts, (1) the known traveling time, $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(C'^{t}_{ij}, TW_j^S - t\right) + ST'_j + WT'_j\right]x'^{mt}_{ij}$, and (2) the unknown traveling time, $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(\bar{C}^{t}_{ij}, TW_j^S - t\right) + ST_j + WT_j\right]x^{mt}_{ij}$.

If by leaving node *i* at time *t* a vehicle reaches node *j* before its time window's start time (meaning $t + C^t_{ij} < TW_j^S$, where $C_{ij}$ is the traveling time for traveling from node *i* to node *j* and $TW_j^S$ customer *j*'s time window's start time), then the vehicle has to wait until the beginning of the time window in order to start serving. Otherwise, it starts serving when it arrives. The time between the time the vehicle left node *i* towards node *j*, denoted as *t*, and the time it starts serving node *j* can be formulated as $\max\left(C^t_{ij}, TW_j^S - t\right)$. If node *j* is a customer, then both service time at customer *j*, $ST_j$, and waiting time at customer *j*, $WT_j$, have to be added to the traveling time. But if node *j* is the depot, then both service time, $ST_j$, and waiting time, $WT_j$, are equal to 0. Therefore, the time passed since a vehicle left node *i* towards node *j* and the time it left node *j* can be defined as $\max\left(C^t_{ij}, TW_j^S - t\right) + ST_j + WT_j$.

For each edge $e \in E = \left\{(i, j) : i, j \in V, i < j\right\}$, there exists a decision variable $x^{mt}_{ij}$, defined as 1 if edge *e* was traveled at time *t* by vehicle *m*, and otherwise it is defined as 0.

Multiplying the above notation, $\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j$, by the decision variable $x_{ij}^{mt}$, gives us the time passed since vehicle *m* left node *i* towards node *j* at time *t* and the time it left node *j*, if such a vehicle exists, otherwise it is 0.

Let's refer to the time passed since vehicle *m* left node *i* towards node *j* at time *t* and the time it left node *j* multiplied by the decision variable, $\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right] x_{ij}^{mt}$, as the true travel time from node *i* to node *j*. By summing all possible true travel times, $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{T}\left[\max\left(C_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right] x_{ij}^{mt}$, we get the total travel time, which we want to minimize.

The total travel time can be decomposed into two parts, the known travel time and the unknown travel time. If the planning time, $t_s$, is not equal to 0, then we are not at the beginning of the day, and some vehicles have already been sent to customers. In this case, information regarding traveled edges, travel costs, service time and waiting time is already known for every edge traveled and for every customer visited before $t_s$.

Let $C'^{t}_{ij}$ donate the known cost from traveling from node *i* to node *j* at time *t*, where $t < t_s$. Similarly, let $x'^{mt}_{ij}$ denote the known decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t < t_s$, and 0, otherwise. The known traveled cost can be defined as $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(C'^{t}_{ij}, TW_j^S - t\right) + ST'_j + WT'_j\right] x'^{mt}_{ij}$.

The unknown traveling cost can be defined as $\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right] x_{ij}^{mt}$, therefore, the total traveling cost is the sum of the known traveling cost and the unknown traveling cost,

$$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(C'^{t}_{ij}, TW_j^S - t\right) + ST'_j + WT'_j\right] x'^{mt}_{ij} +$$

$$+\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(\bar{C}_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right] x_{ij}^{mt}$$

The cost function, $\bar{C}_{ij}^{t}$, and service times, $ST_i$, as well as customers' time windows, $TW^S$ and $TW^E$, are either determined by environmental conditions (such as road conditions, traffic jams, etc.) or by the customer (service time). However, waiting time, defined as the time the vehicle left from customer *i* to customer *j*, $t_2$, minus the time a vehicle finished serving customer *i*, $t_1$ $(t_2 - t_1)$, is totally determined by the algorithm. Therefore, the question is what is the best time range from which the algorithm should select the waiting time so it will converge to the optimal solution as fast as possible (less iterations), .in respect to all objective functions.

In order to find the best waiting time range, several tests were conducted, using Solomon's C101, R101 and RC101 instances for 25, 50 and 100 customers. Each instance was solved 50 times by the algorithm, 10 times with waiting time in the range of 0 to 5 minutes, 10 times with waiting time in the range of 0 to 10, 10 times with waiting time in the range of 0 to 15, 10 times with waiting time in the range of 0 to 20 and 10 times with waiting time in the range of 0 to 25. All instances were solved with respect to all objective functions described in chapter 3.3. Since Solomon's instances were designed for TWVRP, a simple modification had to be done. Solomon's instances provide the location of each customer, assuming that the travel speed is constant. Since this is not the case in this problem, time intervals were added (24 of them) and for each time interval a lognormal random travel time function was assigned for which $\sigma$=0.03 and $\mu$=4.1 (these values may slightly change between time interval) and therefore, the average traveling speed is 60 KM/H. In order to decrease running time, Solomon's instances were solved using the improved VEGA algorithm, using 500 generations and population size of 200.

For each instance, in order to predict the value of each objective function as a function of the waiting time range, linear regression was used. The results are summarized in Table 8.1.

| Problem | Objective | Coefficients | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | Constant | Time range in minutes | | | | |
| | | | 0-10 | 0-15 | 0-20 | 0-25 | |
| C101-25 | Travel Time | 22.573 | 1.933 | 5.037 | 5.804 | 4.57 | 0.473 |
| | Number Of Vehicles | 3.667 | 1 | 0.667 | 0.667 | 0.333 | 0.206 |
| | Tour Balance | 5.4 | 1.537 | 2.848 | -0.26 | 0.79 | 0.225 |
| | Customer's Dissatisfaction | 0.7 | 0.088 | 1.911 | 1.872 | 0.999 | 0.809 |
| | Arrival Time of Last Vehicle | 19.206 | -0.234 | 1.15 | 0.85 | 0.85 | 0.467 |

| Problem | Objective | Coefficients | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | Constant | Time range in minutes | | | | |
| | | | 0-10 | 0-15 | 0-20 | 0-25 | |
| R101-25 | Travel Time | 10.664 | 1.734 | 1.228 | 3.993 | 4.273 | 0.711 |
| | Number Of Vehicles | 8.333 | -1 | -1.333 | 0 | 0 | 0.183 |
| | Tour Balance | 0.386 | 0.179 | 0.348 | -0.123 | 0.093 | 0.149 |
| | Customer's Dissatisfaction | 1.471 | 0.334 | 0.752 | 1.475 | 0.702 | 0.507 |
| | Arrival Time of Last Vehicle | 10.711 | 0.261 | 0.528 | 0.795 | 0.584 | 0.371 |
| RC101-25 | Travel Time | 11.633 | -0.267 | 3.227 | 3.473 | 6.366 | 0.933 |
| | Number Of Vehicles | 7 | -2.333 | -1.667 | 0.667 | 1 | 0.718 |
| | Tour Balance | 0.512 | 2.577 | -0.157 | -0.131 | -0.096 | 0.328 |
| | Customer's Dissatisfaction | 0.982 | -0.343 | 1.072 | 1.201 | 1.156 | 0.609 |
| | Arrival Time of Last Vehicle | 10.933 | 1.289 | 0.489 | 1.033 | 0.639 | 0.178 |
| C101-50 | Travel Time | 52.256 | 2.26 | 11.049 | 17.622 | 14.061 | 0.474 |
| | Number Of Vehicles | 7.667 | 0.333 | 3.333 | 4 | 1 | 0.3 |
| | Tour Balance | 6.814 | -4.465 | -4.349 | -5.079 | -2.461 | 0.363 |
| | Customer's Dissatisfaction | 4.599 | 9.201 | 4.373 | 6.423 | 4.491 | 0.33 |
| | Arrival Time of Last Vehicle | 19.6 | 2.806 | 2.461 | 2.744 | 3.322 | 0.761 |
| R101-50 | Travel Time | 29.101 | 0.799 | 9.831 | 8.339 | 13.843 | 0.777 |
| | Number Of Vehicles | 21 | -1.667 | 6 | 2.333 | 4.667 | 0.515 |
| | Tour Balance | 0.094 | 0.028 | -0.005 | 0.03 | 0.022 | 0.159 |
| | Customer's Dissatisfaction | 2.376 | 0.495 | 0.515 | 0.813 | 1.699 | 0.316 |
| | Arrival Time of Last Vehicle | 10.65 | 0.245 | 0.439 | 0.45 | 0.672 | 0.402 |
| RC101-50 | Travel Time | 36.339 | -0.301 | 2.265 | 6.928 | 12.679 | 0.714 |
| | Number Of Vehicles | 26.667 | -4333 | -3.667 | -1 | -1 | 0.233 |
| | Tour Balance | 0.076 | 0.045 | 0.054 | 0.064 | 0.027 | 0.497 |
| | Customer's Dissatisfaction | 1.191 | 0.9 | 1.181 | 2.09 | 1.703 | 0.533 |
| | Arrival Time of Last Vehicle | 10.861 | 0.578 | 0.489 | 0.678 | 0.861 | 0.592 |
| C101-100 | Travel Time | 328.573 | 39.44 | 14.238 | 65.881 | 37.271 | 0.762 |
| | Number Of Vehicles | 67.333 | 6.333 | -1.333 | 6.667 | 1 | 0.674 |
| | Tour Balance | 0.147 | -0.067 | -0.053 | -0.068 | 0.073 | 0.584 |
| | Customer's Dissatisfaction | 3.552 | -0.266 | 1.659 | 0.734 | 1.117 | 0.226 |
| | Arrival Time of Last Vehicle | 20.8 | -0.317 | 1.205 | 0.577 | -0.095 | 0.363 |
| R101-100 | Travel Time | 120.922 | 10.309 | 18.671 | 29.994 | 40.312 | 0.903 |
| | Number Of Vehicles | 94 | 1 | -0.667 | 0 | 0.333 | 0.027 |
| | Tour Balance | 0.015 | 0.002 | 0.003 | 0.006 | 0.007 | 0.612 |
| | Customer's Dissatisfaction | 0.018 | 0.213 | 0.881 | 1.529 | 2.355 | 0.976 |
| | Arrival Time of Last Vehicle | 10.239 | 0.134 | 0.317 | 0.517 | 0.995 | 0.9 |
| RC101-100 | Travel Time | 119.993 | -5.981 | 24.803 | 25.344 | 40.512 | 0.967 |
| | Number Of Vehicles | 93.333 | -11.333 | 1 | -2.667 | 0 | 0.629 |
| | Tour Balance | 0.016 | 0.008 | -0.001 | 0.002 | 0.004 | 0.343 |
| | Customer's Dissatisfaction | 0.064 | 0.306 | -0.037 | 0.23 | 0.473 | 0.706 |
| | Arrival Time of Last Vehicle | 10.222 | 0.411 | 0.25 | 0.406 | 0.844 | 0.911 |

**Table 8.1 – Linear regression results**

Based on the results of the linear regression the expected value of each objective for every instance was calculated for the different waiting time ranges. The results are summarized in Table 8.2.

| | | $R^2$ | 0-5 | 0-10 | 0-15 | 0-20 | 0-25 |
|---|---|---|---|---|---|---|---|
| 25/C101 | Travel Time | 0.473 | 22.573 | 24.506 | 27.61 | 28.377 | 27.143 |
| | Number of Vehicles | 0.206 | 3.667 | 4.667 | 4.334 | 4.334 | 4 |
| | StdDev | 0.225 | 5.4 | 6.937 | 8.24 | 5.14 | 6.19 |
| | DSF | 0.809 | 0.7 | 0.788 | 2.611 | 2.572 | 1.699 |
| | Latest Arrival | 0.467 | 19.206 | 18.972 | 20.356 | 20.056 | 20.056 |
| 25/R101 | Travel Time | 0.711 | 10.644 | 12.378 | 11.872 | 14.577 | 14.917 |
| | Number of Vehicles | 0.183 | 8.333 | 7.333 | 7 | 8.333 | 8.333 |
| | StdDev | 0.149 | 0.386 | 0.565 | 0.734 | 0.509 | 0.479 |
| | DSF | 0.507 | 1.471 | 1.805 | 2.223 | 2.946 | 2.173 |
| | Latest Arrival | 0.371 | 10.711 | 10.972 | 11.239 | 11.506 | 11.295 |
| 25/RC101 | Travel Time | 0.933 | 11.633 | 11.357 | 14.86 | 15.106 | 17.999 |
| | Number of Vehicles | 0.718 | 7 | 4.667 | 8.667 | 7.667 | 9 |
| | StdDev | 0.328 | 0.512 | 3.089 | 0.335 | 0.381 | 0.416 |
| | DSF | 0.609 | 0.982 | 0.639 | 2.054 | 2.183 | 2.138 |
| | Latest Arrival | 0.178 | 10.933 | 12.222 | 11.422 | 11.966 | 11.572 |
| 50/C101 | Travel Time | 0.474 | 52.256 | 54.516 | 63.305 | 69.878 | 66.317 |
| | Number of Vehicles | 0.301 | 7.667 | 8 | 11 | 11.667 | 8.667 |
| | StdDev | 0.363 | 6.814 | 2.349 | 2.465 | 1.735 | 4.353 |
| | DSF | 0.331 | 4.599 | 13.799 | 8.972 | 11.022 | 9.09 |
| | Latest Arrival | 0.761 | 19.6 | 22.406 | 22.061 | 22.344 | 22.922 |
| 50/R101 | Travel Time | 0.777 | 29.101 | 29.9 | 38.932 | 37.5 | 42.945 |
| | Number of Vehicles | 0.515 | 21 | 19.333 | 27 | 23.333 | 25.667 |
| | StdDev | 0.159 | 0.094 | 0.121 | 0.089 | 0.124 | 0.116 |
| | DSF | 0.316 | 2.376 | 2.871 | 2.891 | 3.189 | 4.075 |
| | Latest Arrival | 0.402 | 10.65 | 10.894 | 11.089 | 11.1 | 11.322 |
| 50/RC101 | Travel Time | 0.714 | 36.339 | 36.039 | 38.604 | 43.267 | 49.018 |
| | Number of Vehicles | 0.233 | 26.667 | 22.333 | 23 | 23.667 | 25.667 |
| | StdDev | 0.497 | 0.076 | 0.121 | 0.131 | 0.14 | 0.104 |
| | DSF | 0.533 | 1.191 | 2.091 | 2.372 | 3.281 | 2.894 |
| | Latest Arrival | 0.592 | 10.861 | 11.439 | 11.35 | 11.539 | 11.722 |
| 100/C101 | Travel Time | 0.762 | 328.573 | 368.013 | 342.81 | 394.454 | 365.843 |
| | Number of Vehicles | 0.674 | 67.333 | 73.667 | 66 | 74 | 68.333 |
| | StdDev | 0.584 | 0.147 | 0.081 | 0.094 | 0.079 | 0.22 |
| | DSF | 0.226 | 3.552 | 3.286 | 5.211 | 4.286 | 4.729 |
| | Latest Arrival | 0.363 | 20.8 | 20.484 | 22.006 | 21.378 | 20.706 |
| 100/R101 | Travel Time | 0.903 | 120.922 | 131.23 | 139.592 | 150.916 | 161.233 |
| | Number of Vehicles | 0.027 | 94 | 95 | 93.333 | 94 | 94.333 |
| | StdDev | 0.612 | 0.015 | 0.016 | 0.018 | 0.02 | 0.022 |
| | DSF | 0.976 | 0.018 | 0.231 | 0.899 | 1.546 | 2.373 |
| | Latest Arrival | 0.9 | 10.239 | 10.372 | 10.556 | 10.756 | 11.233 |
| 100/RC101 | Travel Time | 0.967 | 119.933 | 113.952 | 144.735 | 145.277 | 160.444 |
| | Number of Vehicles | 0.629 | 93.333 | 82 | 97.333 | 90.667 | 93.333 |
| | StdDev | 0.343 | 0.016 | 0.025 | 0.016 | 0.019 | 0.02 |
| | DSF | 0.706 | 0.064 | 0.37 | 0.027 | 0.294 | 0.537 |
| | Latest Arrival | 0.911 | 10.222 | 10.633 | 10.472 | 10.628 | 11.067 |

**Table 8.2 - Predicted values of objective functions, based on regression**

As can see from the results, 25 out of 45 objectives (nine problems, each with five objectives) are best obtained when the waiting time is in the range of 0 to 5 minutes. 11

out of 45 objectives are best obtained when the waiting time is in the range of 0 to 10 minutes. 6 out of 45 objectives are best obtained when the waiting time is in the range of 0 to 15 minutes. 3 out of 45 objectives are best obtained when the waiting time is in the range of 0 to 20 minutes. None of the objectives are best obtained when the waiting time is in the range of 0 to 25 minutes. However, half of functions found by the linear regression (23 out of 45), have an $R^2$ value lower than 0.75. This means that the value of half of the objective functions calculated based on the functions found by the regression, are probably not close to the true value expected.

As a result, averages comparison was done and used as well. The results are summarized in Table 8.3.

| | | 0-5 | 0-10 | 0-15 | 0-20 | 0-25 |
|---|---|---|---|---|---|---|
| 25/C101 | Travel Time | 22.573 | 24.506 | 27.611 | 28.378 | 27.144 |
| | Number of Vehicles | 3.667 | 4.667 | 4.333 | 4.333 | 4.000 |
| | StdDev | 5.400 | 6.936 | 8.248 | 5.140 | 6.189 |
| | DSF | 0.700 | 0.787 | 2.610 | 2.571 | 1.698 |
| | Latest Arrival | 19.206 | 18.972 | 20.355 | 20.055 | 20.056 |
| 25/R101 | Travel Time | 10.644 | 12.378 | 11.872 | 14.577 | 14.917 |
| | Number of Vehicles | 8.333 | 7.333 | 7.000 | 8.333 | 8.333 |
| | StdDev | 0.386 | 0.565 | 0.733 | 0.509 | 0.479 |
| | DSF | 1.471 | 1.804 | 2.223 | 2.946 | 2.173 |
| | Latest Arrival | 10.711 | 10.972 | 11.239 | 11.506 | 11.295 |
| 25/RC101 | Travel Time | 11.633 | 11.366 | 14.860 | 15.106 | 17.999 |
| | Number of Vehicles | 7.000 | 4.667 | 8.667 | 7.667 | 9.000 |
| | StdDev | 0.512 | 3.089 | 0.355 | 0.381 | 0.416 |
| | DSF | 0.982 | 0.639 | 2.053 | 2.183 | 2.138 |
| | Latest Arrival | 10.933 | 12.222 | 11.422 | 11.967 | 11.572 |
| 50/C101 | Travel Time | 52.256 | 54.516 | 63.305 | 69.877 | 66.317 |
| | Number of Vehicles | 7.667 | 8.000 | 11.000 | 11.667 | 8.667 |
| | StdDev | 6.814 | 2.349 | 2.465 | 1.735 | 4.353 |
| | DSF | 4.599 | 13.799 | 8.972 | 11.022 | 9.090 |
| | Latest Arrival | 19.600 | 22.406 | 22.061 | 22.344 | 22.922 |
| 50/R101 | Travel Time | 29.101 | 29.900 | 38.932 | 37.500 | 42.945 |
| | Number of Vehicles | 21.000 | 19.333 | 27.000 | 23.333 | 25.667 |
| | StdDev | 0.094 | 0.121 | 0.089 | 0.124 | 0.116 |
| | DSF | 2.376 | 2.871 | 2.891 | 3.189 | 4.075 |
| | Latest Arrival | 10.650 | 10.894 | 11.089 | 11.100 | 11.322 |
| 50/RC101 | Travel Time | 36.339 | 36.039 | 38.604 | 43.267 | 49.018 |
| | Number of Vehicles | 26.667 | 22.333 | 23.000 | 23.667 | 25.667 |
| | StdDev | 0.076 | 0.121 | 0.131 | 0.140 | 0.104 |
| | DSF | 1.191 | 2.091 | 2.372 | 3.281 | 2.894 |
| | Latest Arrival | 10.861 | 11.439 | 11.350 | 11.539 | 11.722 |
| 100/C101 | Travel Time | 328.573 | 368.013 | 342.810 | 394.454 | 365.843 |
| | Number of Vehicles | 67.333 | 73.667 | 66.000 | 74.000 | 68.333 |
| | StdDev | 0.147 | 0.081 | 0.094 | 0.079 | 0.220 |
| | DSF | 3.552 | 3.286 | 5.211 | 4.286 | 4.729 |

| | | 0-5 | 0-10 | 0-15 | 0-20 | 0-25 |
|---|---|---|---|---|---|---|
| | Latest Arrival | 20.800 | 20.484 | 22.006 | 21.378 | 20.706 |
| 100/R101 | Travel Time | 120.922 | 131.230 | 139.592 | 150.916 | 161.233 |
| | Number of Vehicles | 94.000 | 95.000 | 93.333 | 94.000 | 94.333 |
| | StdDev | 0.015 | 0.016 | 0.018 | 0.020 | 0.022 |
| | DSF | 0.018 | 0.231 | 0.899 | 1.546 | 2.373 |
| | Latest Arrival | 10.239 | 10.372 | 10.556 | 10.756 | 11.233 |
| 100/RC101 | Travel Time | 119.933 | 113.952 | 144.735 | 145.277 | 160.444 |
| | Number of Vehicles | 93.333 | 82.000 | 97.333 | 90.667 | 93.333 |
| | StdDev | 0.016 | 0.025 | 0.016 | 0.019 | 0.020 |
| | DSF | 0.064 | 0.370 | 0.027 | 0.294 | 0.537 |
| | Latest Arrival | 10.222 | 10.633 | 10.472 | 10.628 | 11.067 |

**Table 8.3 – Average values of objective functions, based on experiments**

The results are similar to the results obtained by using the functions found by the linear regression. 25 out of 45 objectives (nine problems, each with five objectives) are best obtained when the waiting time is in the range of 0 to 5 minutes. 11 out of 45 objectives are best obtained when the waiting time is in the range of 0 to 10 minutes. 6 out of 45 objectives are best obtained when the waiting time is in the range of 0 to 15 minutes. 3 out of 45 objectives are best obtained when the waiting time is in the range of 0 to 20 minutes. None of the objectives are best obtained when the waiting time is in the range of 0 to 25 minutes.

When comparing averages, statistical tests usually have to be done in order to verify that there is a difference between two populations. In this case, such tests were not conducted for the following reasons:

1. Assuming that there is a difference between two populations. In this case, it is obvious that the population with the lower average value is better (assuming minimization).
2. On the other hand, if there is no difference between the two populations, then there is no advantage or disadvantage in choosing the population with the lower average.

It can therefore be concluded that in all cases, it is possible , to choose the population whose results have a lower average value.

## 8.1. Summary

This chapter deals with the waiting time parameter. Waiting time is the time a vehicle waits after it has finished serving a customer before it starts driving to the next customer. Service time is determined by the algorithm, and can be any value in a pre-determined

range. Therefore, the question is, What is the best time range from which the algorithm should select the waiting time so it will converge to the optimal solution as fast as possible (less iterations), with respect to all objective functions?

In order to find the best waiting time range, a set of tests was done using Solomon's C101, R101 and RC101 instances for 25, 50 and 100 customers, each solved 10 times. Based on the results of the test instances, for each instance, in order to predict the value of each objective function as a function of the waiting time range, linear regression was used. The results of the linear regression showed that in more than half of the cases, the best results were obtained when the waiting time range was between 0 and 5 minutes. However, half of functions found by the linear regression (23 out of 45), have a value of $R^2 < 0.75$. This means that the value of half of the objective functions calculated based on the functions found by the regression, are probably not close to the true value expected, so therefore, a comparison of averages was conducted and used as well.

The results of the averages comparison were similar to the results obtained by using the functions found by the linear regression. More than half of the objectives (25 out of 45) are best obtained when the waiting time is in the range of 0 to 5 minutes.

Based on the results obtained using linear regression and the results obtained using comparison of averages, it is best to use a waiting time within the range of 0 to 5 minutes.

# 9. The Customer Satisfaction Function

In a traditional VRPTW, a feasible solution must satisfy all time windows. When a customer is served within its specified time window, the supplier's service level is satisfactory; otherwise, it is not satisfactory. Therefore, a customer's satisfaction level (which is equal to the supplier's service level) can be described using a binary variable. The customer satisfaction level is defined as 1 if the service time falls within the specified time window; otherwise, it is defined as 0. The service level function of the customer is described in Figure 9.1.

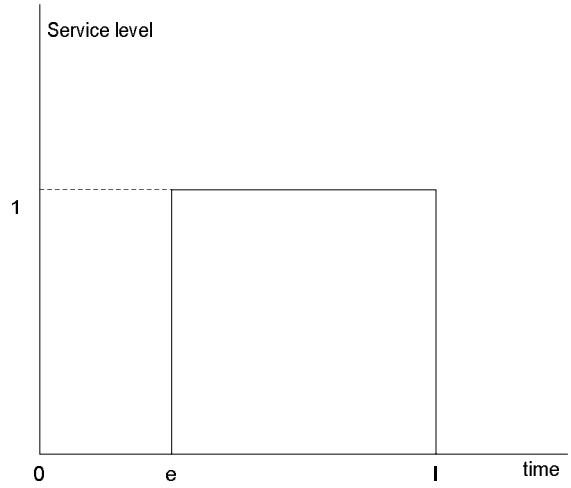



**Figure 9.1 – The service level function of a hard time window**

Time windows may sometimes be violated for economic and operational reasons. However, certain bounds on the violation (earliness or lateness) exist, which a customer can endure. The following two concepts are introduced to describe these bounds.

Let $EET_i$ denote endurable earliness time, the earliest service time that customer *i* can endure when a service starts earlier than $TW_i^S$, and let $ELT_i$ denote endurable lateness time, the latest service time that customer *i* can endure when a service starts later than $TW_i^E$.

The following example describes the relationship of $TW_i^S$, $TW_i^E$, $EET_i$ and $ELT_i$. A factory needs some kind of raw material for its daily production. Every day, the factory

opens at 8:00 and production starts at 10:00. The raw material is sent from an upstream supplier and the process of unloading the raw material requires 30 minutes. The factory specifies its preferred delivery time window to be [8:30, 9:00], because materials delivered within that time window can be directly moved to the workshop without any tardiness. However, the factory is not operating in a just-in-time mode; the delivery can be a little earlier or later than the specified time window. A reasonable combination of EET and ELT could be [8:00, 9:30]. If the materials are delivered within [8:00, 8:30], instead of being moved directly into the workshop, they must be stored in the warehouse because of limited space in the workshop. Of course, this is not what the manager of the factory wishes to see, but it is acceptable. If the materials are delivered within [9:00, 9:30], no inventories have to be held; however, this requires that the execution of the production plan will have a higher accuracy, which will reduce the robustness of the production operations in the factory. Since the factory opens at 8:00, deliveries before 8:00 must wait outside the factory. When production procedure starts at 10:00, delivery after 9:30 is totally unacceptable because of the 30-minutes unloading process. Simply put, although the manager of the factory will be happiest to be served within [8:30, 9:00], the manager will also be reasonably satisfied if served within [8:00, 8:30] or [9:00, 9:30]; however, the result of this is that the customer's satisfaction declines, and deliveries made before 8:00 or after 9:30 are not acceptable. Similar scenarios also appear in dial-a-ride problems.

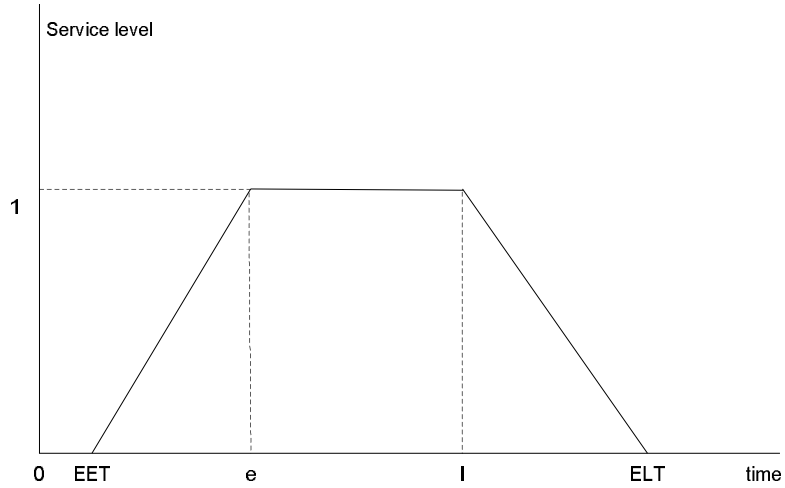



**Figure 9.2 – The service level function of fuzzy time windows**

As discussed, the service may start outside the time window $\left[TW_i^S, TW_i^E\right]$, and the bounds of acceptable earliness and lateness are described by $EET_i$ and $ELT_i$, respectively. Obviously, the earliness and lateness are highly related to the quality of the supplier's service. The customer's level of satisfaction response to a given service time will no longer simply be "good" or "bad"; but between "good" and "bad". For example, the customer may say, "it's alright" to be served within $\left[EET_i, TW_i^S\right]$ or $\left[TW_i^E, ELT_i\right]$. In either case, the service level cannot be described only by two states (0 or 1).

For problems involving personal human emotions, fuzzy set theory is a strong tool. Intuitively, with the concepts of $EET_i$ and $ELT_i$, the supplier's service level for each customer can be described by a fuzzy membership function:

$$S_i(t) = \begin{cases} 0, & t < EET_i \\ f_i(t), & EET_i \le t < TW_i^S \\ 1, & TW_i^S \le t < TW_i^E \\ g_i(t), & TW_i^E \le t < ELT_i \\ 0, & ELT_i \le t \end{cases} \tag{8.2}$$

when in most recent research, $f_i(t)$ is defined as

$$f_i(t) = \frac{t - EET_i}{TW_i^S - EET_i} \tag{8.3}$$

and $g_i(t)$ is defined as

$$g_i(t) = \frac{ELT_i - t}{ELT_i - TW_i^E} \tag{8.4}$$

However, since customer's satisfaction level, as a function of the deviation from the customer's time window, usually cannot be described as a linear function, the following function, which better describes customer's satisfaction, is used.

$$f_i(t) = \frac{\sum_{j=1}^{n} \beta_i^j \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^{\alpha_i}}{\sum_{j=1}^{n} \beta_i^j} \tag{8.5}$$

$$g_i(t) = \frac{\sum_{j=1}^{m} \delta_i^j \left( \frac{ELT - t}{ELT - TW_i^E} \right)^{\gamma_i}}{\sum_{j=1}^{m} \delta_i^j} \tag{8.6}$$

Assuming that each customer has his/her own satisfaction function, $S_i(t)$, and that the service provider assigns as an *importance* factor, $\sigma_i$, to each customer that states how important it is to satisfy customer *i* compared to all other customers, the maximizing customers' satisfaction objective can be described as

$$\min \quad Z = \sum_{i=0}^{n} \sigma_i S_i \left( \sum_{j=0}^{N} \sum_{m=1}^{M} \left( \sum_{t=0}^{t_S - 1} \left( \left( t + \bar{C}_{ji}^t \right) \bar{x}_{ji}^{mt} \right) + \sum_{t=t_S}^{T} \left( \left( t + C_{ij}^t \right) x_{ji}^{mt} \right) \right) \right) \tag{8.7}$$

As stated before, customer satisfaction or dissatisfaction (which is equal to one minus satisfaction value) involves personal human emotions. The satisfaction function may be decomposed to several functions; each focusing on a different factor which influences the overall satisfaction. For example, if a supplier is late and workers have to stay after their regular working hours, one function may consider physiological aspects, such as the dissatisfaction due to workers' loss of family time. Another function may consider financial aspects, such as workers' overtime salary. While there might be several satisfaction (or dissatisfaction) functions, in this study, one satisfaction function is considered, which includes all satisfaction aspects as perceived by the customers.

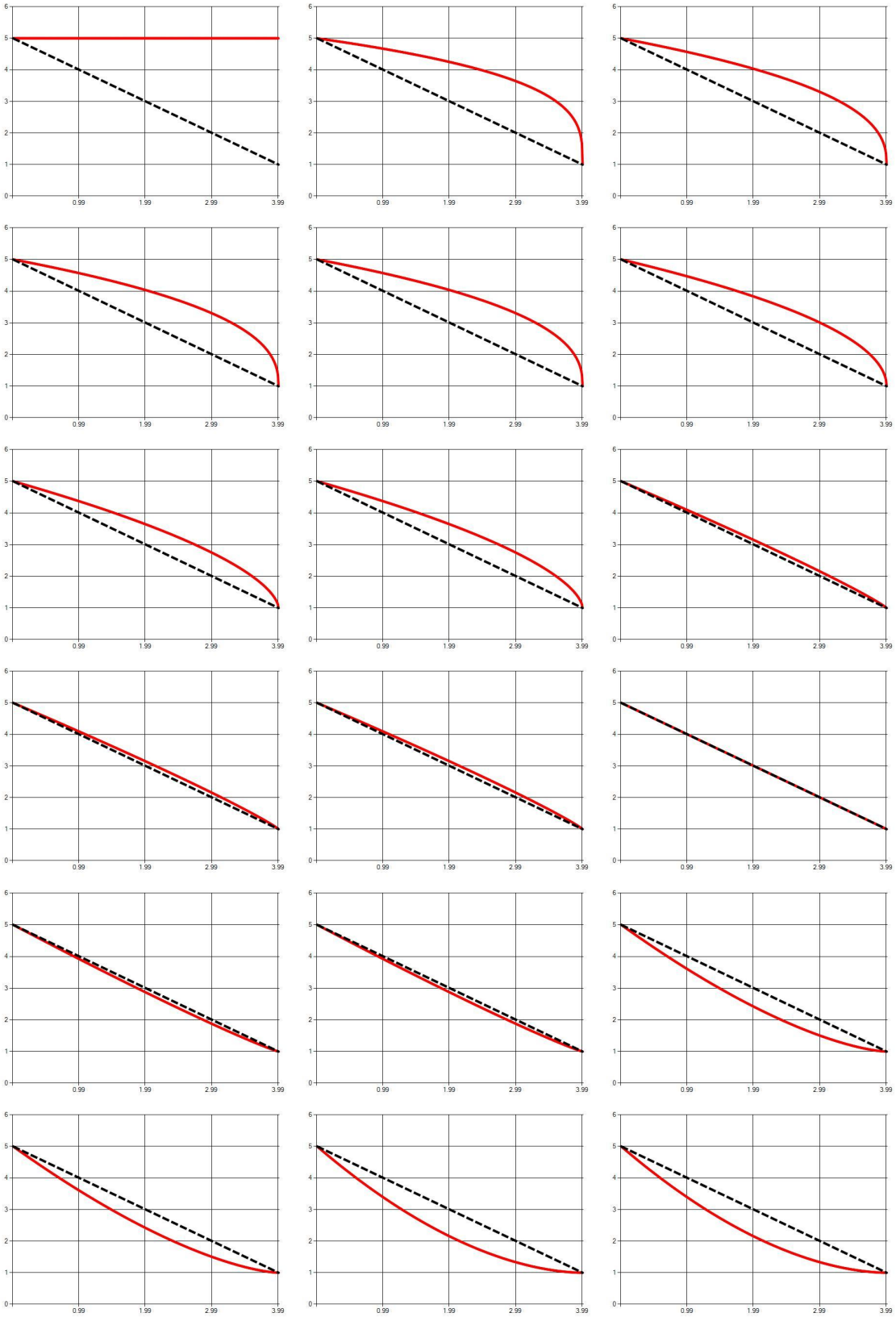
In order to get an impression of the possible values of $\alpha$ and $\gamma$, 38 customers (people), were asked, using questionnaires, about their general satisfaction value when a supplier or other service provider arrives earlier than expected, between 30 minutes to four hours, with 30 minute intervals. Similarly, they were asked for their general satisfaction value

when a supplier or other service provider arrives later than expected, from 30 minutes to four hours, with 30 minute intervals.

Based on the results of the questionnaires, the following analysis was conducted for each customer.

1. Each customer is assigned to a satisfaction function, using curve fitting. CenterSpace Software's NMath numerical library provides object-oriented components for mathematical, engineering, scientific, and financial applications on the .NET platform. One of the components provided by CenterSpace Software's NMath numerical library is the PolynomialLeastSquares class, which performs a least squares fit of a Polynomial to a set of points. The data provided by each customer was used as an input to the PolynomialLeastSquares, which was used to calculate five degree polynomial curve fitting functions (for both earliness and lateness).
2. For each customer, using the five degree polynomials computed, the values of 400 sample points were collected. These sample points represent the satisfaction value of the customer for supplier earliness and lateness, from 0 minutes up to 4 hours, with 0.01 minutes time interval.
3. Since the functions obtained are not presented in the form of the functions described in (8.5) and (8.6), , the least squares method is used in order to find a function in the form of (8.5) and (8.6) for each customer, which best approximates the customer's satisfaction functions. In order to approximate the customer's satisfaction functions, each sample point was evaluated using the $\left[(4-t)/4\right]^{\alpha}$ satisfaction function, with various values of $\alpha$, starting with (-1000) and ending with 1000, with an interval of 0.1. The value of each sample point obtained from the satisfaction function is subtracted from the value obtained from the five degree polynomial. The difference value is then squared, and all the squared values are summed. The value of $\alpha$ is chosen so that the sum of all squared differences is minimal.

The functions calculated for all 38 customers based on the questionnaires are presented in Figure 9.3 and Figure 9.4.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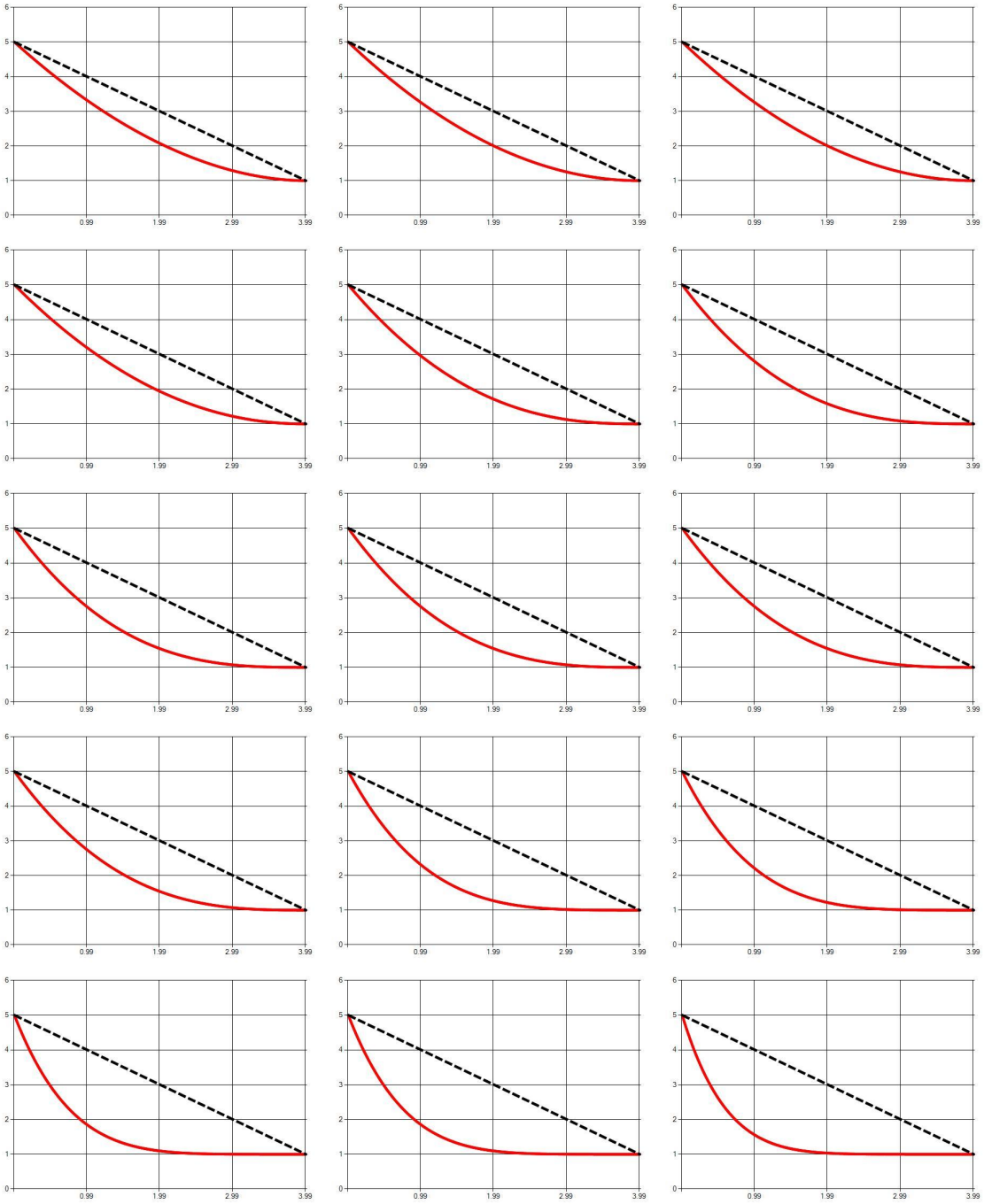
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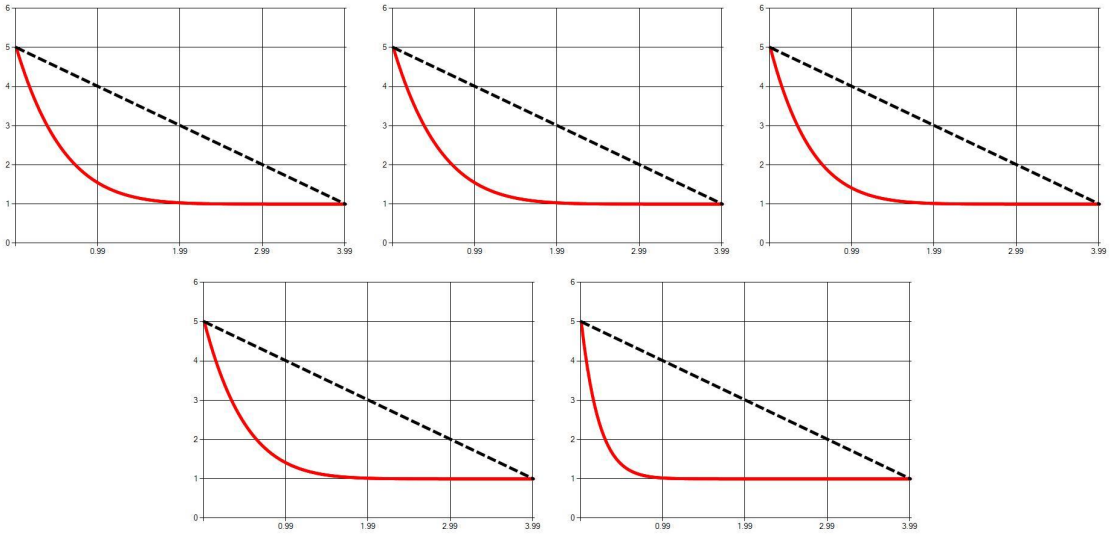
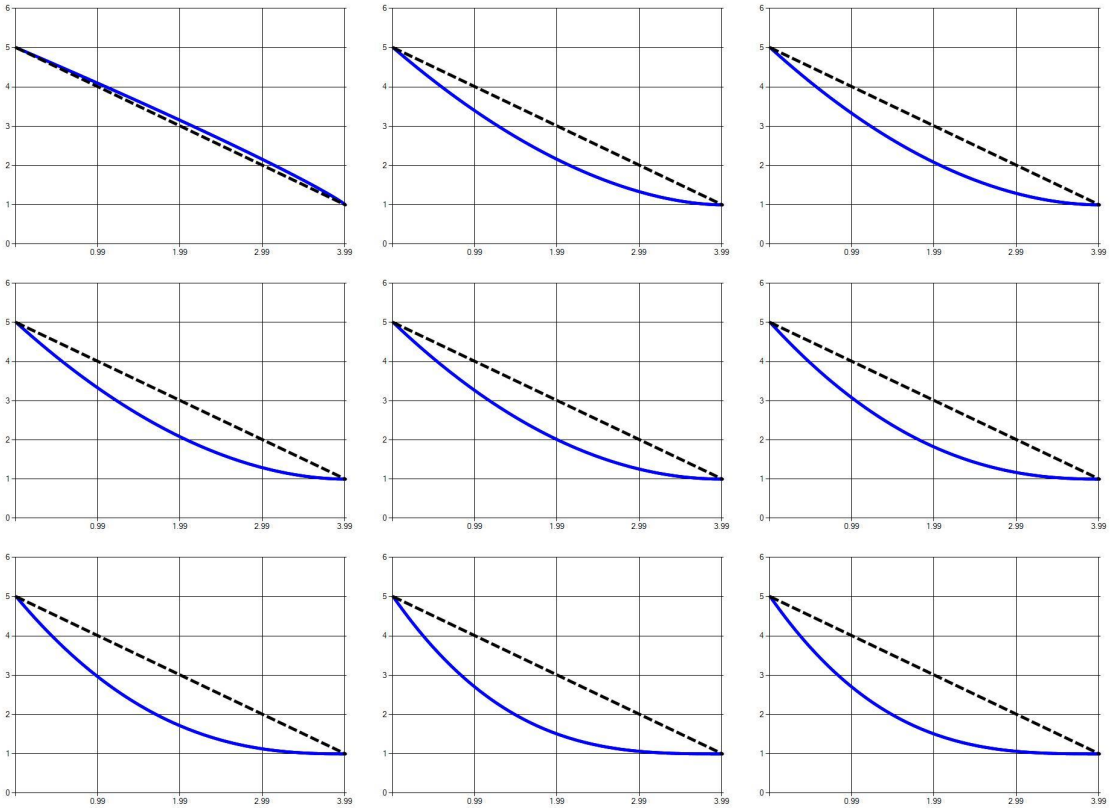
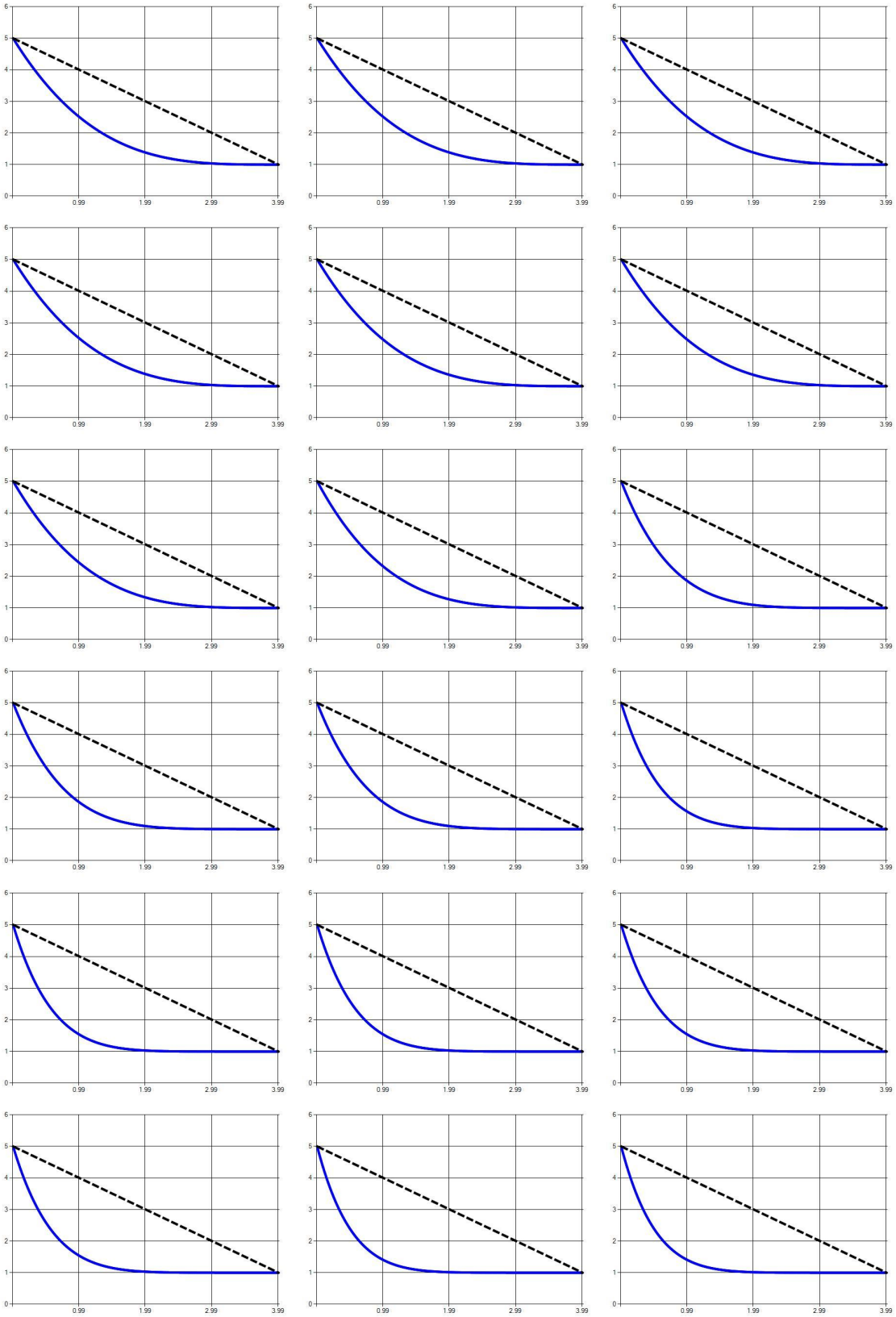


**Figure 9.3 – Various satisfaction functions for supplier earliness**

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2.99
3.99

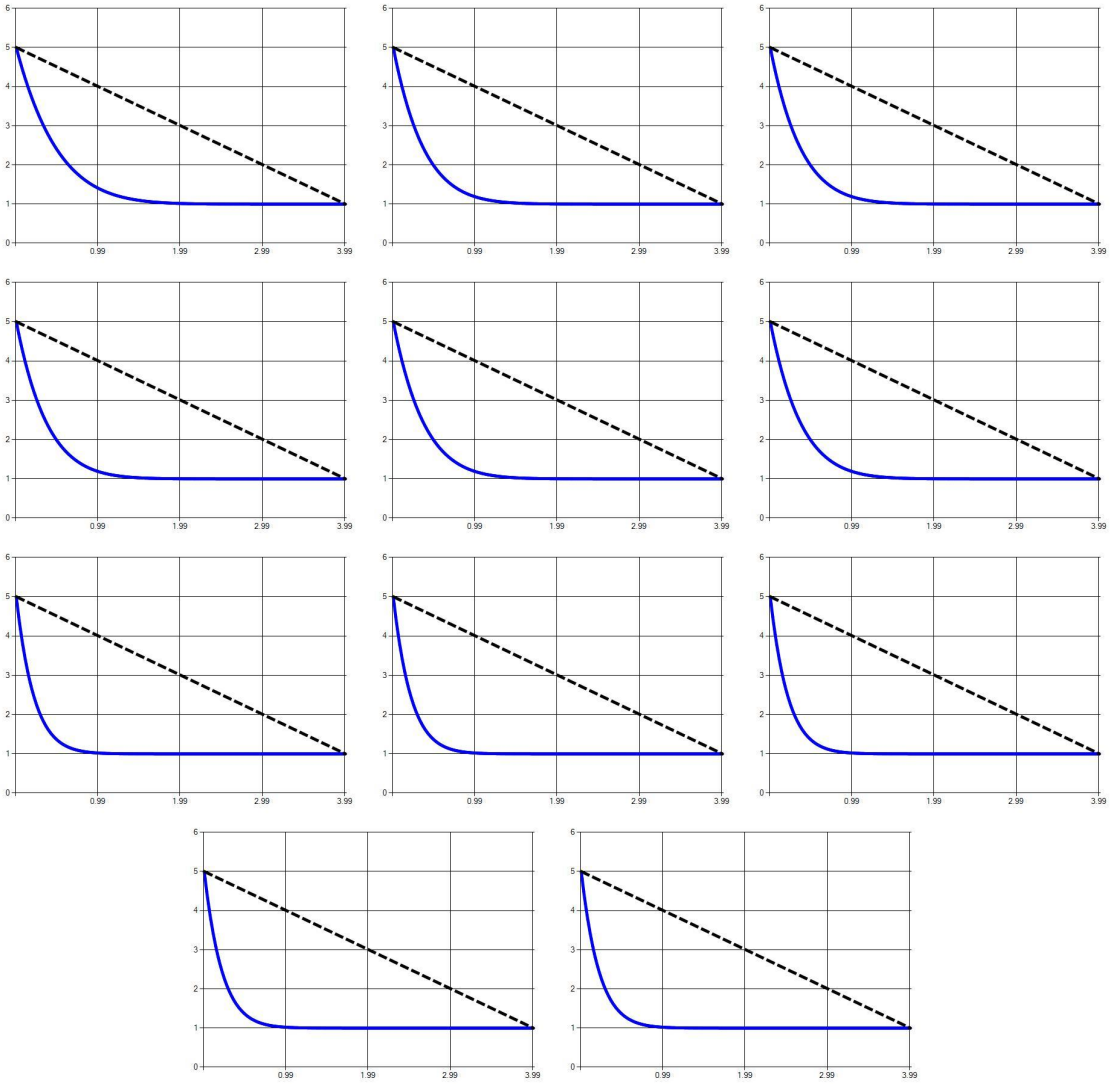

**Figure 9.4 – Various satisfaction functions for supplier lateness**

The results can be summarized as follows:

1. For earliness, 6 out of the 38 customers feel indifferent to the supplier being early. This means that although the supplier arrives much earlier than expected, the customer's satisfaction is minimally influenced.
2. For earliness, for 6 out of the 38 customers, the customer's satisfaction correlates linearly with the supplier being early.
3. For earliness, 18 out of the 38 customers are sensitive to the supplier being early. This means that their satisfaction level declines dramatically when the supplier arrives earlier than expected.

4. For lateness, for 1 out of the 38 customers, the customer's satisfaction changed linearly, and correlates with the supplier being late.
5. For lateness, 34 out of the 38 customers are sensitive to the supplier being late. This means that their satisfaction level declines dramatically when the supplier arrives later than expected.

## 9.1. Summary

In a traditional VRPTW, a feasible solution must satisfy all time windows. When a customer is served within his/her specified time window, the supplier’s service level is satisfactory or equal to 1; otherwise, the service is not satisfactory or equal to 0. Time windows may sometimes be violated for economic and operational reasons. However, certain bounds exist on the violation (earliness or lateness) that a customer can endure. Obviously, the earliness and lateness are closely related to the supplier service level, and therefore, the service level cannot be described by only two states (0 or 1).

To get an impression of how the service level, known as customer's satisfaction, changes as a function of bounds on the violation (earliness or lateness), 38 customers, using questionnaires, were asked for their overall satisfaction value when a supplier or other service provider arrives within 30 minutes to four hours, with 30 minutes intervals, earlier than expected. Similarly, they were asked for their overall satisfaction value when a supplier or other service provider arrives 30 minutes to four hours, in 30 minutes intervals, later than expected.

Each customer, based on the results of his questionnaire, was assigned a satisfaction function. From these functions, it can be concluded that most customers are sensitive to suppliers being either early or late, and their satisfaction level declines dramatically when the supplier arrives earlier/later than expected.

# 10. Case Study

In previous chapters, the vehicle routing problem has been described and various algorithms based on exact methods; heuristics and meta-heuristics have been reviewed. The real-time multi-objective vehicle routing problem has been described and formulated, and three evolutionary algorithms for solving it were also presented. Various tests using randomly generated networks were done on the three algorithms, mainly for calibration purposes. These tests showed that each of the three algorithms when applied on a randomly generated network converge towards a better solution.

The goal of this chapter is to compare the results of the three algorithms using a case study. The case study is based on two networks that are based on a real-world transportation network, including the locations of the depot, the customers and information about travel time between the different customers. The case study is performed using simulation.

## 10.1. Network

Two transportation networks, each based on real-file information, each with different characteristics, were generated. Both networks are based on Israel's road network.

The first network is based on the greater Tel-Aviv metropolitan area's urban road network. In this network, there are 45 customers (not including the depot).

"Mega Ba'ir" (Mega in the city) is a super-market chain store, with more than 80 branches (as reported in their Internet site). In order to make the network as realistic as possible, the locations of each one of the 45 customers were chosen according to the locations of the stores throughout the greater Tel-Aviv metropolitan area. Using "Google Maps", the shortest distance (based on actual network) between every two customers was found.

Next, for each edge in the network, the traveling time for different times of the day was collected (see 10.1.1).

Each customer is also associated with a time window. The time windows are randomly generated, and are based on the following assumptions:

1. The minimum possible time window start time, *PSTW*, is equal to 8:00 am plus the travel time it takes to get from the depot to the customer (when leaving the depot at 8:00 am), assuming that the distance from the depot to the customer is known, and the travel speed is 15 kilometers per hour.
2. The time window start time, *STW*, is based on the possible time windows start time, and is a random value within the range of *PSTW* to *PSTW*+1.5 (plus one and a half hour).
3. The time window end time, *ETW*, is based on the time windows start time, and is a random value within the range of *STW*+0.5 to *STW*+3.

Each customer is also associated with a randomly generated demand, in the range of 10 to 50, similar to the demands used in Solomon's instances.


**Figure 10.1 – Locations of 45 customers in the greater Tel-Aviv metropolitan area**

The second network is based on Israel's interurban road network. In this network, there are 34 customers (not including the depot). The 34 customers are major cities in Israel.

Using “Google Maps”, the shortest distance (based on actual network) between every two customers was found.

Next, for each day, the traveling times at different hours during the day were collected. Each customer is also associated with a time window. The time windows are randomly generated, and are based on the following assumptions:

1. The minimum possible time window start time, *PSTW*, is equal to 8:00 am plus the travel time it takes to get from the depot to the customer (when leaving the depot at

8:00 am), assuming that the distance from the depot to the customer is known, and the travel speed is 70 kilometers per hour.

2. The time window start time, *STW*, is based on the possible time windows start time, and is a random value within the range of *PSTW* to *PSTW*+1.5 (plus one and a half hour).
3. The time window end time, *ETW*, is based on the time windows start time, and is a random value within the range of *STW*+0.5 to *STW*+3.

Each customer is also associated with a randomly generated demand, in the range of 10 to 50, once again, similar to the demands used in Solomon's instances.


**Figure 10.2 - Locations of 39 customers in Israel**

In each test problem, half of the customers are considered as customers with unknown demands. The customers who are considered as customers with unknown demands are the customers with the latest time window start time. Each unknown demand is revealed to the simulation at least two hours before the beginning of the time window.

### 10.1.1. Collecting Real-World Travel Time Information

The main problem in building a real-world network is knowing how travel time changes during the day for each of the edges in the network. In recent years, "Google" started providing real-time traffic information in their "Google Maps" service.

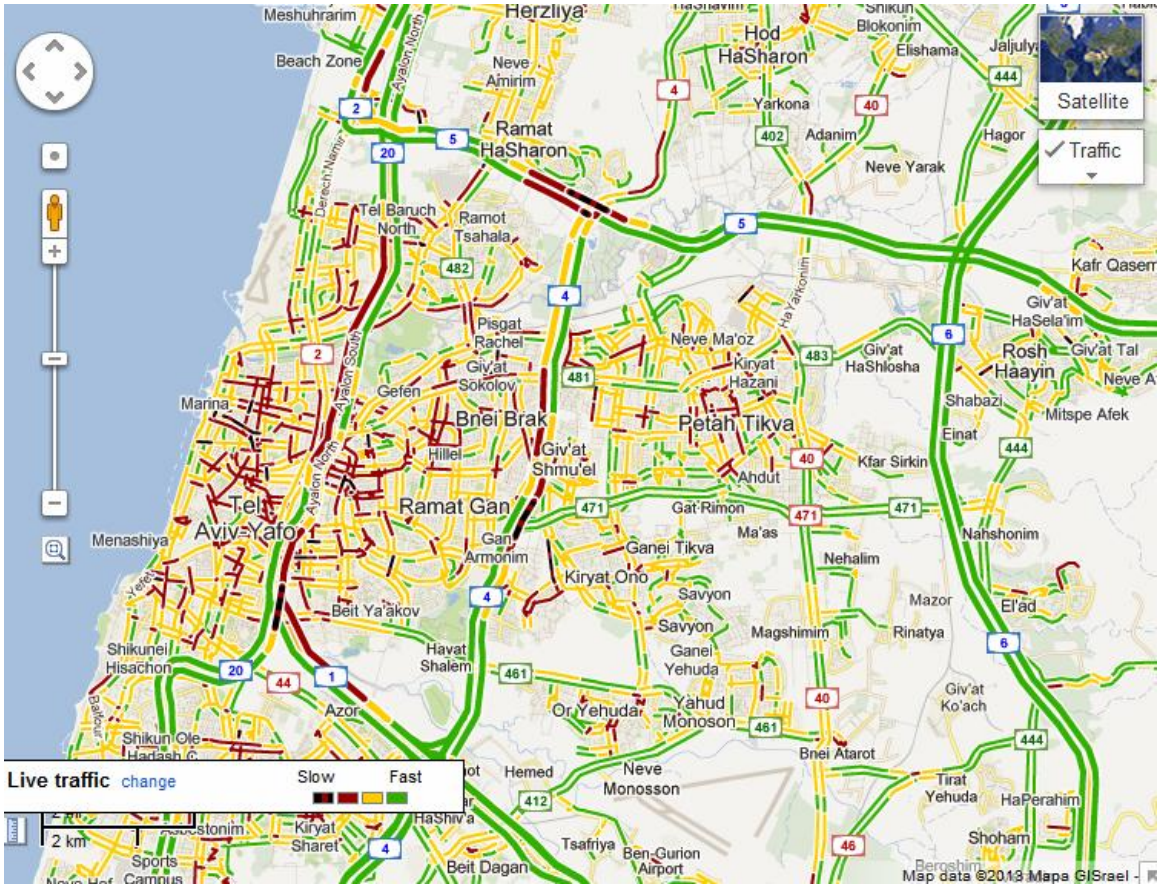


**Figure 10.3 – "Google Maps" with real-time traffic information**

When querying "Google Map" service for a route between two locations, it now provides several possible routes, for each of which it provides its length, its average travel time and its travel time in current traffic conditions. Based on this information, it is possible to calculate the travel speed for each route based on its length and travel time.

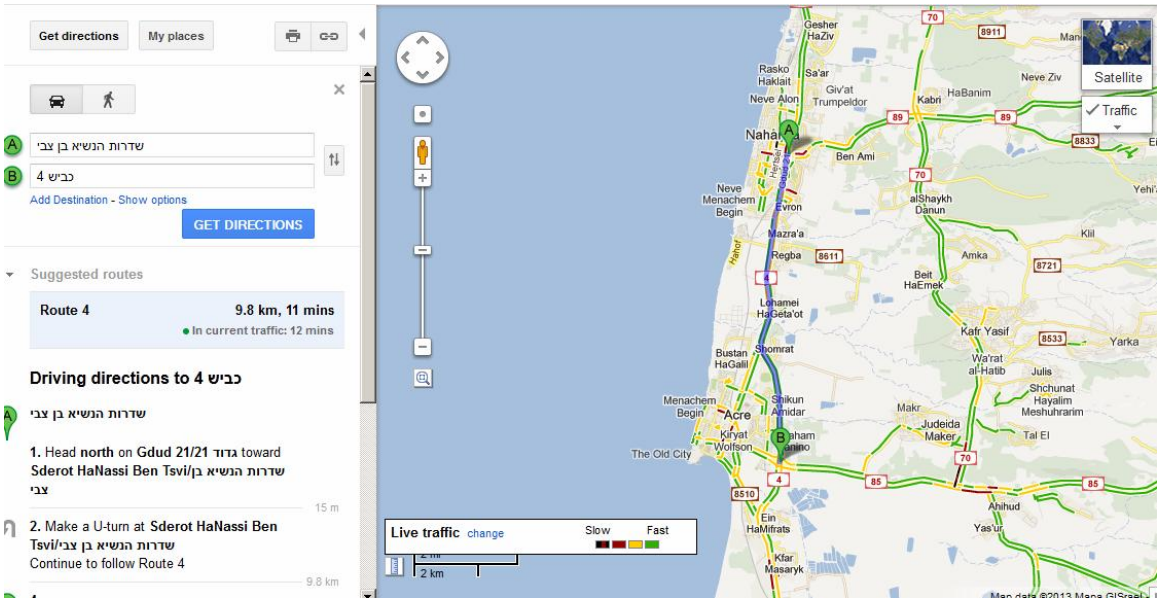


**Figure 10.4 – "Google Maps" with shortest route between two points**

In order to collect travel time information using Google maps service, a small application has been written. This application works as follows:

1. A text file containing a list of routes is read by the application.
2. In turn each route queries “Google maps” service.
3. The result of each query is then analyzed, and the information is saved (time of day, travel time and speed).
4. After all routes have been queried, the process starts from the beginning (stage 2).

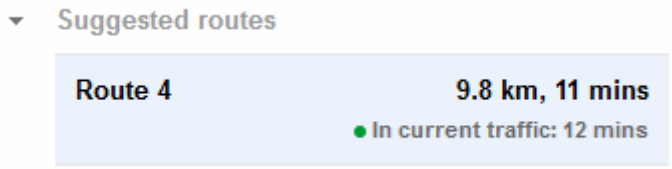


**Figure 10.5 – "Google Maps" with route travel time information**

It was noticed that when querying "Google Map" service for a route between two locations, at different times of the day, “Google Maps” may sometimes provide two or more different routes. This happens because “Google Maps" considers traffic conditions when calculating and suggesting a route. This behavior is problematic when collecting data for statistical analysis. To avoid this problem, mid-points were used. Mid-points are

points that must be included in the route. Using a mid-point guarantees that when there are multiple possible routes between two points, the same route will be selected at any time of the day.

The travel time information collecting application collected information during a period of six months, for work days only (in order to avoid statistical bais caused by different usage of the road network on work days and weekends). The application has been used on six different computers, in order to increase the number of samples collected for each edge.

## 10.2. The Time-Dependent Shortest Path Algorithm (TDSP)

This section describes the shortest path algorithm used to calculate the travel time between two customers, assuming that the travel time depends upon the departure time and the edges traveled. Development of a new efficient TDSP algorithm is not within the scope of this research. Therefore, the TDSP algorithm described next can be replaced with any other TDSP algorithm, if the algorithm is more efficient than the proposed TDSP algorithm.

The proposed algorithm is an extended version of the Dijkstra's algorithm, similar to the one used by Malandraki (1986) to calculate TDSP with travel time functions.

Each customer is associated with two properties: (1) previous customer (node) and (2) arrival time.

Dijkstra($G$,$s$,$t$)
    for all $u$ in $V$-$s$ set a($u$)=$\infty$, $p$($u$)=Unknown
    $a$($s$)=$t$, $p$($s$)=None
    $R$={}
    While $R \neq V$
        pick $u$ from $V$, $u$ is not in $R$ with smallest a($u$)
        $R$=$R$+$u$
        for all nodes $v$ adjacent to $u$
        if $d$($v$)>$d$($u$)+TravelTime($u$,$v$,$a$($u$))
            $d$($v$)=$d$($u$)+ TravelTime ($u$,$v$,$a$($u$))
            $p$($v$)=$u$

The first stage of the Dijkstra's algorithm is an initialization stage. In the initialization stage each node ($u$), except the first node, is assigned with the following values: (1) previous customer ($p(u)$): unknown and (2) arrival time ($a(u)$): unknown, represented by the value of positive infinity. The first node, which is the origin node is assigned with the following values: (1) previous customer: none and (2) arrival time: the time that a vehicle leaves the current node ($t$).

After the initialization stage, an iterative stage begins. In each iteration, a node is selected. The selected node, $u$, is the node with the smallest arrival time, which has not been selected earlier (not present in $R$) .Next, the selected node, $u$, is added to the set of selected nodes, $R$. For all nodes $v$ adjacent to $u$, the arrival time is calculated using the *TravelTime* function described next. If the known arrival time of node $v$ is higher than the calculated arrival time, then the arrival time of node $v$ is set to the calculated arrival time and the value of the previous customer associated with node $v$ is set to $u$. This process continues until the set $R$ contains all the nodes of the graph.

```
TravelTime(u,v,t)
    TempLength = length of edge (u,v)
    TimeIntervalIndex = ⌊t/ LengthOfTimeInterval ⌋
    TimeLeft = LengthOfTimeInterval*(TimeIntervalIndex)-t
    TravelTime = 0
    Start loop
        Speed = Get random speed of edge in time period = TimeIntervalIndex
        TravelTime = TravelTime+TempLength/Speed
        if TempLength/Speed <= TimeLeft then
            exit loop and return TravelTime as edge travel time
        TempLength = TempLength-(TimeLeft*Speed)
        TimeIntervalIndex  = TimeIntervalIndex +1
        TimeLeft = LengthOfTimeInterval
        repeat loop
```

Several aiding variables are used to calculate the travel time. The first variable is *TempLength*. This variable contains the length of the traveled edge. The *TimeIntervalIndex* variable represents the time interval which corresponds to start time, *t* (in hours). *LengthOfTimeInterval* represents the duration, in hours, of a single time interval. *TimeIntervalIndex* equal to the time interval corresponds to the start time; therefore, *TimeIntervalIndex*+1 is the next time interval. Multiplying *TimeIntervalIndex*+1 by *LengthOfTimeInterval* and subtracting the start time, *t*, from the result, results in the time left from the start time, *t*, until the end of the current time interval, denoted by *TimeLeft*. The last aiding variable is *TravelTime*. This variable contains the edge travel time as calculated by the function.

It is now possible to start calculating the travel time. The calculation of the travel time begins with assigning the *Speed* variable with a random speed based on the statistical information collected to edge (*u*,*v*), based upon the relationship described in 3.1.2, in the time interval *TimeIntervalIndex*. *TravelTime* is then equal to the current *TravelTime* plus *TempLength* divided by *Speed*. This means that the travel time along the edge (u,v) is equal to the travel time known so far, plus the travel time of the untraveled part of the edge. If the added travel time is equal or less than the time left in the current time interval, the calculation is done, and the travel time is equal to *TravelTime*. Otherwise, it means that the edge is also traveled during the next time interval. In this case, a correction has to be made, because the travel speed in the next time interval is not necessarily equal to the travel speed in the current time interval. To correct the travel time, the *TempLength* variable is assigned with the part of the edge needed to be traveled in the next time interval; simultaneously, the *TimeIntervalIndex* is increased by 1. The calculation can now be repeated until the *TravelTime* is fully calculated.

## 10.3. Assumptions

Several assumptions are made in simulation of a full-day operation as follows.

1. The planning period starts at 7:00 am and ends at 8:00 am. Planning period refers to the time that the algorithm runs before the first vehicle has to leave the depot.
2. Service starts at 8:00 am, when the first vehicle leaves the depot, and ends when the last vehicle returns to the depot.

3. During the planning period, new information about customers' demand is not acceptable.
4. The workday is divided into 24 time intervals, each one hour long, starting at 0:00 am.
5. For each edge in the transportation network, the travel time is given using log-normal distribution functions, for each time internal.
6. Information about real travel times is known two in advance (i.e., for the next two time intervals), and is updated 15 minutes before the beginning of the hour.
7. Every half an hour on the hour, new vehicles that have to leave the depot, leave the depot to their first customers (this can happen due to new customers demands or due to route splitting made by the algorithm).
8. New customers demands are acceptable only if there is at least one vehicle which is not driving to the depot.
9. If all vehicles are either at the depot or driving to the depot, the algorithm stops working (end of the case study).
10. The capacity of a single vehicle is equal to 200 units, as in Solomon's instances.

## 10.4. Simulation

In order to perform the case study, simulation was used. The simulation is based on two processes running in parallel, that are exchanging information between each other. The two processes are the algorithm process and the simulation process.

The algorithm process is an implementation of each of the three EAs described earlier for exchanging information with the simulation process.

The steps of the algorithm process are as follows:

1. Generate set of initial solutions.
2. If there are pending operations in the operations queue, process the operations.
3. Generate the next set of solutions from current solutions.
4. If there are pending operations in the operations queue, process the operations.
5. If stop condition is met, finish, otherwise go to step 3.

Operations are requests raised by the simulation process, and stored in a special shared memory known as the operations queue, that have to be performed by the algorithm process. The algorithm process can handle four such operations there:

1. Add customer – when the simulation process identifies a new demand request from a new customer it raises an “Add customer” request. Adding a new customer means that in all solutions generated by the algorithm, there should be a route which includes the newly added customers. The simplest way to ensure that is by adding a new route to each one of the solutions. This new route is a simple route which starts at the depot, visits the newly added customer, and returns to the depot. Since various operations are then applied on each of the solutions, the newly added customer will be quickly managed in two other routes.
2. Remove customer - The simulation process keeps track of each of the vehicles that has left the depot. Each vehicle that has left the depot can be in one of the following two states, (1) "driving" and (2) "at the customer". When the vehicle changes its state from "at the customer" to "driving", it means that the previous customer has already been served, and therefore does not have to be visited anymore. As a result, the previously visited customer has to be removed from all the solutions evolved by the algorithm. The "remove customer" operation gets the number of the customer who has to be removed and then removes it from all the solutions evolved by the algorithm. It also makes sure that by removing the customer, there are no empty routes left in any of the solutions.
3. Request route - When a vehicle has finished serving a customer it is required to continue with its route toward the next customer. However, the vehicle, when leaving the depot, is not given a route, but instead is given only the first customer in the route. This is done since while the vehicle is driving towards its destination, the algorithm continues improving the solutions, and by providing a route to the vehicle it is likely to be given a route which will later be replaced with a better one. For that reason, whenever a vehicle has to start driving towards its next destination, the simulation process asks the algorithm process for the best routes known so far. From the routes, the simulation process determines the vehicle's next destination.
4. Fix solutions based on current route - The simulation process supervises all available vehicles. This is done using information provided by the algorithm process. When

choosing routes using the "request routes" operation, it is necessary that the selected routes will also appear in all other solutions evolved by the algorithm. The "fix solutions based on current routes" operation goes over each of the solutions evolved by the algorithm, and makes any necessary changes according to the selected routes.

The simulation process simulates an entire workday. It does so by handling each of the vehicles, collecting data about travel times and new customers' demands. The simulation process uses a timer to simulate an entire workday. A single timer event simulates one second in the real-world. When a timer event is fired, a special time variable, called *SimTime*, is increased by one second. Next, *SimTime* is compared against another variable called *NextDepartureTime*. The *NextDepartureTime* variable represents the time when new vehicles have to leave the depot on the new routes (usually, every half an hour on the hour). If *SimTime* is greater than *NextDepartureTime*, then the value of *NextDepartureTime* is recalculated. *CalcTime* is another variable maintained by the simulation process. *CalcTime* is the time that an operation has to be performed, and it can either be equal to the value of the *NextDepartureTime* variable, or to the earliest time that an assigned vehicle (a vehicle currently located at a customer) has to leave the customer to drive to another customer, whichever is earlier. If *SimTime* is equal or greater than *CalcTime*, and there are no pending operations in the operations queue, the following is done:

If *CalcTime* equals *NextDepartureTime*, then the "Request route", "Assign routes" and the "Fix solutions based on current route" operators are added to the operations queue.

Otherwise, for each assigned vehicle, the following is done:

1. If a vehicle is located at the customer, and according to *SimTime*, it has to leave and start driving to the next customer, then "Request route", "Assign routes" and the "Fix solutions based on current route" operators are added to the operations queue.
2. If a vehicle is driving to a customer, and according to SimTime it has arrived at the customer, then the vehicle's number is added to the "change vehicle status from driving to at customer" queue, and a "change vehicle status from driving to at customer" operator is added to the operators queue.
3. If a vehicle is located at the depot and is due to leave the depot at *SimTime*, then the vehicle's number is added to the "change vehicle status from waiting to driving"

queue, and a "change vehicle status from waiting to queue" operator is added to the operators queue.

As seen earlier, pending operations have to be processed before the simulation process can continue. There are several operators stored in the operators queue, that can be processed by the simulation process:

1. Assign route - the "assign routes" to vehicles operator is performed after a "request routes" operator has been performed by the algorithm process. After receiving a set of routes from the algorithm process, the simulation process must make sure that a vehicle is assigned to each route in the set of routes received from the algorithm process. Some of the routes already have vehicles assigned to them, while all the other routes, which are new routes, have to be assigned to new vehicles.
2. Change status from driving to “at a customer” - The "change status from driving to at customer operator" does the following for each vehicle whose status is changed from "driving" to "at a customer": first, it changes the status of the vehicle from "driving" to "at a customer", next the customer to whom the vehicle was driving is marked for deletion by adding it to the "remove customers" queue, and the "remove customer" and "fix solution based on current routes" are added to the operators queue.
3. Change status from waiting to driving– The "change status from waiting to driving" is a simple operator which changes the status of a waiting vehicle from waiting at the depot to driving.
4. New customer - The "new customer" operator gets a demand request from the user, adds the new customer and its request to the "new customers" queue, and adds an "add customer" operator to the operators queue.
5. Stop algorithm – The "stop algorithm" operator tells the algorithm process to stop. It is called when all vehicles are back at the depot.
6. Update time interval – The "update time interval" operator gets travel time information from the the algorithm process used and updated with it.

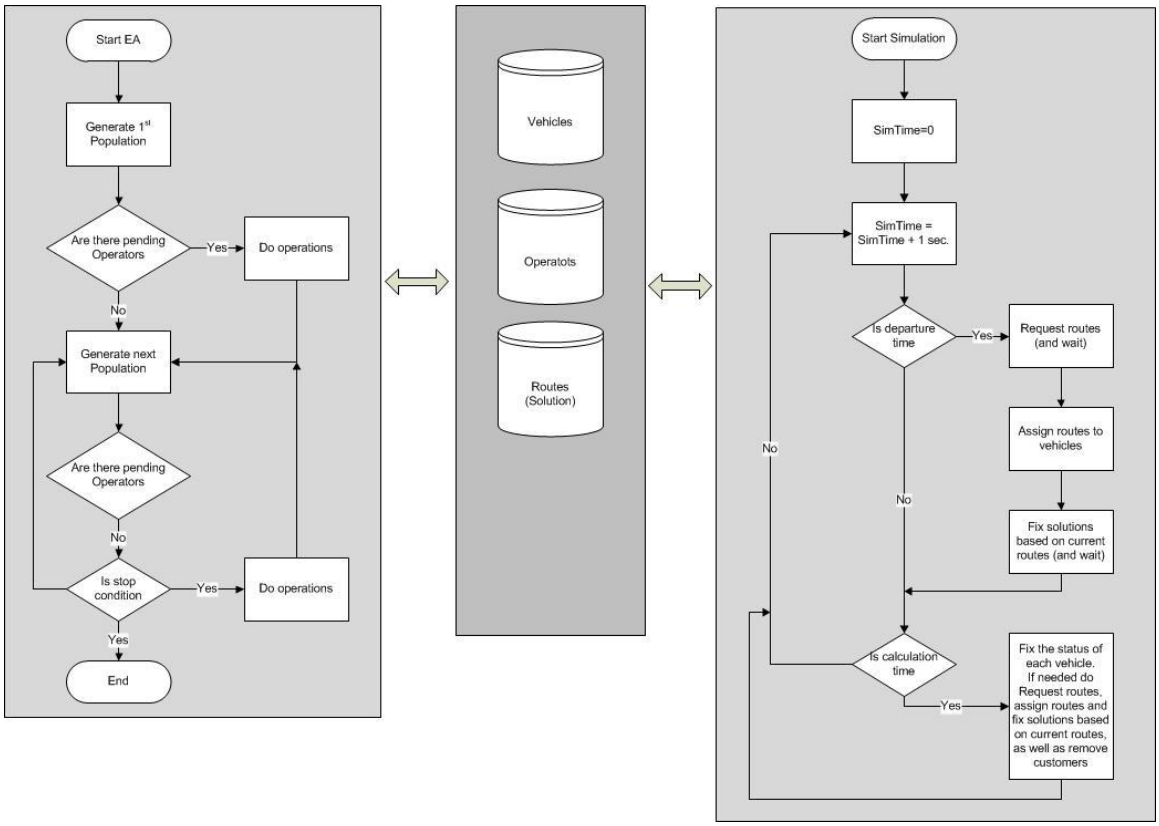


**Figure 10.6 – The relationsheep between the algorithm process and the simulation process**

## 10.5. Strategy of the Case Study

In this case study, five strategies for constructing the routing plan are considered. The five strategies are as follows:

1. The first strategy assumes that all information, including customers' demands and traveling time, is known in advanced. Based on this information, the algorithm runs for a pre-defined period of time, after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. All vehicles follow this set of routes.
2. The second strategy assumes that all information, including customer's demands and traveling time, is known in advanced. Based on this information, the algorithm runs for a pre-defined period of time; after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. Vehicles start driving according to this set of routes, while the algorithm continues to run. Whenever a vehicle arrives sat a customer, the customer is removed from all

solutions evolved by the algorithm. Whenever a vehicle has to leave a customer and drive to the next customer (or the depot), or at pre-defined time intervals, using the TOPSIS mechanism, a new set of routes is selected from the set of non-dominated solutions found by the algorithm. Driving vehicles are then rerouted according to the new set of routes, and new vehicles are assigned as needed. This operation is repeated until all customers have been served, and all vehicles have returned to the depot.

3. In the third strategy traveling time is unknown; however, all other information, including customers' demands, is known in advance. However, at pre-defined intervals, traveling time information for the next pre-defined time period is revealed to the algorithm. Based on this information, the algorithm runs for a pre-defined period of time, after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. Vehicles start driving according to this set of routes, while the algorithm continues to run. Whenever a vehicle at a customer, the customer is removed from all solutions evolved by the algorithm. Whenever a vehicle has to leave a customer and drive to the next customer (or the depot), or at pre-defined time intervals, using the TOPSIS mechanism, a new set of routes is selected from the set of non-dominated solutions found by the algorithm. Driving vehicles are then rerouted according to the new set of routes, and new vehicles are assigned as needed. This operation is repeated until all customers have been served, and all vehicles have returned to the depot.
4. In the fourth strategy customers' demands are unknown, while all other information, including traveling time, is known in advance. When the algorithm starts, demands of some of the customers are known. Based on this information, the algorithm runs for a pre-defined period of time; after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. Vehicles start driving according to this set of routes, while the algorithm continues to run. Whenever a vehicle arrives at a customer, the customer is removed from all solutions evolved by the algorithm. Simultaneously, new customers' demands are revealed to the algorithm, which, accordingly adds the new customers to the evolved solutions. Whenever a vehicle has to leave a customer and drive to the next customer (or the depot), or at pre-defined time intervals, using the TOPSIS mechanism, a new set of routes is selected from the set of non-dominated solutions found by the algorithm.

Driving vehicles are then rerouted according to the new set of routes, and new vehicles are assigned as needed. This operation is repeated until all customers have been served, and all vehicles returned to the depot.

5. In the fifth strategy, neither customers' demands nor traveling times are known in advance. When the algorithm starts, some of the customers' demands are known. In regard to traveling time, at pre-defined intervals, traveling time information for the next pre-defined time period is revealed to the algorithm. Based on this information, the algorithm runs for a pre-defined period of time, after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. Vehicles start driving according to this set of routes, while the algorithm continues to run. Whenever a vehicle arrives at a customer, the customer is removed from all solutions evolved by the algorithm. Simultaneously, new customers' demands are revealed to the algorithm, which accordingly adds the new customers to the evolved solutions. Whenever a vehicle has to leave a customer and drive to the next customer (or the depot), or at pre-defined time intervals, using the TOPSIS mechanism, a new set of routes is selected from the set of non-dominated solutions found by the algorithm. Driving vehicles are then rerouted according to the new set of routes, and new vehicles are assigned as needed. This operation is repeated until all customers have been served, and all vehicles have returned to the depot.

## 10.6. Case Study 1

In the first case study the test scenario is defined as follows:

1. Network: The greater Tel-Aviv metropolitan area transportation network.
2. Dissatisfaction function: It is assumed that the dissatisfaction functions of all customers are linear, meaning $f_i(t) = 1 - \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^1$ and $g_i(t) = 1 - \left( \frac{ELT - t}{ELT - TW_i^E} \right)^1$.

The test scenario is solved 100 times. In the first 50 times, it is assumed that all customers have the same priority. Under this assumption, the test scenario is solved 100 times using each of the 5 strategies described earlier. In the next 50 times, it is assumed

that each customer has a priority equal to its demand. Under this assumption, the test scenario is solved 10 times using each of the 5 strategies described earlier.

### 10.6.1. Priority Comparison

For each of our three algorithms, five paired-samples t-test were conducted to compare the total travel time obtained when all customers have the same priority vs. the travel time obtained when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.1.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 50.976 | 2.143 | 49.608 | 1.75 | 4.871 | 99 | 0 |
| | 2 | 58.415 | 1.462 | 57.645 | 3.042 | 2.298 | 99 | 0.024 |
| | 3 | 55.77 | 1.924 | 53.942 | 1.923 | 7.074 | 99 | 0 |
| | 4 | 60.557 | 2.205 | 57.922 | 3.496 | 6.013 | 99 | 0 |
| | 5 | 57.174 | 1.853 | 52.378 | 4.118 | 10.82 | 99 | 0 |
| SPEA2 | 1 | 59.176 | 3.406 | 54.554 | 3.264 | 9.323 | 99 | 0 |
| | 2 | 73.706 | 3.816 | 70.538 | 1.762 | 6.91 | 99 | 0 |
| | 3 | 58.394 | 12.22 | 64.69 | 1.25 | -5.063 | 99 | 0 |
| | 4 | 72.598 | 2.949 | 68.079 | 8.238 | 5.406 | 99 | 0 |
| | 5 | 64.424 | 0.914 | 58.762 | 14.57 | 3.876 | 99 | 0 |
| VE-ABC | 1 | 64.833 | 3.769 | 77.627 | 6.807 | -15.764 | 99 | 0 |
| | 2 | 81.972 | 2.272 | 80.89 | 2.937 | 3.01 | 99 | 0.003 |
| | 3 | 64.121 | 3.223 | 33.935 | 0.834 | 89.904 | 99 | 0 |
| | 4 | 78.618 | 2.689 | 81.198 | 2.284 | -7.596 | 99 | 0 |
| | 5 | 62.831 | 3.226 | 32.985 | 1.133 | 86.239 | 99 | 0 |

**Table 10.1 – Paired T-Test resuls for comparison of the total travel time for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for all strategies there is a significant difference in the travel time, which is lower when each customer has a different priority. For the SPEA2 algorithm, strategy 3 shows a significant difference in the travel time, which is lower when all customers have the same priorith. A significant difference in the travel time also exists for all other strategies, which is lower when each customer has a different priority. As for the VE-ABC algorithm, for strategies 1 and 4 there is a significant difference in the travel time, which is lower when all customers have the same priority. Similarly, for strategies 2, 3 and 5 there is a significant difference in the travel time, which becomes lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the number of vehicles needed when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.2.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 11 | 0.853 | 10.44 | 0.499 | 5.789 | 99 | 0 |
| | 2 | 19.91 | 2.332 | 20.29 | 2.775 | -1.023 | 99 | 0.309 |
| | 3 | 26.81 | 5.59 | 25.78 | 3.126 | 1.552 | 99 | 0.124 |
| | 4 | 21.95 | 2.928 | 20.6 | 3.83 | 2.834 | 99 | 0.006 |
| | 5 | 29.85 | 2.267 | 24.75 | 3.825 | 12.333 | 99 | 0 |
| SPEA2 | 1 | 15.4 | 1.463 | 13.46 | 1.167 | 9.779 | 99 | 0 |
| | 2 | 32.55 | 3.937 | 31.99 | 2.385 | 1.176 | 99 | 0.242 |
| | 3 | 34.25 | 6.838 | 37.47 | 1.784 | -4.509 | 99 | 0 |
| | 4 | 31.84 | 2.905 | 31.62 | 5.548 | 0.375 | 99 | 0.709 |
| | 5 | 37.63 | 1.412 | 31.56 | 7.296 | 7.984 | 99 | 0 |
| VE-ABC | 1 | 15.33 | 1.67 | 35.42 | 8.761 | -22.177 | 99 | 0 |
| | 2 | 34.21 | 2.54 | 38.98 | 0.804 | -17.667 | 99 | 0 |
| | 3 | 34.67 | 3.785 | 19.63 | 1.125 | 36.232 | 99 | 0 |
| | 4 | 31.62 | 3.09 | 38.9 | 1.567 | -20.261 | 99 | 0 |
| | 5 | 33.15 | 5.456 | 18.26 | 1.515 | 24.625 | 99 | 0 |

**Table 10.2 – Paired T-Test resuls for comparison of the number of vehicles needed for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results for strategies 1, 4 and 5, there is a significant difference in the number of vehicles needed, which is lower when each customer has a different priority. For the SPEA2 algorithm, for strategies 1, 3 and 5 there is a significant difference in the travel time, which is lower when each customer has a different priority. As for the VE-ABC algorithm, for all strategies there is a significant difference in the travel time. For strategies 1, 2 and 4, there is a significant difference in the travel time, which is lower when all customers have the same priority. Similarly, for strategies 3 and 5 there is a significant difference in the travel time, which becomes lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the balance of the tours when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.3.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 0.768 | 0.311 | 0.632 | 0.056 | 4.237 | 99 | 0 |
| | 2 | 0.394 | 0.262 | 0.368 | 0.284 | 0.715 | 99 | 0.476 |
| | 3 | 0.334 | 0.387 | 0.259 | 0.266 | 1.533 | 99 | 0.128 |
| | 4 | 0.49 | 0.324 | 0.46 | 0.31 | 0.732 | 99 | 0.466 |
| | 5 | 0.235 | 0.289 | 0.451 | 0.356 | -5.038 | 99 | 0 |
| SPEA2 | 1 | 0.365 | 0.179 | 0.363 | 0.2 | 0.053 | 99 | 0.958 |
| | 2 | 0.288 | 0.295 | 0.287 | 0.305 | 0.031 | 99 | 0.975 |
| | 3 | 0.286 | 0.319 | 0.19 | 0.229 | 2.337 | 99 | 0.021 |
| | 4 | 0.309 | 0.35 | 0.34 | 0.31 | -0.666 | 99 | 0.507 |
| | 5 | 0.258 | 0.307 | 0.214 | 0.273 | 1.175 | 99 | 0.243 |
| VE-ABC | 1 | 0.64 | 0.342 | 0.351 | 0.392 | 5.628 | 99 | 0 |
| | 2 | 0.294 | 0.288 | 0.145 | 0.188 | 4.646 | 99 | 0 |
| | 3 | 0.252 | 0.269 | 0.326 | 0.311 | -1.891 | 99 | 0.062 |
| | 4 | 0.343 | 0.304 | 0.255 | 0.332 | 2.254 | 99 | 0.026 |
| | 5 | 0.294 | 0.307 | 0.122 | 0.021 | 5.537 | 99 | 0 |

**Table 10.3 – Paired T-Test results for comparison of the balance of the tours for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategies 1 and 5 there is a significant difference in the tour balance. In strategy 1 the tour balance is lower (meaning more balanced) when each customer has a different priority. However, in strategy 5 the tour balance is lower when all customers have the same priority. For the SPEA2 algorithm, only strategy 3 shows a significant difference in the tour balance, which is lower when each customer has different priority. As for the VE-ABC algorithm, for all strategies but stratedy 3, there is a significant difference in the tour balance, which is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total dissatisfaction of the customers when all customers have the same priority vs. the total dissatisfaction of the customers when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.4.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 58.685 | 22.198 | 64.729 | 15.394 | -2.147 | 99 | 0.034 |
| | 2 | 7.512 | 8.336 | 3.539 | 2.047 | 4.769 | 99 | 0 |
| | 3 | 9.518 | 9.492 | 5.218 | 3.125 | 4.154 | 99 | 0 |
| | 4 | 12.981 | 10.097 | 2.796 | 1.567 | 9.879 | 99 | 0 |
| | 5 | 7.317 | 9.767 | 5.01 | 4.866 | 2.078 | 99 | 0.04 |
| SPEA2 | 1 | 40.797 | 10.131 | 0.147 | 0.225 | 40.139 | 99 | 0 |
| | 2 | 68.786 | 48.423 | 0.24 | 0.298 | 14.159 | 99 | 0 |
| | 3 | 53.119 | 25.344 | 0.174 | 0.229 | 20.894 | 99 | 0 |

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| | 4 | 56.901 | 27.795 | 0.3 | 0.271 | 20.411 | 99 | 0 |
| | 5 | 38.33 | 9.592 | 0.249 | 0.356 | 39.645 | 99 | 0 |
| VE-ABC | 1 | 1785.667 | 538.197 | 0.549 | 0.609 | 33.167 | 99 | 0 |
| | 2 | 73.677 | 24.78 | 0.306 | 0.345 | 29.598 | 99 | 0 |
| | 3 | 38.366 | 19.664 | 0.188 | 0.269 | 19.404 | 99 | 0 |
| | 4 | 78.169 | 37.324 | 0.236 | 0.255 | 20.875 | 99 | 0 |
| | 5 | 78.448 | 71.603 | 0.032 | 0.025 | 10.951 | 99 | 0 |

**Table 10.4 – Paired T-Test results for comparison of the total dissatisfaction of the customers for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategy 1, there is a significant difference in the total dissatisfaction of the customers, which is lower when all customers have the same priority. For all other strategies, there is also a significant difference in the total dissatisfaction of the customers, which is lower when each customer has a different priority. For both the SPEA2 algorithm and the VE-ABC algorithm, in all strategies there is a significant difference in the total dissatisfaction of the customers obtained when all customers have the same priority and when each customer has a different priority, which is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the arrival time of the last vehicle when all customers have the same priority vs. the arrival time of the last vehicle when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.5.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 14.262 | 0.343 | 14.728 | 0.575 | -7.619 | 99 | 0 |
| | 2 | 12.866 | 0.4 | 12.841 | 0.414 | 0.454 | 99 | 0.651 |
| | 3 | 12.342 | 0.571 | 12.753 | 0.546 | -4.914 | 99 | 0 |
| | 4 | 12.817 | 0.462 | 12.867 | 0.555 | -0.677 | 99 | 0.5 |
| | 5 | 12.179 | 0.469 | 12.605 | 0.587 | -6.298 | 99 | 0 |
| SPEA2 | 1 | 13.041 | 0.407 | 12.943 | 0.329 | 1.845 | 99 | 0.068 |
| | 2 | 12.176 | 0.451 | 12.18 | 0.315 | -0.059 | 99 | 0.953 |
| | 3 | 11.637 | 0.491 | 11.996 | 0.427 | -5.401 | 99 | 0 |
| | 4 | 12.163 | 0.35 | 12.098 | 0.439 | 1.202 | 99 | 0.232 |
| | 5 | 11.816 | 0.423 | 11.979 | 0.472 | -2.669 | 99 | 0.009 |
| VE-ABC | 1 | 14.364 | 0.555 | 12.491 | 1.022 | 16.089 | 99 | 0 |
| | 2 | 12.345 | 0.442 | 11.868 | 0.288 | 8.783 | 99 | 0 |
| | 3 | 12.069 | 0.454 | 11.16 | 0.328 | 14.824 | 99 | 0 |
| | 4 | 12.338 | 0.549 | 11.855 | 0.305 | 7.502 | 99 | 0 |
| | 5 | 12.078 | 0.535 | 11.246 | 0.216 | 14.655 | 99 | 0 |

**Table 10.5 – Paired T-Test results for comparison of the arrival time of the last vehicle for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategies 1, 3 and 5 there is a significant difference in the arrival time of the last vehicle, which is earlier when all customers have the same priority. For the SPEA2 algorithm, for strategies 3 and 5 there is a significant difference in the arrival time of the last vehicle, which is earlier when all customers have the same priority. As for the VE-ABC algorithm, in all strategies there is a significant difference in the arrival time of the last vehicle, which is later when each customer has a different priority.

#### 10.6.1.1 Conclusions

For the first objective function, total travel time, a significant difference in the solutions was found for the improved VEGA and SPEA2 algorithms, which is better when each customer has a different priority. However when using the VE-ABC algorithm, no significant difference in the solutions was found. Similar results were found for the 2$^{nd}$ objective function, number of vehicles needed and 4$^{th}$ objective function, customers' dissatisfaction.

For the 3$^{rd}$ objective function, tour balance, a significant difference in the solutions was found only for the VE-ABC algorithm, which is better when each customer has a different priority.

For the fifth objective, arrival time at the last customer, the best solutions are obtained when when all customers have the same priority, for the improved VEGA and SPEA2 algorithms, and when each customer has a different objective function, when using the VE-ABC algorithm.

### 10.6.2. Strategies Comparison – VEGA algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.6.

| Customes Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 50.976 | 2.143 | 58.415 | 1.462 | -27.741 | 99 | 0 |
| | 2 | 11 | 0.853 | 19.91 | 2.332 | -33.745 | 99 | 0 |
| | 3 | 0.768 | 0.311 | 0.394 | 0.262 | 9.104 | 99 | 0 |
| | 4 | 58.685 | 22.198 | 0.231 | 0.256 | 26.325 | 99 | 0 |
| | 5 | 14.262 | 0.343 | 12.866 | 0.4 | 26.671 | 99 | 0 |
| Different | 1 | 49.608 | 1.75 | 57.645 | 3.042 | -22.181 | 99 | 0 |
| | 2 | 10.44 | 0.499 | 20.29 | 2.775 | -34.765 | 99 | 0 |
| | 3 | 0.632 | 0.056 | 0.368 | 0.284 | 9.145 | 99 | 0 |
| | 4 | 2104.407 | 500.488 | 3.539 | 2.047 | 41.977 | 99 | 0 |
| | 5 | 14.728 | 0.575 | 12.841 | 0.414 | 25.481 | 99 | 0 |

**Table 10.6 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same and different priorities**

As it can be seen from the results, no matter whether all customers have the same priority or not, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether or not all customers have the same priority, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.7.

| Customes Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 50.976 | 2.143 | 55.77 | 1.924 | -16.39 | 99 | 0 |
| | 2 | 11 | 0.853 | 26.81 | 5.59 | -28.348 | 99 | 0 |
| | 3 | 0.768 | 0.311 | 0.334 | 0.387 | 9.407 | 99 | 0 |
| | 4 | 58.685 | 22.198 | 0.293 | 0.292 | 26.247 | 99 | 0 |
| | 5 | 14.262 | 0.343 | 12.342 | 0.571 | 30.748 | 99 | 0 |
| Different | 1 | 49.608 | 1.75 | 53.942 | 1.923 | -16.937 | 99 | 0 |
| | 2 | 10.44 | 0.499 | 25.78 | 3.126 | -47.969 | 99 | 0 |
| | 3 | 0.632 | 0.056 | 0.259 | 0.266 | 13.68 | 99 | 0 |
| | 4 | 2104.407 | 500.488 | 5.218 | 3.125 | 41.957 | 99 | 0 |
| | 5 | 14.728 | 0.575 | 12.753 | 0.546 | 27.621 | 99 | 0 |

**Table 10.7 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same and different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether or not all customers have the same priority, strategy 3 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.8.

| Customes Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 58.415 | 1.462 | 55.77 | 1.924 | 10.989 | 99 | 0 |
| | 2 | 19.91 | 2.332 | 26.81 | 5.59 | -10.734 | 99 | 0 |
| | 3 | 0.394 | 0.262 | 0.334 | 0.387 | 1.281 | 99 | 0.203 |
| | 4 | 7.512 | 8.336 | 0.293 | 0.292 | 8.664 | 99 | 0 |
| | 5 | 12.866 | 0.4 | 12.342 | 0.571 | 7.751 | 99 | 0 |
| Different | 1 | 57.645 | 3.042 | 53.942 | 1.923 | 9.862 | 99 | 0 |
| | 2 | 20.29 | 2.775 | 25.78 | 3.126 | -13.635 | 99 | 0 |
| | 3 | 0.368 | 0.284 | 0.259 | 0.266 | 2.682 | 99 | 0.009 |
| | 4 | 115.061 | 66.547 | 5.218 | 3.125 | 16.43 | 99 | 0 |
| | 5 | 12.841 | 0.414 | 12.753 | 0.546 | 1.206 | 99 | 0.231 |

**Table 10.8 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same and different priorities**

As it can be seen from the results, whether or not all customers have the same priority, strategy 2 provides better solutions only in terms of the number of vehicles needed. Strategy 3 provides better solutions in terms of traveling time and customers' dissatisfaction. When all customers have the same priority, strategy 3 provides better solutions in terms of arrival time of the last vehicle. When each customer has a different priority, strategy 3 provides better solutions in terms of tour balance.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.9.

| Customes Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 58.415 | 1.462 | 60.557 | 2.205 | -8.099 | 99 | 0 |
| | 2 | 19.91 | 2.332 | 21.95 | 2.928 | -5.452 | 99 | 0 |
| | 3 | 0.394 | 0.262 | 0.49 | 0.324 | -2.216 | 99 | 0.029 |
| | 4 | 7.512 | 8.336 | 0.399 | 0.311 | 8.589 | 99 | 0 |
| | 5 | 12.866 | 0.4 | 12.817 | 0.462 | 0.78 | 99 | 0.437 |
| Different | 1 | 57.645 | 3.042 | 57.922 | 3.496 | -0.574 | 99 | 0.567 |
| | 2 | 20.29 | 2.775 | 20.6 | 3.83 | -0.636 | 99 | 0.526 |
| | 3 | 0.368 | 0.284 | 0.46 | 0.31 | -2.259 | 99 | 0.026 |
| | 4 | 115.061 | 66.547 | 2.796 | 1.567 | 16.858 | 99 | 0 |
| | 5 | 12.841 | 0.414 | 12.867 | 0.555 | -0.368 | 99 | 0.714 |

**Table 10.9 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same and different priorities**

As it can be seen from the results, whether or not all customers have the same priority, strategy 2 provides better solutions in terms of tour balance, which strategy 4 provides better solutions in terms of customers' dissatisfaction. When all customers have the same priority, strategy 2 provides better solutions in terms of travel time and number of vehicles needed.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.10.

| Customes Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 55.77 | 1.924 | 57.174 | 1.853 | -5.075 | 99 | 0 |
| | 2 | 26.81 | 5.59 | 29.85 | 2.267 | -4.875 | 99 | 0 |
| | 3 | 0.334 | 0.387 | 0.235 | 0.289 | 1.978 | 99 | 0.051 |
| | 4 | 9.518 | 9.492 | 0.225 | 0.3 | 9.761 | 99 | 0 |
| | 5 | 12.342 | 0.571 | 12.179 | 0.469 | 2.128 | 99 | 0.036 |
| Different | 1 | 53.942 | 1.923 | 52.352 | 4.076 | 3.57 | 99 | 0.001 |
| | 2 | 25.78 | 3.126 | 24.78 | 3.863 | 2.033 | 99 | 0.045 |
| | 3 | 0.259 | 0.266 | 0.456 | 0.362 | -4.887 | 99 | 0 |
| | 4 | 169.626 | 101.601 | 5.008 | 4.867 | 16.206 | 99 | 0 |
| | 5 | 12.753 | 0.546 | 12.603 | 0.587 | 1.826 | 99 | 0.071 |

**Table 10.10 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same and different priorities**

As it can be seen from the results, whether or not all customers have the same priority, strategy 5 provides better solutions in terms of customers' dissatisfaction. When all customers have the same priority, strategy 3 provides better solutions in terms of travel time and number of vehicles needed and strategy 5 provides better solutions in terms of arrival time of last vehicle. When each customer has different priority, strategy 3 provides better solutions in terms of route balance and strategy 5 provides better solutions in terms of travel time and number of vehicles needed.

#### 10.6.2.1 Conclusions

Table 10.11 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customes Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | | + | + | | + |
| | 5 | | | | | + |
| Different | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | |
| | 4 | | | | + | |
| | 5 | | | + | | + |

**Table 10.11 - Best strategy for each of the objective functions**

As it can be seen from Table 10.11, whether or not all customers have the same priority, objective functions 1, travel time, and 2, number of vehicles needed, are achieved best using strategy 1. Objective functions 3, tour balance, is best obtained by using strategy 3. Objective functions 5, arrival time of the last vehicle, is best obtained by using strategy 5 (which means that knowing all customers' demands in advance does not improve the algorithm's results).

## 10.6.3. Strategies Comparison – SPEA2 algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.12.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 59.176 | 3.406 | 73.706 | 3.816 | -30.053 | 99 | 0 |
| | 2 | 15.4 | 1.463 | 32.55 | 3.937 | -41.45 | 99 | 0 |
| | 3 | 0.365 | 0.179 | 0.288 | 0.295 | 2.105 | 99 | 0.038 |
| | 4 | 40.797 | 10.131 | 2.116 | 1.489 | 37.012 | 99 | 0 |
| | 5 | 13.041 | 0.407 | 12.176 | 0.451 | 14.332 | 99 | 0 |
| Different | 1 | 54.554 | 3.264 | 70.538 | 1.762 | -40.851 | 99 | 0 |
| | 2 | 13.46 | 1.167 | 31.99 | 2.385 | -69.541 | 99 | 0 |
| | 3 | 0.363 | 0.2 | 0.287 | 0.305 | 2.126 | 99 | 0.036 |
| | 4 | 4.778 | 7.324 | 0.24 | 0.298 | 6.202 | 99 | 0 |
| | 5 | 12.943 | 0.329 | 12.18 | 0.315 | 15.897 | 99 | 0 |

**Table 10.12 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether or not all customers have the same priority, strategy 2 provides better solutions in terms of tour balance, customers dissatisfaction and arrival time at the last customer.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.13.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 59.176 | 3.406 | 58.394 | 12.22 | 0.637 | 99 | 0.526 |
| | 2 | 15.4 | 1.463 | 34.25 | 6.838 | -27.833 | 99 | 0 |
| | 3 | 0.365 | 0.179 | 0.286 | 0.319 | 2.177 | 99 | 0.032 |
| | 4 | 40.797 | 10.131 | 1.634 | 0.78 | 38.148 | 99 | 0 |
| | 5 | 13.041 | 0.407 | 11.637 | 0.491 | 23.595 | 99 | 0 |
| Different | 1 | 54.554 | 3.264 | 64.69 | 1.25 | -27.896 | 99 | 0 |
| | 2 | 13.46 | 1.167 | 37.47 | 1.784 | -109.384 | 99 | 0 |
| | 3 | 0.363 | 0.2 | 0.19 | 0.229 | 5.941 | 99 | 0 |
| | 4 | 4.778 | 7.324 | 0.174 | 0.229 | 6.313 | 99 | 0 |
| | 5 | 12.943 | 0.329 | 11.996 | 0.427 | 17.656 | 99 | 0 |

**Table 10.13 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As can be seen from the results, if all customers have the same priority, a significant difference is found in the number of vehicles needed, which is lower in strategy 1. A significant difference is also found in the balance of the tour, customers' dissatisfaction and arrival time of the last vehicle, which are lower in strategy 3. When each customer has a different priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.14.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 73.706 | 3.816 | 58.394 | 12.22 | 11.79 | 99 | 0 |
| | 2 | 32.55 | 3.937 | 34.25 | 6.838 | -2.186 | 99 | 0.031 |
| | 3 | 0.288 | 0.295 | 0.286 | 0.319 | 0.048 | 99 | 0.962 |
| | 4 | 68.786 | 48.423 | 1.634 | 0.78 | 13.893 | 99 | 0 |
| | 5 | 12.176 | 0.451 | 11.637 | 0.491 | 8.629 | 99 | 0 |
| Different | 1 | 70.538 | 1.762 | 64.69 | 1.25 | 27.642 | 99 | 0 |
| | 2 | 31.99 | 2.385 | 37.47 | 1.784 | -16.867 | 99 | 0 |

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| | 3 | 0.287 | 0.305 | 0.19 | 0.229 | 2.534 | 99 | 0.013 |
| | 4 | 7.809 | 9.672 | 0.174 | 0.229 | 7.904 | 99 | 0 |
| | 5 | 12.18 | 0.315 | 11.996 | 0.427 | 3.484 | 99 | 0.001 |

**Table 10.14 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, when all customers have the same priority, a significant difference was found for travel time, customers' dissatisfaction and arrival time of the last vehicle, all of which are lower in strategy 3. Similarly, when each customer has his own priority, a significant difference was found for travel time, tour balance customers' dissatisfaction and arrival time of last vehicle, all of which are lower in strategy 3. A significant difference was also found for the number of vehicles needed, which is lower in strategy 2.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.15.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 73.706 | 3.816 | 72.598 | 2.949 | 2.159 | 99 | 0.033 |
| | 2 | 32.55 | 3.937 | 31.84 | 2.905 | 1.4 | 99 | 0.165 |
| | 3 | 0.288 | 0.295 | 0.309 | 0.35 | -0.434 | 99 | 0.666 |
| | 4 | 68.786 | 48.423 | 1.75 | 0.855 | 13.868 | 99 | 0 |
| | 5 | 12.176 | 0.451 | 12.163 | 0.35 | 0.223 | 99 | 0.824 |
| Different | 1 | 70.538 | 1.762 | 68.079 | 8.238 | 2.984 | 99 | 0.004 |
| | 2 | 31.99 | 2.385 | 31.62 | 5.548 | 0.616 | 99 | 0.54 |
| | 3 | 0.287 | 0.305 | 0.34 | 0.31 | -1.263 | 99 | 0.21 |
| | 4 | 7.809 | 9.672 | 0.3 | 0.271 | 7.762 | 99 | 0 |
| | 5 | 12.18 | 0.315 | 12.098 | 0.439 | 1.564 | 99 | 0.121 |

**Table 10.15 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As can be seen from the results, whether all customers have the same priority or each customer has a different priority, a significant difference was found for travel time and customers' dissatisfaction, which are lower in strategy 4.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.16.

| Customer's | Objective | Strategy 3 | Strategy 5 | t | df | Sig. |
|---|---|---|---|---|---|---|

| | | M | SD | M | SD | | | |
|---|---|---|---|---|---|---|---|---|
| The Same | 1 | 58.394 | 12.22 | 64.424 | 0.914 | -4.914 | 99 | 0 |
| | 2 | 34.25 | 6.838 | 37.63 | 1.412 | -4.92 | 99 | 0 |
| | 3 | 0.286 | 0.319 | 0.258 | 0.307 | 0.66 | 99 | 0.511 |
| | 4 | 53.119 | 25.344 | 1.179 | 0.295 | 20.493 | 99 | 0 |
| | 5 | 11.637 | 0.491 | 11.816 | 0.423 | -2.847 | 99 | 0.005 |
| Different | 1 | 64.69 | 1.25 | 58.762 | 14.57 | 4.019 | 99 | 0 |
| | 2 | 37.47 | 1.784 | 31.56 | 7.296 | 7.899 | 99 | 0 |
| | 3 | 0.19 | 0.229 | 0.214 | 0.273 | -0.638 | 99 | 0.525 |
| | 4 | 5.661 | 7.442 | 0.249 | 0.356 | 7.311 | 99 | 0 |
| | 5 | 11.996 | 0.427 | 11.979 | 0.472 | 0.266 | 99 | 0.791 |

**Table 10.16 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As can be seen from the results, whether all customers have the same priority or not, a significant difference was found for customers' dissatisfaction, which is lower in strategy 5. When all customers have the same priority, a significant difference was found for travel time, number of vehicles needed and arrival time of the last vehicle, all of which are lower in strategy 3. Similarly, when each customer has his own priority, a significant difference was found for travel time and number of vehicles, all of which are lower in strategy 5.

### 10.6.3.1 Conclusions

Table 10.17 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | + | | |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | + | | | | + |
| | 5 | | | + | | |
| Different | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | + | | + | | |
| | 5 | | | + | + | + |

**Table 10.17 - Best strategy for each of the objective functions**

As can be seen from Table 10.17, whether or not all customers have the same priority, objective functions 1, travel time, 2, number of vehicles needed and 4, customers' dissatisfaction, are best obtained using strategy 1. Objective functions 3, tour balance, and 5, arrival time of the last vehicle, are best obtained by using strategy 3.

### 10.6.4. Strategies Comparison – VE-ABC algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.18.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 64.833 | 3.769 | 81.972 | 2.272 | -39.757 | 99 | 0 |
| | 2 | 15.33 | 1.67 | 34.21 | 2.54 | -65.027 | 99 | 0 |
| | 3 | 0.64 | 0.342 | 0.294 | 0.288 | 7.515 | 99 | 0 |
| | 4 | 1785.667 | 538.197 | 2.266 | 0.762 | 33.136 | 99 | 0 |
| | 5 | 14.364 | 0.555 | 12.345 | 0.442 | 30.018 | 99 | 0 |
| Different | 1 | 77.627 | 6.807 | 80.89 | 2.937 | -4.596 | 99 | 0 |
| | 2 | 35.42 | 8.761 | 38.98 | 0.804 | -4.027 | 99 | 0 |
| | 3 | 0.351 | 0.392 | 0.145 | 0.188 | 5.17 | 99 | 0 |
| | 4 | 17.861 | 19.803 | 0.306 | 0.345 | 8.895 | 99 | 0 |
| | 5 | 12.491 | 1.022 | 11.868 | 0.288 | 5.588 | 99 | 0 |

**Table 10.18 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As can be seen from the results, whether all customers have the same priority or not, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.19.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 64.833 | 3.769 | 64.118 | 3.202 | 1.494 | 99 | 0.138 |
| | 2 | 15.33 | 1.67 | 34.68 | 3.792 | -48.151 | 99 | 0 |
| | 3 | 0.64 | 0.342 | 0.251 | 0.27 | 9.047 | 99 | 0 |
| | 4 | 1785.667 | 538.197 | 1.157 | 0.562 | 33.159 | 99 | 0 |
| | 5 | 14.364 | 0.555 | 12.073 | 0.451 | 29.452 | 99 | 0 |

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| Different | 1 | 77.627 | 6.807 | 33.935 | 0.834 | 62.504 | 99 | 0 |
| | 2 | 35.42 | 8.761 | 19.63 | 1.125 | 17.839 | 99 | 0 |
| | 3 | 0.351 | 0.392 | 0.326 | 0.311 | 0.522 | 99 | 0.603 |
| | 4 | 17.861 | 19.803 | 0.188 | 0.269 | 8.931 | 99 | 0 |
| | 5 | 12.491 | 1.022 | 11.16 | 0.328 | 12.066 | 99 | 0 |

**Table 10.19 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As can be seen from the results, a significant difference was found in customers' dissatisfaction and arrival time of last vehicle, which is lower in 3, whether all customers have the same priority or not. In the case of the number of vehicles needed, a significant difference also exiss using strategy 1, when all customers have the same priority. When each customer has a different priority, strategy 3 provides a better solution in terms of tour balance.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.20.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 81.972 | 2.272 | 63.612 | 3.995 | 35.333 | 99 | 0 |
| | 2 | 34.21 | 2.54 | 34.49 | 4.089 | -0.559 | 99 | 0.578 |
| | 3 | 0.294 | 0.288 | 0.251 | 0.27 | 1.083 | 99 | 0.281 |
| | 4 | 73.677 | 24.78 | 1.146 | 0.573 | 29.267 | 99 | 0 |
| | 5 | 12.345 | 0.442 | 12.066 | 0.458 | 4.292 | 99 | 0 |
| Different | 1 | 80.89 | 2.937 | 33.935 | 0.834 | 152.175 | 99 | 0 |
| | 2 | 38.98 | 0.804 | 19.63 | 1.125 | 138.676 | 99 | 0 |
| | 3 | 0.145 | 0.188 | 0.326 | 0.311 | -4.904 | 99 | 0 |
| | 4 | 9.942 | 11.218 | 0.188 | 0.269 | 8.713 | 99 | 0 |
| | 5 | 11.868 | 0.288 | 11.16 | 0.328 | 17.427 | 99 | 0 |

**Table 10.20 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, strategy 2 provides a better solution in terms of balance of the tour, when each customer has a different priority. Strategy 3 provides better solutions in terms of travel time customers' dissatisfaction and arrival time of last vehicle, wether all customers have the same priority or not.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.21.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 81.972 | 2.272 | 78.618 | 2.689 | 9.745 | 99 | 0 |
| | 2 | 34.21 | 2.54 | 31.62 | 3.09 | 7.508 | 99 | 0 |
| | 3 | 0.294 | 0.288 | 0.343 | 0.304 | -1.223 | 99 | 0.224 |
| | 4 | 73.677 | 24.78 | 2.404 | 1.148 | 28.725 | 99 | 0 |
| | 5 | 12.345 | 0.442 | 12.338 | 0.549 | 0.1 | 99 | 0.92 |
| Different | 1 | 80.89 | 2.937 | 81.198 | 2.284 | -0.788 | 99 | 0.433 |
| | 2 | 38.98 | 0.804 | 38.9 | 1.567 | 0.478 | 99 | 0.634 |
| | 3 | 0.145 | 0.188 | 0.255 | 0.332 | -2.817 | 99 | 0.006 |
| | 4 | 9.942 | 11.218 | 0.236 | 0.255 | 8.629 | 99 | 0 |
| | 5 | 11.868 | 0.288 | 11.855 | 0.305 | 0.306 | 99 | 0.76 |

**Table 10.21 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As can be seen from the results, strategy 2 provides better solutions in terms of tour balance, when all customers have the same priority. Strategy 4 provides better solutions in terms of travel time, number of vehicles and customers' dissatisfaction, when each customer has a different priority.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.22.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 63.612 | 3.995 | 62.831 | 3.226 | 1.661 | 99 | 0.1 |
| | 2 | 34.49 | 4.089 | 33.15 | 5.456 | 2.162 | 99 | 0.033 |
| | 3 | 0.251 | 0.27 | 0.294 | 0.307 | -1.101 | 99 | 0.274 |
| | 4 | 37.265 | 18.641 | 2.413 | 2.202 | 18.425 | 99 | 0 |
| | 5 | 12.066 | 0.458 | 12.078 | 0.535 | -0.153 | 99 | 0.879 |
| Different | 1 | 33.935 | 0.834 | 32.985 | 1.133 | 6.789 | 99 | 0 |
| | 2 | 19.63 | 1.125 | 18.26 | 1.515 | 7.466 | 99 | 0 |
| | 3 | 0.326 | 0.311 | 0.122 | 0.021 | 6.51 | 99 | 0 |
| | 4 | 6.126 | 8.733 | 0.032 | 0.025 | 6.98 | 99 | 0 |
| | 5 | 11.16 | 0.328 | 11.246 | 0.216 | -2.115 | 99 | 0.037 |

**Table 10.22 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the sameor different priorities**

As can be seen from the results, strategy 5 provides better solutions in terms of number of vehicles and customers' dissatisfaction, when all customers have the same priority or not. Strategy 5 also provides better solutions in terms of travel time and tour balance, when each customer has a different priority. Strategy 3 provides better solutions in terms of arrival time of last vehicle, when all customers have the same priority.

#### 10.6.4.1 Conclusions

Table 10.23 describes which of the five strategies used provides the best value for each of the objective functions, whether all customers have the same priority or each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | | | | | + |
| | 2 | + | | | | |
| | 3 | | + | + | | + |
| | 4 | | | + | | |
| | 5 | | | + | | + |
| Different | 1 | | | | | + |
| | 2 | | | | | + |
| | 3 | | + | | | + |
| | 4 | | | | | + |
| | 5 | | | + | | |

**Table 10.23 - Best strategy for each of the objective functions**

As can be seen from Table 10.23, when all customers have the same priority, objective functions 1, travel time, and 3, tour balance are best obtained using strategy 5. Objective function 5, arrival time of the last vehicle, is best obtained either using strategy 3.

When all customers have the same priority, objective function 2, number of vehicles needed, is best obtained using strategy 1. Objective function 4, customers' dissatisfaction, is best obtained by using strategy 3.

When each customer has different priority, objective functions 2, number of vehicles needed and 4, customers' dissatisfaction, are best obtained using strategy 5.

### 10.6.5. Algorithms Comparison

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 5 has to be used. Table 10.24 compares the results obtained for each of the five objectives, using paired t-tests, functions by each one of the three algorithms when applying the 3$^{rd}$ strategy. For each objective function, the best value obtained is highlighted in red.

| Customer's Priority | Objective Function | Algorithm | | |
|---|---|---|---|---|
| | | Imp. VEGA | SPEA2 | VE-ABC |

| Customer's Priority | Objective Function | Algorithm | | |
|---|---|---|---|---|
| | | Imp. VEGA | SPEA2 | VE-ABC |
| The Same | 1 | 57.174 | 64.424 | 62.831 |
| | 2 | 29.85 | 37.63 | 33.15 |
| | 3 | 0.235 | 0.258 | 0.294 |
| | 4 | 0.225 | 1.179 | 2.413 |
| | 5 | 12.179 | 11.816 | 12.078 |
| Different | 1 | 52.377 | 58.762 | 32.985 |
| | 2 | 24.67 | 31.56 | 18.26 |
| | 3 | 0.446 | 0.214 | 0.122 |
| | 4 | 4.997 | 0.249 | 0.032 |
| | 5 | 12.589 | 11.979 | 11.246 |

**Table 10.24 - Comparison of the 5th strategy used in all three algorithms**

As can be seen, when all customers have the same priority, most objective functions are best obtained by using the improved VEGA algorithm. However, if used when each customer has a different priority, all objective functions are best obtained first by using the improved VE-VEGA algorithm.

### 10.6.6. Conclusions

For all three algorithms, whether all customers have the same priority or not, objective functions 1, travel time, and 2, number of vehicles needed, are best obtained using strategy 1. Objective functions 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 3 or by using strategy 5 (which means that knowing all customers' demands in advance does not improve the algorithm's results).

Also, in all objective function, except the $5^{th}$, arrival time of last vehicle, better solutions are obtained when each customer has a different priority.

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 3 has to be used. From the results obtained, it is not clear whether one algorithm can provide the best solution in all cases.

## 10.7. Case Study 2

In the second case study the test scenario is defined as follows:

1. Network: The greater Tel-Aviv metropolitan area transportation network.

2. Dissatisfaction function: It is assumed that all customers dislike it when the supplier is either early or late. Therefore, the dissatisfaction functions of all customers are in the form of $f_i(t) = 1 - \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^5$ and $g_i(t) = 1 - \left( \frac{ELT - t}{ELT - TW_i^E} \right)^5$.

The test scenario is solved 100 times. In the first 50 times, it is assumed that all customers have the same priority. Under this assumption, the test scenario is solved 100 times using each of the 5 strategies described earlier. In the next 50 times, it is assumed that each customer has a priority equal to his demand. Under this assumption, the test scenario is solved 10 times using each of the 5 strategies described earlier.

### 10.7.1. Priority Comparison

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total travel time obtained when all customers have the same priority vs. the travel time obtained when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.25.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 50.235 | 2.12 | 49.383 | 2.051 | 2.77 | 99 | 0.007 |
| | 2 | 56.78 | 2.381 | 58.053 | 2.34 | -3.782 | 99 | 0 |
| | 3 | 53.754 | 2.328 | 56.329 | 1.853 | -8.907 | 99 | 0 |
| | 4 | 58.366 | 2.827 | 57.412 | 1.972 | 2.762 | 99 | 0.007 |
| | 5 | 53.321 | 3.497 | 54.425 | 6.048 | -1.734 | 99 | 0.086 |
| SPEA2 | 1 | 59.456 | 3.87 | 57.281 | 2.432 | 4.527 | 99 | 0 |
| | 2 | 72.54 | 2.967 | 72.854 | 1.718 | -0.905 | 99 | 0.368 |
| | 3 | 63.069 | 1.61 | 58.479 | 12.283 | 3.74 | 99 | 0 |
| | 4 | 72.937 | 2.7 | 72.343 | 3.561 | 1.306 | 99 | 0.194 |
| | 5 | 65.885 | 1.274 | 64.9 | 2.162 | 3.675 | 99 | 0 |
| VE-ABC | 1 | 62.574 | 2.528 | 51.812 | 17.456 | 6.141 | 99 | 0 |
| | 2 | 78.716 | 2.862 | 79.773 | 2.033 | -2.971 | 99 | 0.004 |
| | 3 | 63.547 | 2.61 | 63.372 | 2.33 | 0.481 | 99 | 0.631 |
| | 4 | 79.326 | 1.982 | 74.508 | 10.636 | 4.591 | 99 | 0 |
| | 5 | 63.902 | 2.405 | 63.653 | 1.841 | 0.782 | 99 | 0.436 |

**Table 10.25 – Paired T-Test results for comparison of the total travel time for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategies 2 and 3, a significant difference exists in the solution obtained when all customers have the same priority vs. the solution obtained when each customer has a different priority: it is

better when all customers have the same priority. While for strategies 1 and 4 a better solution is obtained when each customer has a different priority.

For the SPEA2 algorithm, for strategies 1, 3 and 5, a significant difference exists in the solution. The best solutions are obtained when each customer has a different priority.

As for the VE-ABC algorithm, for strategies 1 and 4, the best solutions are obtained when each customer has a different priority, while for strategy 2 the best solution is obtained when all customers have the same priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the number of vehicles needed when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.26.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 9.92 | 0.787 | 9.98 | 0.778 | -0.537 | 99 | 0.593 |
| | 2 | 19.61 | 3.025 | 18.99 | 2.97 | 1.584 | 99 | 0.116 |
| | 3 | 24.18 | 2.794 | 28.48 | 2.402 | -12.096 | 99 | 0 |
| | 4 | 20.67 | 3.327 | 19.46 | 1.684 | 3.286 | 99 | 0.001 |
| | 5 | 24.32 | 4.075 | 28.06 | 4.419 | -6.475 | 99 | 0 |
| SPEA2 | 1 | 14.73 | 2.265 | 14.01 | 1.396 | 2.578 | 99 | 0.011 |
| | 2 | 33.04 | 4.122 | 33.64 | 2.41 | -1.178 | 99 | 0.242 |
| | 3 | 37.38 | 1.797 | 33.77 | 7.558 | 4.815 | 99 | 0 |
| | 4 | 32.16 | 3.936 | 32.92 | 1.947 | -1.754 | 99 | 0.082 |
| | 5 | 39.5 | 1.078 | 36.86 | 2.697 | 9.534 | 99 | 0 |
| VE-ABC | 1 | 14.6 | 1.583 | 13.77 | 2.733 | 2.657 | 99 | 0.009 |
| | 2 | 34.23 | 1.874 | 34.4 | 2.361 | -0.588 | 99 | 0.558 |
| | 3 | 34.71 | 3.361 | 33.47 | 3.096 | 2.714 | 99 | 0.008 |
| | 4 | 34.1 | 2.607 | 31.93 | 5.044 | 3.967 | 99 | 0 |
| | 5 | 33.93 | 4.344 | 32.2 | 4.355 | 2.778 | 99 | 0.007 |

**Table 10.26 – Paired T-Test results for comparison of the number of vehicles needed for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategies 3 and 5 there are significant differences in the number of vehicles needed, which is lower when all customers have the same priority, while for strategy 4 it is lower when each customer has a different priority. For the SPEA2 algorithm, for strategy 1, 3 and 5 there is a significant difference in the number of vehicles needed, which is lower when each customer has a different priority. As for the VE-ABC algorithm, a significant difference

was found for all strategies except 2, which is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the balance of the tours when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.27.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 0.643 | 0.204 | 0.656 | 0.227 | -0.392 | 99 | 0.696 |
| | 2 | 0.4 | 0.248 | 0.533 | 0.338 | -3.261 | 99 | 0.002 |
| | 3 | 0.367 | 0.344 | 0.254 | 0.267 | 2.696 | 99 | 0.008 |
| | 4 | 0.428 | 0.348 | 0.359 | 0.254 | 1.496 | 99 | 0.138 |
| | 5 | 0.354 | 0.349 | 0.371 | 0.335 | -0.358 | 99 | 0.721 |
| SPEA2 | 1 | 0.449 | 0.298 | 0.412 | 0.216 | 1.012 | 99 | 0.314 |
| | 2 | 0.276 | 0.271 | 0.309 | 0.351 | -0.795 | 99 | 0.429 |
| | 3 | 0.147 | 0.286 | 0.294 | 0.323 | -3.501 | 99 | 0.001 |
| | 4 | 0.276 | 0.261 | 0.181 | 0.244 | 2.686 | 99 | 0.008 |
| | 5 | 0.047 | 0.005 | 0.37 | 0.339 | -9.544 | 99 | 0 |
| VE-ABC | 1 | 0.505 | 0.273 | 0.659 | 0.353 | -3.419 | 99 | 0.001 |
| | 2 | 0.278 | 0.301 | 0.288 | 0.27 | -0.262 | 99 | 0.794 |
| | 3 | 0.326 | 0.308 | 0.32 | 0.331 | 0.119 | 99 | 0.906 |
| | 4 | 0.206 | 0.197 | 0.356 | 0.263 | -4.453 | 99 | 0 |
| | 5 | 0.268 | 0.257 | 0.216 | 0.277 | 1.463 | 99 | 0.147 |

**Table 10.27 – Paired T-Test results for comparison of the balance of the tours for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, no significant differences were found between the two strategies. For the SPEA2 algorithm, for strategies 3 and 5, there is a significant difference in the tour balance, which is lower when all customers have the same priority. For strategy 4, a significant difference was found, and the best solution is obtained when each customer has a different priority. As for the VE-ABC algorithm, for strategies 1 and 4, there is a significant difference in the tour balance, which is lower when all customers have the same priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total dissatisfaction of the customers when all customers have the same priority vs. the total dissatisfaction of the customers when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.28.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 6377.836 | 1493.335 | 5.007 | 1.144 | 42.673 | 99 | 0 |
| | 2 | 711.196 | 471.646 | 0.659 | 0.479 | 15.068 | 99 | 0.001 |
| | 3 | 883.465 | 543.889 | 0.556 | 0.36 | 16.235 | 99 | 0.001 |
| | 4 | 523.829 | 236.472 | 0.418 | 0.267 | 22.131 | 99 | 0.001 |
| | 5 | 1067.894 | 584.697 | 0.41 | 0.338 | 18.256 | 99 | 0.001 |
| SPEA2 | 1 | 279.938 | 131.424 | 0.28 | 0.168 | 21.282 | 99 | 0 |
| | 2 | 240.18 | 121.072 | 0.418 | 0.302 | 19.796 | 99 | 0 |
| | 3 | 217.02 | 87.821 | 0.31 | 0.294 | 24.678 | 99 | 0 |
| | 4 | 183.016 | 19.873 | 0.283 | 0.148 | 92.092 | 99 | 0 |
| | 5 | 168.64 | 25.725 | 0.41 | 0.375 | 65.543 | 99 | 0 |
| VE-ABC | 1 | 5625.337 | 1649.478 | 6.916 | 2.743 | 34.061 | 99 | 0 |
| | 2 | 357.507 | 218.147 | 0.427 | 0.289 | 16.369 | 99 | 0.001 |
| | 3 | 340.457 | 291.438 | 0.406 | 0.299 | 11.668 | 99 | 0.007 |
| | 4 | 331.339 | 111.103 | 0.48 | 0.322 | 29.774 | 99 | 0 |
| | 5 | 224.793 | 72.874 | 0.359 | 0.376 | 30.77 | 99 | 0 |

**Table 10.28 – Paired T-Test results for comparison of the total dissatisfaction of the customers for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For all algorithms, the improved VEGA algorithm, the SPEA2 and the VE-ABC algorithm, in all strategies there is a significant difference in the total dissatisfaction of the customers obtained when all customers have the same priority vs. when each customer has a different priority. Total dissatisfaction is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the arrival time of the last vehicle when all customers have the same priority vs. the arrival time of the last vehicle when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.29.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 14.674 | 0.453 | 14.349 | 0.51 | 4.957 | 99 | 0 |
| | 2 | 12.885 | 0.398 | 12.916 | 0.485 | -0.483 | 99 | 0.63 |
| | 3 | 12.914 | 0.541 | 12.408 | 0.382 | 7.643 | 99 | 0 |
| | 4 | 12.789 | 0.5 | 12.806 | 0.458 | -0.246 | 99 | 0.806 |
| | 5 | 12.908 | 0.725 | 12.361 | 0.485 | 6.251 | 99 | 0 |
| SPEA2 | 1 | 13.046 | 0.647 | 12.928 | 0.294 | 1.753 | 99 | 0.083 |
| | 2 | 12.375 | 0.344 | 12.03 | 0.342 | 7.962 | 99 | 0 |
| | 3 | 11.851 | 0.29 | 11.802 | 0.562 | 0.781 | 99 | 0.437 |
| | 4 | 12.322 | 0.404 | 12.088 | 0.246 | 4.667 | 99 | 0 |
| | 5 | 11.937 | 0.143 | 12.053 | 0.422 | -2.457 | 99 | 0.016 |
| VE-ABC | 1 | 14.828 | 0.707 | 14.345 | 2.26 | 1.969 | 99 | 0.052 |
| | 2 | 12.265 | 0.453 | 12.12 | 0.409 | 2.276 | 99 | 0.025 |

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| | 3 | 12.069 | 0.477 | 12.281 | 0.444 | -3.184 | 99 | 0.002 |
| | 4 | 12.28 | 0.373 | 12.139 | 0.476 | 2.275 | 99 | 0.025 |
| | 5 | 11.941 | 0.412 | 12.165 | 0.341 | -4.176 | 99 | 0 |

**Table 10.29 – Paired T-Test results for comparison of the arrival time of the last vehicle for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as can be seen from the results, for strategies 1, 3 and 5 there is a significant difference in the arrival time of the last vehicle, which is earlier when each customer has a different priority. For the SPEA2 algorithm, for strategies 1, 3 and 5 there is a significant difference in the arrival time of the last vehicle, which is earlier when each customer has a different priority. Strategy 5 provides better solutions when all customers have the same priority. As for the VE-ABC algorithm, strategies 3 and 5 provide better solutions when all customers have the same priority, while strategies 2 and 4 provide better solutions when each customer has a different priority.

#### 10.7.1.1 Conclusions

For the first objective function, total travel time, a significant difference in the solutions was found for the improved VEGA, SPEA2 and VE-ABC algorithms, which is better when each customer has a different priority. Similar results were found for the 2nd objective function, number of vehicles needed, 4$^{th}$ objective function, customers' dissatisfaction and 5th objective function, arrival time at the last vehicle.

For the 3$^{rd}$ objective function, tour balance, no significant difference in the solutions was found.

## 10.7.2. Strategies Comparison – VEGA algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.30.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 50.235 | 2.12 | 56.78 | 2.381 | -21.565 | 99 | 0 |
| | 2 | 9.92 | 0.787 | 19.61 | 3.025 | -31.832 | 99 | 0 |
| | 3 | 0.643 | 0.204 | 0.4 | 0.248 | 8.047 | 99 | 0 |
| | 4 | 6377.836 | 1493.335 | 21.875 | 14.507 | 42.487 | 99 | 0 |
| | 5 | 14.674 | 0.453 | 12.885 | 0.398 | 30.295 | 99 | 0 |
| Different | 1 | 49.383 | 2.051 | 58.053 | 2.34 | -27.2 | 99 | 0 |
| | 2 | 9.98 | 0.778 | 18.99 | 2.97 | -29.572 | 99 | 0 |
| | 3 | 0.656 | 0.227 | 0.533 | 0.338 | 2.999 | 99 | 0.003 |
| | 4 | 162.778 | 37.18 | 0.659 | 0.479 | 43.575 | 99 | 0 |
| | 5 | 14.349 | 0.51 | 12.916 | 0.485 | 19.518 | 99 | 0 |

**Table 10.30 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in term of tour balance, customers' dissatisfaction and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.31.

| Customes Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 50.235 | 2.12 | 53.754 | 2.328 | -10.918 | 99 | 0 |
| | 2 | 9.92 | 0.787 | 24.18 | 2.794 | -47.002 | 99 | 0 |
| | 3 | 0.643 | 0.204 | 0.367 | 0.344 | 6.803 | 99 | 0 |
| | 4 | 6377.836 | 1493.335 | 27.174 | 16.729 | 42.597 | 99 | 0 |
| | 5 | 14.674 | 0.453 | 12.914 | 0.541 | 24.223 | 99 | 0 |
| Different | 1 | 49.383 | 2.051 | 56.329 | 1.853 | -25.204 | 99 | 0 |
| | 2 | 9.98 | 0.778 | 28.48 | 2.402 | -74.225 | 99 | 0 |
| | 3 | 0.656 | 0.227 | 0.254 | 0.267 | 11.818 | 99 | 0 |
| | 4 | 162.778 | 37.18 | 0.556 | 0.36 | 43.714 | 99 | 0 |
| | 5 | 14.349 | 0.51 | 12.408 | 0.382 | 30.631 | 99 | 0 |

**Table 10.31 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether or not all customers have the same priority, strategy 3 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.32.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 56.78 | 2.381 | 53.754 | 2.328 | 8.758 | 99 | 0 |
| | 2 | 19.61 | 3.025 | 24.18 | 2.794 | -10.121 | 99 | 0 |
| | 3 | 0.4 | 0.248 | 0.367 | 0.344 | 0.792 | 99 | 0.43 |
| | 4 | 711.196 | 471.646 | 27.174 | 16.729 | 14.463 | 99 | 0 |
| | 5 | 12.885 | 0.398 | 12.914 | 0.541 | -0.42 | 99 | 0.675 |
| Different | 1 | 58.053 | 2.34 | 56.329 | 1.853 | 5.372 | 99 | 0 |
| | 2 | 18.99 | 2.97 | 28.48 | 2.402 | -25.12 | 99 | 0 |
| | 3 | 0.533 | 0.338 | 0.254 | 0.267 | 6.773 | 99 | 0 |
| | 4 | 21.429 | 15.564 | 0.556 | 0.36 | 13.412 | 99 | 0 |
| | 5 | 12.916 | 0.485 | 12.408 | 0.382 | 8.316 | 99 | 0 |

**Table 10.32 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether all customers have the same priority or not, strategy 2 provides better solutions only in terms of number of vehicles needed and strategy 3 provides better solutions in terms of travel time and customers' dissatisfaction. When when each customer has a different priority, strategy 3 provides better solutions in terms of tour balance and arrival time of last vehicle as well.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.33.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 56.78 | 2.381 | 58.366 | 2.827 | -4.402 | 99 | 0 |
| | 2 | 19.61 | 3.025 | 20.67 | 3.327 | -2.458 | 99 | 0.016 |
| | 3 | 0.4 | 0.248 | 0.428 | 0.348 | -0.635 | 99 | 0.527 |
| | 4 | 711.196 | 471.646 | 16.112 | 7.274 | 14.697 | 99 | 0 |
| | 5 | 12.885 | 0.398 | 12.789 | 0.5 | 1.519 | 99 | 0.132 |
| Different | 1 | 58.053 | 2.34 | 57.412 | 1.972 | 1.952 | 99 | 0.054 |
| | 2 | 18.99 | 2.97 | 19.46 | 1.684 | -1.27 | 99 | 0.207 |
| | 3 | 0.533 | 0.338 | 0.359 | 0.254 | 3.932 | 99 | 0 |
| | 4 | 21.429 | 15.564 | 0.418 | 0.267 | 13.517 | 99 | 0 |
| | 5 | 12.916 | 0.485 | 12.806 | 0.458 | 1.7 | 99 | 0.092 |

**Table 10.33 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As can be seen from the results, when all customers have the same priority, strategy 2 provides better solutions in terms of travel time and number of vehicles needed and

strategy 4 provides better solutions in terms of customers' dissatisfaction. When each customer has a different priority, strategy 4 provides better solutions in terms of tour banalce and customers' dissatisfaction.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.34.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 53.754 | 2.328 | 53.321 | 3.497 | 0.971 | 99 | 0.334 |
| | 2 | 24.18 | 2.794 | 24.32 | 4.075 | -0.271 | 99 | 0.787 |
| | 3 | 0.367 | 0.344 | 0.354 | 0.349 | 0.28 | 99 | 0.78 |
| | 4 | 883.465 | 543.889 | 32.847 | 17.985 | 15.594 | 99 | 0 |
| | 5 | 12.914 | 0.541 | 12.908 | 0.725 | 0.06 | 99 | 0.952 |
| Different | 1 | 56.329 | 1.853 | 54.425 | 6.048 | 3.104 | 99 | 0.002 |
| | 2 | 28.48 | 2.402 | 28.06 | 4.419 | 0.847 | 99 | 0.399 |
| | 3 | 0.254 | 0.267 | 0.371 | 0.335 | -2.68 | 99 | 0.009 |
| | 4 | 18.074 | 11.721 | 0.41 | 0.338 | 15.116 | 99 | 0 |
| | 5 | 12.408 | 0.382 | 12.361 | 0.485 | 0.796 | 99 | 0.428 |

**Table 10.34 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As can be seen from the results, when all customers have the same priority, strategy 5 provides better solutions in terms of customers' dissatisfaction. When each customer has a different priority, strategy 3 provides better solutions in terms of tour banalce and strategy 5 provides better solutions in terms of travel time and customers' dissatisfaction.

### 10.7.2.1 Conclusions

Table 10.35 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | | | | + | |
| | 5 | | + | + | + | + |
| Different | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | |
| | 4 | | | | + | + |

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| | 5 | | | + | | + |

**Table 10.35 - Best strategy for each of the objective functions**

As can be seen from Table 10.35, whether all customers have the same priority or, objective functions 1, travel time, and 2, nember of vehicles needed, are best obtained using strategy 1. Objective function 3, tour balance, is best obtained by using strategy 3. Objective functions 4, customers' dissatisfaction, is best obtained by using strategy 4, and 5, arrival time of last vehicle, is best obtained by using strategy 3 or 5.

## 10.7.3. Strategies Comparison – SPEA2 algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.36.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 59.456 | 3.87 | 72.54 | 2.967 | -27.904 | 99 | 0 |
| | 2 | 14.73 | 2.265 | 33.04 | 4.122 | -40.107 | 99 | 0 |
| | 3 | 0.449 | 0.298 | 0.276 | 0.271 | 4.177 | 99 | 0 |
| | 4 | 279.938 | 131.424 | 7.388 | 3.724 | 20.771 | 99 | 0 |
| | 5 | 13.046 | 0.647 | 12.375 | 0.344 | 8.982 | 99 | 0 |
| Different | 1 | 57.281 | 2.432 | 72.854 | 1.718 | -51.266 | 99 | 0 |
| | 2 | 14.01 | 1.396 | 33.64 | 2.41 | -64.488 | 99 | 0 |
| | 3 | 0.412 | 0.216 | 0.309 | 0.351 | 2.528 | 99 | 0.013 |
| | 4 | 9.089 | 5.448 | 0.418 | 0.302 | 16.04 | 99 | 0 |
| | 5 | 12.928 | 0.294 | 12.03 | 0.342 | 19.491 | 99 | 0 |

**Table 10.36 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.37.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 59.456 | 3.87 | 63.069 | 1.61 | -8.65 | 99 | 0 |
| | 2 | 14.73 | 2.265 | 37.38 | 1.797 | -81.176 | 99 | 0 |
| | 3 | 0.449 | 0.298 | 0.147 | 0.286 | 7.78 | 99 | 0 |
| | 4 | 279.938 | 131.424 | 6.675 | 2.701 | 20.758 | 99 | 0 |
| | 5 | 13.046 | 0.647 | 11.851 | 0.29 | 16.604 | 99 | 0 |
| Different | 1 | 57.281 | 2.432 | 58.479 | 12.283 | -0.978 | 99 | 0.331 |
| | 2 | 14.01 | 1.396 | 33.77 | 7.558 | -25.865 | 99 | 0 |
| | 3 | 0.412 | 0.216 | 0.294 | 0.323 | 2.85 | 99 | 0.005 |
| | 4 | 9.089 | 5.448 | 0.31 | 0.294 | 16.033 | 99 | 0 |
| | 5 | 12.928 | 0.294 | 11.802 | 0.562 | 17.82 | 99 | 0 |

**Table 10.37 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction and arrival time of the last vehicle. When all customers have the same priority, strategy 1 provides better solutions in terms of travel time as well.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.38.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 72.54 | 2.967 | 63.069 | 1.61 | 28.59 | 99 | 0 |
| | 2 | 33.04 | 4.122 | 37.38 | 1.797 | -9.467 | 99 | 0 |
| | 3 | 0.276 | 0.271 | 0.147 | 0.286 | 3.293 | 99 | 0.001 |
| | 4 | 240.18 | 121.072 | 6.675 | 2.701 | 19.382 | 99 | 0 |
| | 5 | 12.375 | 0.344 | 11.851 | 0.29 | 11.543 | 99 | 0 |
| Different | 1 | 72.854 | 1.718 | 58.479 | 12.283 | 11.703 | 99 | 0 |
| | 2 | 33.64 | 2.41 | 33.77 | 7.558 | -0.163 | 99 | 0.871 |
| | 3 | 0.309 | 0.351 | 0.294 | 0.323 | 0.34 | 99 | 0.735 |
| | 4 | 13.603 | 9.815 | 0.31 | 0.294 | 13.555 | 99 | 0 |
| | 5 | 12.03 | 0.342 | 11.802 | 0.562 | 3.305 | 99 | 0.001 |

**Table 10.38 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers share the same priority, strategy 3 provides better solutions in terms of travel time, customers' dissatisfaction and arrival time of the last vehicle. If all customers have the same priority, strategy 2 also

provides better solutions in terms of number of vehicles needed, and strategy 3 provides better solutions in terms of tour balance.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.39.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 72.54 | 2.967 | 72.937 | 2.7 | -0.925 | 99 | 0.357 |
| | 2 | 33.04 | 4.122 | 32.16 | 3.936 | 1.532 | 99 | 0.129 |
| | 3 | 0.276 | 0.271 | 0.276 | 0.261 | 0 | 99 | 1 |
| | 4 | 240.18 | 121.072 | 5.629 | 0.611 | 19.372 | 99 | 0 |
| | 5 | 12.375 | 0.344 | 12.322 | 0.404 | 1.027 | 99 | 0.307 |
| Different | 1 | 72.854 | 1.718 | 72.419 | 3.433 | 1.144 | 99 | 0.255 |
| | 2 | 33.64 | 2.41 | 32.82 | 2.091 | 2.638 | 99 | 0.01 |
| | 3 | 0.309 | 0.351 | 0.187 | 0.247 | 2.836 | 99 | 0.006 |
| | 4 | 13.603 | 9.815 | 0.345 | 0.616 | 13.481 | 99 | 0 |
| | 5 | 12.03 | 0.342 | 12.1 | 0.272 | -1.726 | 99 | 0.087 |

**Table 10.39 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers share the same priority, strategy 4 provides better solutions in terms of customers' dissatisfaction. When each customer has a different priority strategy 4 provides better solutions in term of number of vehicles needed and tour balance, as well.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.40.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 63.069 | 1.61 | 65.885 | 1.274 | -14.827 | 99 | 0 |
| | 2 | 37.38 | 1.797 | 39.5 | 1.078 | -9.228 | 99 | 0 |
| | 3 | 0.147 | 0.286 | 0.047 | 0.005 | 3.501 | 99 | 0.001 |
| | 4 | 217.02 | 87.821 | 5.187 | 0.791 | 24.103 | 99 | 0 |
| | 5 | 11.851 | 0.29 | 11.937 | 0.143 | -2.658 | 99 | 0.009 |
| Different | 1 | 58.479 | 12.283 | 64.9 | 2.162 | -5.216 | 99 | 0 |
| | 2 | 33.77 | 7.558 | 36.86 | 2.697 | -3.959 | 99 | 0 |
| | 3 | 0.294 | 0.323 | 0.37 | 0.339 | -1.635 | 99 | 0.105 |
| | 4 | 10.079 | 9.55 | 0.41 | 0.375 | 10.08 | 99 | 0 |
| | 5 | 11.802 | 0.562 | 12.053 | 0.422 | -3.634 | 99 | 0 |

**Table 10.40 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As can be seen from the results, when all customers have the same priority, strategy 3 provides better solutions in terms of travel time, number of vehicles needed and arrival time of last vehicle, while strategy 5 provides better solutions in term of customers' dissatisfaction.

#### 10.7.3.1 Conclusions

Table 10.41 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | | | + |
| | 4 | | | | | + |
| | 5 | | | + | | |
| Different | 1 | + | | + | | |
| | 2 | + | | | | |
| | 3 | | | | + | |
| | 4 | + | | + | + | |
| | 5 | | | + | | |

**Table 10.41 - Best strategy for each of the objective functions**

As can be seen from Table 10.41, whether all customers have the same priority or not, objective functions 1, travel time, and 2, number of vehicles needed, are obtained using strategy 1. Objective function 3 is best obtained by using strategy 3.

### 10.7.4. Strategies Comparison – VE-ABC algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.42.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 62.574 | 2.528 | 78.716 | 2.862 | -40.741 | 99 | 0 |
| | 2 | 14.6 | 1.583 | 34.23 | 1.874 | -79.126 | 99 | 0 |
| | 3 | 0.505 | 0.273 | 0.278 | 0.301 | 5.172 | 99 | 0 |

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| | 4 | 5625.337 | 1649.478 | 10.996 | 6.71 | 34.049 | 99 | 0 |
| | 5 | 14.828 | 0.707 | 12.265 | 0.453 | 30.287 | 99 | 0 |
| Different | 1 | 51.812 | 17.456 | 79.773 | 2.033 | -16.1 | 99 | 0 |
| | 2 | 13.77 | 2.733 | 34.4 | 2.361 | -57.21 | 99 | 0 |
| | 3 | 0.659 | 0.353 | 0.288 | 0.27 | 7.668 | 99 | 0 |
| | 4 | 224.849 | 89.19 | 0.427 | 0.289 | 25.153 | 99 | 0 |
| | 5 | 14.345 | 2.26 | 12.12 | 0.409 | 9.784 | 99 | 0 |

**Table 10.42 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.43.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 62.574 | 2.528 | 63.547 | 2.61 | -2.888 | 99 | 0.005 |
| | 2 | 14.6 | 1.583 | 34.71 | 3.361 | -54.262 | 99 | 0 |
| | 3 | 0.505 | 0.273 | 0.326 | 0.308 | 4.606 | 99 | 0 |
| | 4 | 5625.337 | 1649.478 | 10.472 | 8.964 | 34.004 | 99 | 0 |
| | 5 | 14.828 | 0.707 | 12.069 | 0.477 | 30.767 | 99 | 0 |
| Different | 1 | 51.812 | 17.456 | 63.372 | 2.33 | -6.5 | 99 | 0 |
| | 2 | 13.77 | 2.733 | 33.47 | 3.096 | -45.959 | 99 | 0 |
| | 3 | 0.659 | 0.353 | 0.32 | 0.331 | 7.002 | 99 | 0 |
| | 4 | 224.849 | 89.19 | 0.406 | 0.299 | 25.16 | 99 | 0 |
| | 5 | 14.345 | 2.26 | 12.281 | 0.444 | 8.982 | 99 | 0 |

**Table 10.43 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 3 provides better solutions in terms of tour balance, customers' dissatisfaction, and arrival time of the last vehicle.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.44.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 78.716 | 2.862 | 63.547 | 2.61 | 38.245 | 99 | 0 |
| | 2 | 34.23 | 1.874 | 34.71 | 3.361 | -1.223 | 99 | 0.224 |
| | 3 | 0.278 | 0.301 | 0.326 | 0.308 | -1.143 | 99 | 0.256 |
| | 4 | 357.507 | 218.147 | 10.472 | 8.964 | 15.929 | 99 | 0 |
| | 5 | 12.265 | 0.453 | 12.069 | 0.477 | 2.865 | 99 | 0.005 |
| Different | 1 | 79.773 | 2.033 | 63.372 | 2.33 | 49.484 | 99 | 0 |
| | 2 | 34.4 | 2.361 | 33.47 | 3.096 | 2.457 | 99 | 0.016 |
| | 3 | 0.288 | 0.27 | 0.32 | 0.331 | -0.787 | 99 | 0.433 |
| | 4 | 13.89 | 9.408 | 0.406 | 0.299 | 14.285 | 99 | 0 |
| | 5 | 12.12 | 0.409 | 12.281 | 0.444 | -2.887 | 99 | 0.005 |

**Table 10.44 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, strategy 3 provides better solutions in terms of travel time and customers' dissatisfaction, whether all customers have the same priority or not.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.45.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 78.716 | 2.862 | 79.326 | 1.982 | -1.701 | 99 | 0.092 |
| | 2 | 34.23 | 1.874 | 34.1 | 2.607 | 0.4 | 99 | 0.69 |
| | 3 | 0.278 | 0.301 | 0.206 | 0.197 | 1.997 | 99 | 0.049 |
| | 4 | 357.507 | 218.147 | 10.192 | 3.417 | 15.923 | 99 | 0 |
| | 5 | 12.265 | 0.453 | 12.28 | 0.373 | -0.252 | 99 | 0.801 |
| Different | 1 | 79.773 | 2.033 | 74.508 | 10.636 | 4.972 | 99 | 0 |
| | 2 | 34.4 | 2.361 | 31.93 | 5.044 | 4.604 | 99 | 0 |
| | 3 | 0.288 | 0.27 | 0.356 | 0.263 | -1.862 | 99 | 0.066 |
| | 4 | 13.89 | 9.408 | 0.48 | 0.322 | 14.189 | 99 | 0 |
| | 5 | 12.12 | 0.409 | 12.139 | 0.476 | -0.292 | 99 | 0.771 |

**Table 10.45 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As can be seen from the results, when all customers have the same priority, Strategy 4 provides better solutions in terms of tour balance and customers' dissatisfaction. When each customer has a different priority, strategy 4 provides better solutions in terms of travel time, number of vehicles and customers' dissatisfaction.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.46.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 63.547 | 2.61 | 63.902 | 2.405 | -0.956 | 99 | 0.341 |
| | 2 | 34.71 | 3.361 | 33.93 | 4.344 | 1.363 | 99 | 0.176 |
| | 3 | 0.326 | 0.308 | 0.268 | 0.257 | 1.622 | 99 | 0.108 |
| | 4 | 340.457 | 291.438 | 6.914 | 2.242 | 11.464 | 99 | 0 |
| | 5 | 12.069 | 0.477 | 11.941 | 0.412 | 2.03 | 99 | 0.045 |
| Different | 1 | 63.372 | 2.33 | 63.653 | 1.841 | -0.909 | 99 | 0.365 |
| | 2 | 33.47 | 3.096 | 32.2 | 4.355 | 2.298 | 99 | 0.024 |
| | 3 | 0.32 | 0.331 | 0.216 | 0.277 | 2.692 | 99 | 0.008 |
| | 4 | 13.196 | 9.725 | 0.359 | 0.376 | 13.268 | 99 | 0 |
| | 5 | 12.281 | 0.444 | 12.165 | 0.341 | 2.033 | 99 | 0.045 |

**Table 10.46 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As can be seen from the results, whether all customers have the same priority or each customer has a different priority, for objective function 4 and 5, the best solutions are obtained using strategy 5. When each customer has a different priority, for objectives 2 and 3, the best solutions are obtained using strategy 5, as well.

### 10.7.4.1 Conclusions

Table 10.47 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customes Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | | + | + |
| | 4 | | | | | + |
| | 5 | | | | | + |
| Different | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | + | | | + |
| | 4 | | + | + | | + |
| | 5 | | + | | + | + |

**Table 10.47 - Best strategy for each of the objective functions**

As can be seen from Table 10.47, whether or not all customers have the same priority, objective functions 1, travel time, and 2, number of vehicles needed, are best obtained by using strategy 1. Objective functions 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are obtained by using strategy 5.

## 10.7.5. Algorithms Comparison

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 5 has to be used. Table 10.24 compares the results obtained for each of the five objectives, using paired t-tests, functions by each one of the three algorithms when applying the 3rd strategy. For each objective function, the best value obtained is highlighted in red.

| Customer's Priority | Objective Function | Algorithm | | |
|---|---|---|---|---|
| | | Imp. VEGA | SPEA2 | VE-ABC |
| The Same | 1 | 64.833 | 50.235 | 59.456 |
| | 2 | 15.33 | 9.92 | 14.73 |
| | 3 | 0.64 | 0.643 | 0.449 |
| | 4 | 54.925 | 196.174 | 8.611 |
| | 5 | 14.364 | 14.674 | 13.046 |
| Different | 1 | 77.627 | 49.383 | 57.281 |
| | 2 | 35.42 | 9.98 | 14.01 |
| | 3 | 0.351 | 0.656 | 0.412 |
| | 4 | 0.549 | 5.007 | 0.28 |
| | 5 | 12.491 | 14.349 | 12.928 |

**Table 10.48 - Comparison of the 5th strategy used in all three algorithms**

As it can be seen, whether all customers have the same priority or not, objective function 1, travel time, and objective function 2, number of vehicles needed, , are best obtained using the SPEA2 algorithm, while objective functions 3, tour balance, and 4, customers' dissatisfaction are best obtained using the VE-ABC algorithm.

When all customers have the same priority, objective function 5, arrival time of last vehicle, is best obtained by using the VE-ABC algorithm. When each customer has a different priority, objective function 5 is best obtained by using the improved improved VEGA algorithm.

### 10.7.6. Conclusions

For all three algorithms, whether all customers have the same priority or not, objective functions 1, travel time, and 2, number of vehicles needed, are best obtained using strategy 1. Objective functions 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained either by using strategy 3 or by using strategy 5.

Also, in all objective function, except the 3rd, tour balance, better solutions are obtained when each customer has a different priority.

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 5 has to be used.

## 10.8. Case Study 3 – Israel

In the first case study the test scenario is defined as follows:

1. Network: Israeli transportation network.
2. Dissatisfaction function: It is assumed that the dissatisfaction functions of all customers are linear, meaning $f_i(t) = 1 - \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^1$ and $g_i(t) = 1 - \left( \frac{ELT - t}{ELT - TW_i^E} \right)^1$.

The test scenario is solved 100 times. For the first 50 times, it is assumed that all customers have the same priority. Under this assumption, the test scenario is solved 100 times using each of the 5 strategies described earlier. In the next 50 times, it is assumed that each customer has a priority equal to his demand. Under this assumption, the test scenario is solved 10 times using each of the 5 strategies described earlier.

### 10.8.1. Priority Comparison

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total travel time obtained when all customers have the same priority vs. the travel time obtained when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.49.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 74.974 | 5.924 | 72.845 | 3.976 | 0.901 | 99 | 0.391 |
| | 2 | 96.128 | 3.969 | 90.521 | 23.406 | 0.703 | 99 | 0.5 |
| | 3 | 86.641 | 8.965 | 95.254 | 7.626 | -2.329 | 99 | 0.044 |
| | 4 | 95.456 | 7.829 | 93.787 | 6.484 | 0.5 | 99 | 0.626 |
| | 5 | 84.384 | 7.974 | 90.053 | 16.162 | -1.204 | 99 | 0.256 |
| SPEA2 | 1 | 98.488 | 9.378 | 91.669 | 3.735 | 2.743 | 99 | 0.022 |
| | 2 | 104.88 | 23.569 | 106.365 | 12.749 | -0.147 | 99 | 0.888 |
| | 3 | 92.095 | 19.482 | 99.452 | 2.965 | -1.157 | 99 | 0.277 |
| | 4 | 115.649 | 7.46 | 107.252 | 12.717 | 1.761 | 99 | 0.111 |
| | 5 | 86.673 | 15.228 | 97.353 | 8.663 | -1.946 | 99 | 0.084 |
| VE-ABC | 1 | 103.968 | 16.345 | 115.347 | 6.932 | -1.98 | 99 | 0.08 |
| | 2 | 134.079 | 6.567 | 131.377 | 6.612 | 0.793 | 99 | 0.45 |
| | 3 | 90.844 | 17.594 | 95.083 | 18.005 | -0.432 | 99 | 0.677 |
| | 4 | 131.438 | 6.422 | 128.261 | 9.603 | 0.786 | 99 | 0.454 |
| | 5 | 93.226 | 18.392 | 100.054 | 3.977 | -1.106 | 99 | 0.299 |

**Table 10.49 – Paired T-Test results for comparison of the total travel time for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as evident from the results, for strategy 3 there exists a significant difference in travel time, which is lower when all customers have the same priority. For the SPEA2 algorithm, there is a significant difference in the travel time only for strategy 1, which is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the number of vehicles needed when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.50.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 8.622 | 1.153 | 8.01 | 0.787 | 1.244 | 99 | 0.243 |
| | 2 | 14.747 | 1.334 | 14.284 | 2.861 | 0.478 | 99 | 0.646 |
| | 3 | 15.991 | 2.464 | 17.878 | 1.116 | -2.374 | 99 | 0.044 |
| | 4 | 15.154 | 1.789 | 14.45 | 1.504 | 1.172 | 99 | 0.273 |
| | 5 | 15.584 | 1.686 | 16.791 | 2.981 | -1.208 | 99 | 0.261 |
| SPEA2 | 1 | 13.339 | 1.825 | 12.707 | 1.202 | 1.154 | 99 | 0.28 |
| | 2 | 18.105 | 2.346 | 18.789 | 2.727 | -0.459 | 99 | 0.655 |
| | 3 | 19.503 | 3.212 | 19.296 | 0.854 | 0.17 | 99 | 0.869 |
| | 4 | 18.662 | 1.019 | 17.738 | 2.529 | 0.863 | 99 | 0.41 |
| | 5 | 17.628 | 2.596 | 19.914 | 1.13 | -2.724 | 99 | 0.021 |
| VE-ABC | 1 | 13.105 | 2.53 | 14.513 | 1.754 | -1.497 | 99 | 0.169 |
| | 2 | 21.824 | 1.367 | 21.11 | 1.288 | 1.261 | 99 | 0.238 |
| | 3 | 18.419 | 4.148 | 18.366 | 3.572 | 0.101 | 99 | 0.921 |
| | 4 | 21.181 | 1.33 | 20.657 | 1.409 | 0.898 | 99 | 0.393 |
| | 5 | 18.592 | 3.484 | 19.341 | 1.144 | -0.811 | 99 | 0.437 |

**Table 10.50 – Paired T-Test results for comparison of the number of vehicles needed for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as apparent from the results for strategy 3, there is a significant difference in the number of vehicles needed, which is lower when all customers have the same priority. For the SPEA2 algorithm, for strategy 5 there is a significant difference in travel time, which, again, is lower when all customers have the same priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the balance of the tours when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.51.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 2.323 | 1.089 | 1.714 | 0.667 | 1.066 | 99 | 0.313 |
| | 2 | 1.535 | 0.539 | 1.428 | 0.752 | 0.284 | 99 | 0.782 |
| | 3 | 1.007 | 0.531 | 0.8 | 0.554 | 0.388 | 99 | 0.705 |
| | 4 | 1.602 | 0.928 | 1.495 | 0.381 | 0.463 | 99 | 0.655 |
| | 5 | 1.048 | 0.525 | 0.715 | 0.147 | 1.641 | 99 | 0.134 |
| SPEA2 | 1 | 0.947 | 0.488 | 0.929 | 0.597 | -0.442 | 99 | 0.668 |
| | 2 | 1.091 | 0.485 | 1.163 | 0.424 | -0.03 | 99 | 0.976 |
| | 3 | 0.446 | 0.081 | 0.713 | 0.117 | -2.989 | 99 | 0.016 |
| | 4 | 1.303 | 0.456 | 1.319 | 0.51 | 0.267 | 99 | 0.796 |
| | 5 | 0.722 | 0.26 | 0.694 | 0.414 | -0.716 | 99 | 0.49 |
| VE-ABC | 1 | 1.681 | 1.06 | 1.513 | 0.526 | 0.17 | 99 | 0.868 |
| | 2 | 0.691 | 0.458 | 1.005 | 0.244 | -1.249 | 99 | 0.242 |
| | 3 | 0.713 | 0.697 | 0.855 | 0.174 | -0.346 | 99 | 0.74 |
| | 4 | 1.107 | 0.411 | 0.681 | 0.404 | 2.158 | 99 | 0.057 |
| | 5 | 5.463 | 9.673 | 0.873 | 0.382 | 1.466 | 99 | 0.176 |

**Table 10.51 – Paired T-Test results for comparison of the balance of the tours for all three algorithms when all customers have the same priority vs. each customer has a different priority**

Only for the SPEA2 algorithm with strategy 3 shows a significant difference in the tour balance, which is lower when all customers have the same priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total dissatisfaction of the customers when all customers have the same priority vs. the total dissatisfaction of the customers when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.52.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 3026.267 | 1079.539 | 4.606 | 2.096 | 8.851 | 99 | 0 |
| | 2 | 180.668 | 185.374 | 0.121 | 0.154 | 3.079 | 99 | 0.011 |
| | 3 | 989.097 | 510.017 | 0.115 | 0.174 | 6.13 | 99 | 0.002 |
| | 4 | 173.122 | 241.857 | 0.177 | 0.135 | 2.264 | 99 | 0.049 |
| | 5 | 990.146 | 476.367 | 0.218 | 0.288 | 6.572 | 99 | 0.002 |
| SPEA2 | 1 | 81.914 | 76 | 0.179 | 0.17 | 3.407 | 99 | 0.009 |
| | 2 | 38.053 | 20.122 | 0.008 | 0.058 | 5.989 | 99 | 0.001 |
| | 3 | 348.95 | 175.098 | 0.122 | -0.021 | 6.303 | 99 | 0.002 |
| | 4 | 34.289 | 7.813 | 0.059 | 0.07 | 13.919 | 99 | 0.001 |
| | 5 | 516.48 | 338.355 | 0.075 | 0.122 | 4.826 | 99 | 0.003 |
| VE-ABC | 1 | 3094.087 | 1660.215 | 2.947 | 1.204 | 5.89 | 99 | 0 |
| | 2 | 48.242 | 10.91 | 0.151 | 0.033 | 13.881 | 99 | 0.001 |
| | 3 | 949.718 | 594.273 | 0.013 | 0.234 | 5.052 | 99 | 0.001 |
| | 4 | 66.352 | 56.454 | 0.074 | 0.037 | 3.721 | 99 | 0.006 |
| | 5 | 777.402 | 518.056 | 0.027 | 0.135 | 4.746 | 99 | 0 |

**Table 10.52 – Paired T-Test results for comparison of the total dissatisfaction of the customers for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For all algorithms and for all strategies, there are significant differences in the total dissatisfaction of the customers, which are lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-test were conducted to compare the arrival time of the last vehicle when all customers have the same priority vs. the arrival time of the last vehicle when each customer has a different priority for each of the five strategies. The results are summarized in Table 10.53.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 21.31 | 0.807 | 21.348 | 1.001 | 0.068 | 99 | 0.949 |
| | 2 | 20.092 | 0.682 | 19.275 | 2.021 | 1.065 | 99 | 0.316 |
| | 3 | 21.682 | 0.631 | 21.199 | 0.71 | 1.635 | 99 | 0.136 |
| | 4 | 19.986 | 1.206 | 20.17 | 0.947 | -0.399 | 99 | 0.696 |
| | 5 | 21.241 | 1.517 | 20.682 | 1.781 | 0.775 | 99 | 0.461 |
| SPEA2 | 1 | 19.45 | 0.167 | 19.448 | 0.003 | 0.302 | 99 | 0.772 |
| | 2 | 18.346 | 2.258 | 19.282 | 1.394 | -0.966 | 99 | 0.362 |
| | 3 | 20.571 | 2.678 | 21.083 | 0.53 | -0.537 | 99 | 0.604 |
| | 4 | 19.268 | 1.2 | 19.103 | 1.261 | 0.241 | 99 | 0.814 |
| | 5 | 20.972 | 2.753 | 20.792 | 1.994 | 0.106 | 99 | 0.918 |
| VE-ABC | 1 | 21.764 | 1.463 | 21.158 | 1.35 | 0.7 | 99 | 0.502 |
| | 2 | 19.823 | 0.733 | 19.812 | 0.686 | -0.019 | 99 | 0.986 |
| | 3 | 20.283 | 2.173 | 20.438 | 1.976 | -0.145 | 99 | 0.886 |
| | 4 | 19.746 | 0.941 | 19.666 | 0.273 | 0.071 | 99 | 0.944 |
| | 5 | 20.794 | 2.415 | 20.999 | 0.662 | -0.418 | 99 | 0.686 |

**Table 10.53 – Paired T-Test results for comparison of the arrival time of the last vehicle for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For all algorithms and for all strategies, there are no significant differences in the arrival time of the last vehicle.

#### 10.8.1.1 Conclusions

For the fourth objective function, customers' dissatisfaction, the best solution is obtained when each customer has a different objective function, for all strategies and all algorithms.

For the other objectives, no significant differences were found between the results obtained when all customers have the same priority, and the results obtained when each customer has a different priority, for all strategies and algorithms

## 10.8.2. Strategies Comparison – VEGA algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.54.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 74.917 | 5.738 | 96.128 | 3.976 | -7.58 | 99 | 0.002 |
| | 2 | 8.539 | 1.113 | 14.739 | 1.428 | -10.464 | 99 | 0 |
| | 3 | 2.33 | 1.112 | 1.367 | 0.627 | 2.475 | 99 | 0.033 |
| | 4 | 93.131 | 33.104 | 5.591 | 5.781 | 8.558 | 99 | 0 |
| | 5 | 21.304 | 0.749 | 19.984 | 0.577 | 4.192 | 99 | 0.004 |
| Different | 1 | 72.816 | 3.989 | 90.538 | 23.562 | -2.345 | 99 | 0.043 |
| | 2 | 8.177 | 0.859 | 14.223 | 2.919 | -6.017 | 99 | 0.001 |
| | 3 | 1.693 | 0.67 | 1.421 | 0.806 | 1.102 | 99 | 0.297 |
| | 4 | 4.631 | 2.078 | 0.115 | 0.109 | 7.178 | 99 | 0.001 |
| | 5 | 21.347 | 0.945 | 19.4 | 2.045 | 2.867 | 99 | 0.02 |

**Table 10.54 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As evident from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of customers' dissatisfaction and arrival time of the last vehicle. When all customers have the same priority, strategy 2 also provides better solutions in terms of tour balance.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.55.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 74.998 | 5.735 | 86.641 | 9.06 | -2.946 | 99 | 0.015 |
| | 2 | 8.668 | 1.047 | 15.887 | 2.608 | -6.66 | 99 | 0 |
| | 3 | 2.289 | 1.21 | 0.908 | 0.576 | 4.61 | 99 | 0.002 |
| | 4 | 93.073 | 33.216 | 30.392 | 15.752 | 5.641 | 99 | 0.001 |
| | 5 | 21.396 | 0.759 | 21.722 | 0.652 | -0.957 | 99 | 0.363 |
| Different | 1 | 72.759 | 4.011 | 95.29 | 7.676 | -7.574 | 99 | 0.001 |
| | 2 | 8.167 | 0.909 | 17.985 | 1.197 | -25.211 | 99 | 0.002 |
| | 3 | 1.845 | 0.646 | 0.936 | 0.486 | 4.146 | 99 | 0 |
| | 4 | 4.732 | 2.031 | 0.106 | 0.072 | 7.059 | 99 | 0.001 |
| | 5 | 21.293 | 1.068 | 21.173 | 0.717 | 0.413 | 99 | 0.687 |

**Table 10.55 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As may be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles

needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of tour balance and customers' dissatisfaction.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.56.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 96.015 | 3.923 | 86.604 | 9.079 | 3.516 | 99 | 0.009 |
| | 2 | 14.702 | 1.358 | 15.968 | 2.535 | -1.16 | 99 | 0.275 |
| | 3 | 1.385 | 0.582 | 0.974 | 0.501 | 2.405 | 99 | 0.039 |
| | 4 | 5.482 | 5.633 | 30.411 | 15.647 | -5.492 | 99 | 0.001 |
| | 5 | 20.003 | 0.688 | 21.591 | 0.734 | -5.28 | 99 | 0.001 |
| Different | 1 | 90.435 | 23.557 | 95.391 | 7.713 | -0.609 | 99 | 0.557 |
| | 2 | 14.319 | 2.882 | 17.997 | 1.211 | -3.553 | 99 | 0.005 |
| | 3 | 1.34 | 0.729 | 0.883 | 0.429 | 1.87 | 99 | 0.094 |
| | 4 | 0.038 | 0.184 | 0.228 | 0.117 | -2.626 | 99 | 0.027 |
| | 5 | 19.319 | 1.894 | 21.123 | 0.764 | -2.729 | 99 | 0.023 |

**Table 10.56 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As evident from the results, whether or not all customers have the same priority, strategy 2 provides better solutions only in terms of customer dissatisfaction and arrival time of the last vehicle. When all customers have the same priority, strategy 3 provides better solutions in terms of travel time and tour balance. When each customer has different priorities, strategy 2 provides better solutions in terms of the number of vehicles needed.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.57.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | Df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 96.13 | 3.931 | 95.468 | 7.887 | 0.211 | 99 | 0.838 |
| | 2 | 14.751 | 1.381 | 15.093 | 1.781 | -0.52 | 99 | 0.618 |
| | 3 | 1.442 | 0.548 | 1.623 | 1.004 | -0.464 | 99 | 0.653 |
| | 4 | 5.536 | 5.691 | 5.361 | 7.345 | 0.068 | 99 | 0.949 |
| | 5 | 19.944 | 0.606 | 19.967 | 1.16 | 0.184 | 99 | 0.858 |
| Different | 1 | 90.565 | 23.404 | 93.932 | 6.541 | -0.431 | 99 | 0.676 |
| | 2 | 14.352 | 2.793 | 14.386 | 1.653 | -0.094 | 99 | 0.928 |
| | 3 | 1.429 | 0.796 | 1.373 | 0.282 | -0.106 | 99 | 0.92 |
| | 4 | 0.106 | 0.081 | 0.047 | 0.144 | -0.44 | 99 | 0.668 |
| | 5 | 19.349 | 2.022 | 20.213 | 0.924 | -1.146 | 99 | 0.281 |

**Table 10.57 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As may be seen from the results, no significant difference was found for any of the objective functions.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.58.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 86.78 | 8.919 | 84.353 | 8.035 | 0.648 | 99 | 0.535 |
| | 2 | 15.923 | 2.528 | 15.607 | 1.861 | 0.26 | 99 | 0.801 |
| | 3 | 0.954 | 0.519 | 0.935 | 0.478 | -0.098 | 99 | 0.924 |
| | 4 | 30.452 | 15.672 | 30.4 | 14.666 | -0.007 | 99 | 0.993 |
| | 5 | 21.619 | 0.687 | 21.203 | 1.508 | 0.647 | 99 | 0.535 |
| Different | 1 | 95.238 | 7.577 | 89.941 | 16.223 | 1.058 | 99 | 0.316 |
| | 2 | 17.85 | 1.28 | 16.752 | 2.833 | 1.177 | 99 | 0.268 |
| | 3 | 0.83 | 0.533 | 0.664 | 0.148 | 1.127 | 99 | 0.29 |
| | 4 | 0.16 | 0.17 | 0.107 | 0.197 | 0.018 | 99 | 0.986 |
| | 5 | 21.142 | 0.622 | 20.654 | 1.811 | 0.637 | 99 | 0.54 |

**Table 10.58 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As seen from the results, no significant difference was found for any of the objective functions.

#### 10.8.2.1 Conclusions

Table 10.59 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | + | + |
| | 4 | | + | | + | |
| | 5 | | + | | + | |
| Different | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | | + | | + | + |
| | 5 | | + | | + | + |

**Table 10.59 - Best strategy for each of the objective functions**

As can be seen from Table 10.59, when all customers have the same priority, objective functions 1, travel time, and 2, number of vehicles needed, are obtained by using strategy 1. Objective 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, is obtained by using strategy 4.

When each customer has a different priority, objective functions 1, travel time, and 2, number of vehicles needed, are obtained by using strategy 1. Objective 3, tour balance, 4, customer's dissatisfaction, and 5, arrival time of the last vehicle, are obtained by using strategy 5.

### 10.8.3. Strategies Comparison – SPEA2 algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.60.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 98.328 | 9.458 | 104.846 | 23.48 | -0.777 | 99 | 0.459 |
| | 2 | 13.254 | 1.786 | 18.214 | 2.458 | -4.298 | 99 | 0.002 |

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| | 3 | 0.764 | 0.421 | 1.187 | 0.547 | -1.913 | 99 | 0.087 |
| | 4 | 2.514 | 2.364 | 1.223 | 0.521 | 1.647 | 99 | 0.133 |
| | 5 | 19.635 | 0.227 | 18.208 | 2.198 | 1.73 | 99 | 0.12 |
| Different | 1 | 91.628 | 3.761 | 106.35 | 12.629 | -3.561 | 99 | 0.006 |
| | 2 | 12.755 | 1.306 | 18.684 | 2.75 | -6.902 | 99 | 0.002 |
| | 3 | 0.893 | 0.46 | 1.222 | 0.465 | -0.619 | 99 | 0.553 |
| | 4 | 0.055 | 0.15 | 0.011 | -0.06 | 1.869 | 99 | 0.096 |
| | 5 | 19.549 | 0.058 | 19.174 | 1.382 | 0.686 | 99 | 0.509 |

**Table 10.60 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As reflected in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. When each customer has a different priority, strategy 1 provides better solutions in terms of travel time as well.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.61.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 98.349 | 9.374 | 92.17 | 19.5 | 0.935 | 99 | 0.376 |
| | 2 | 13.222 | 1.73 | 19.537 | 3.158 | -5.424 | 99 | 0.002 |
| | 3 | 0.901 | 0.434 | 0.581 | 0.229 | 2.55 | 99 | 0.032 |
| | 4 | 2.493 | 2.288 | 10.69 | 5.37 | -3.961 | 99 | 0.004 |
| | 5 | 19.642 | 0.306 | 20.585 | 2.643 | -1.268 | 99 | 0.239 |
| Different | 1 | 91.589 | 3.763 | 99.389 | 2.978 | -4.847 | 99 | 0.001 |
| | 2 | 12.687 | 1.259 | 19.394 | 1.033 | -13.863 | 99 | 0.002 |
| | 3 | 0.929 | 0.592 | 0.802 | 0.126 | 1.162 | 99 | 0.275 |
| | 4 | 0.185 | 0.041 | 0.143 | 0.056 | 2.009 | 99 | 0.074 |
| | 5 | 19.437 | 0.165 | 21.098 | 0.362 | -11.042 | 99 | 0.001 |

**Table 10.61 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As may be seen from the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. When all customers have the same priority, strategy 1 provides better results in terms of customers' dissatisfaction. When each customer has a different priority, strategy 1 provides better solutions in terms of travel time and arrival time of the last vehicle.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.62.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 104.895 | 23.389 | 92.185 | 19.457 | 1.551 | 99 | 0.154 |
| | 2 | 18.285 | 2.421 | 19.471 | 3.248 | -1.243 | 99 | 0.246 |
| | 3 | 1.181 | 0.447 | 0.418 | 0.069 | 3.858 | 99 | 0.004 |
| | 4 | 1.141 | 0.571 | 10.672 | 5.361 | -5.644 | 99 | 0.001 |
| | 5 | 18.314 | 2.272 | 20.669 | 2.636 | -2.116 | 99 | 0.061 |
| Different | 1 | 106.281 | 12.711 | 99.516 | 2.964 | 1.582 | 99 | 0.146 |
| | 2 | 18.781 | 2.795 | 19.276 | 0.873 | -0.618 | 99 | 0.553 |
| | 3 | 1.137 | 0.462 | 0.668 | 0.202 | 2.565 | 99 | 0.03 |
| | 4 | 0.125 | 0.097 | 0.029 | 0.038 | -0.045 | 99 | 0.964 |
| | 5 | 19.182 | 1.453 | 20.99 | 0.378 | -4.409 | 99 | 0.002 |

**Table 10.62 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As seen from the results, whether all customers have the same priority or each customer has a different priority, a significant difference was found for tour balance, which is lower in strategy 3. When each customer has his own priority, a significant difference was found for the arrival time at the last customer, which is earlier in strategy 2. A significant difference was also found for customers' dissatisfaction, which is lower in strategy 2 when all customers have the same priority.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.15.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 104.934 | 23.486 | 115.673 | 7.429 | -1.422 | 99 | 0.188 |
| | 2 | 18.202 | 2.377 | 18.578 | 0.884 | -0.425 | 99 | 0.683 |
| | 3 | 1.078 | 0.462 | 1.258 | 0.559 | -0.91 | 99 | 0.387 |
| | 4 | 1.216 | 0.576 | 1.018 | 0.334 | 0.548 | 99 | 0.598 |
| | 5 | 18.399 | 2.2 | 19.273 | 1.103 | -1.715 | 99 | 0.12 |
| Different | 1 | 106.252 | 12.613 | 107.272 | 12.635 | -0.236 | 99 | 0.82 |
| | 2 | 18.78 | 2.7 | 17.776 | 2.548 | 0.937 | 99 | 0.372 |
| | 3 | 1.206 | 0.585 | 1.232 | 0.441 | -0.644 | 99 | 0.538 |
| | 4 | 0.007 | 0.13 | 0.064 | 0.066 | 1.28 | 99 | 0.232 |
| | 5 | 19.185 | 1.369 | 19.099 | 1.173 | 0.245 | 99 | 0.814 |

**Table 10.63 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As demonstrated by the results, whether all customers have the same priority or each customer has a different priority, no significant difference was found for any of the objective functions.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.16.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 92.027 | 19.434 | 86.593 | 15.166 | 0.562 | 99 | 0.591 |
| | 2 | 19.466 | 3.112 | 17.766 | 2.537 | 1.129 | 99 | 0.289 |
| | 3 | 0.51 | 0.099 | 0.671 | 0.377 | -1.268 | 99 | 0.237 |
| | 4 | 10.722 | 5.297 | 15.877 | 10.311 | -1.336 | 99 | 0.214 |
| | 5 | 20.66 | 2.695 | 20.791 | 2.761 | -0.197 | 99 | 0.849 |
| Different | 1 | 99.35 | 3.041 | 97.33 | 8.619 | 0.727 | 99 | 0.488 |
| | 2 | 19.335 | 0.917 | 20.097 | 1.089 | -1.411 | 99 | 0.191 |
| | 3 | 0.686 | 0.272 | 0.843 | 0.386 | -0.034 | 99 | 0.974 |
| | 4 | 0.038 | 0.01 | 0.04 | 0.167 | -0.737 | 99 | 0.48 |
| | 5 | 21.038 | 0.537 | 20.729 | 2.118 | 0.373 | 99 | 0.719 |

**Table 10.64 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As reflected by the results, whether all customers have the same priority or each customer has a different priority, no significant difference was found for any of the objective functions.

### 10.8.3.1 Conclusions

Table 10.65 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | + | + | | + |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | + | + | | + | |
| | 5 | + | + | + | + | + |
| Different | 1 | + | | | | + |
| | 2 | + | | | | |
| | 3 | + | | + | | + |
| | 4 | | + | + | + | + |
| | 5 | + | + | | + | |

**Table 10.65 - Best strategy for each of the objective functions**

As can be seen from Table 10.65, when all customers have the same priority, objective functions 1, travel time, and 2, number of vehicles needed, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are obtained by using strategy 1. Objective 3, tour balance, is obtained by using either strategy 3 or strategy 5.

When each customer has a different priority, objective functions 1, travel time, and 2, number of vehicles, 3, tour balance, and 5, arrival time of the last vehicle needed, are obtained by using strategy 1. Objective 4, customers' dissatisfaction, is obtained by using either strategy 3 or strategy 5.

### 10.8.4. Strategies Comparison – VE-ABC algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.66.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 103.991 | 16.358 | 133.988 | 6.604 | -5.079 | 99 | 0 |
| | 2 | 13.053 | 2.565 | 21.795 | 1.266 | -14.567 | 99 | 0 |
| | 3 | 1.646 | 0.965 | 0.789 | 0.474 | 2.268 | 99 | 0.05 |
| | 4 | 95.189 | 50.974 | 1.575 | 0.304 | 5.797 | 99 | 0.002 |
| | 5 | 21.765 | 1.512 | 19.756 | 0.783 | 3.306 | 99 | 0.009 |
| Different | 1 | 115.402 | 7.002 | 131.389 | 6.51 | -5.009 | 99 | 0.002 |
| | 2 | 14.529 | 1.618 | 21.148 | 1.265 | -9.849 | 99 | 0 |
| | 3 | 1.6 | 0.647 | 0.915 | 0.161 | 2.577 | 99 | 0.03 |
| | 4 | 3.047 | 1.223 | 0.135 | -0.037 | 7.67 | 99 | 0.002 |
| | 5 | 21.279 | 1.26 | 19.948 | 0.662 | 3.129 | 99 | 0.011 |

**Table 10.66 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As demonstrated by the results, when all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, strategy 2 provides better solutions in terms of tour balance, customers' dissatisfaction and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.67.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 104.116 | 16.328 | 90.888 | 17.502 | 1.724 | 99 | 0.117 |
| | 2 | 13.051 | 2.536 | 18.468 | 4.107 | -3.389 | 99 | 0.009 |
| | 3 | 1.539 | 0.944 | 0.829 | 0.529 | 2.01 | 99 | 0.076 |
| | 4 | 95.186 | 51.045 | 29.172 | 18.299 | 3.374 | 99 | 0.01 |
| | 5 | 21.601 | 1.524 | 20.208 | 2.26 | 1.603 | 99 | 0.142 |
| Different | 1 | 115.243 | 6.968 | 94.899 | 18.112 | 3.46 | 99 | 0.009 |
| | 2 | 14.545 | 1.74 | 18.317 | 3.51 | -3.784 | 99 | 0.005 |
| | 3 | 1.492 | 0.686 | 0.812 | 0.213 | 3.579 | 99 | 0.005 |
| | 4 | 2.994 | 1.166 | 0.073 | 0.132 | 7.279 | 99 | 0 |
| | 5 | 21.3 | 1.297 | 20.403 | 2.01 | 0.884 | 99 | 0.399 |

**Table 10.67 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

Based on the results, when all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. Under the same conditions, strategy 3 provides better solutions in terms of customers' dissatisfaction. When each customer has different priority, strategy 1 provides better solutions in terms of the number of vehicles needed and strategy 3 provides better solutions for tour balance and customers' dissatisfaction.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.68.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 133.994 | 6.586 | 90.899 | 17.496 | 8.41 | 99 | 0.002 |
| | 2 | 21.893 | 1.41 | 18.491 | 4.158 | 2.684 | 99 | 0.026 |
| | 3 | 0.798 | 0.599 | 0.792 | 0.563 | -0.131 | 99 | 0.899 |
| | 4 | 1.539 | 0.239 | 29.21 | 18.353 | -4.838 | 99 | 0.001 |
| | 5 | 19.836 | 0.683 | 20.375 | 2.346 | -0.593 | 99 | 0.567 |
| Different | 1 | 131.349 | 6.508 | 95.055 | 18.105 | 4.948 | 99 | 0.003 |
| | 2 | 21.118 | 1.274 | 18.325 | 3.559 | 2.169 | 99 | 0.057 |
| | 3 | 1.034 | 0.076 | 0.806 | 0.215 | 1.561 | 99 | 0.154 |
| | 4 | 0.095 | 0.105 | 0.06 | 0.229 | -0.674 | 99 | 0.517 |
| | 5 | 19.783 | 0.627 | 20.46 | 2.106 | -0.755 | 99 | 0.471 |

**Table 10.68 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As shown in the results, whether or not all customers have the same priority, strategy 3 provides better solutions in terms of travel time. Strategy 2 provides better solutions in terms of customers' dissatisfaction, when all customers have the same priority and strategy 3 provides better solutions in terms of number of vehicles needed.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.69.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 134.024 | 6.593 | 131.596 | 6.335 | 0.817 | 99 | 0.433 |
| | 2 | 21.838 | 1.244 | 21.109 | 1.243 | 0.969 | 99 | 0.357 |
| | 3 | 0.835 | 0.617 | 1.077 | 0.393 | -1.863 | 99 | 0.093 |
| | 4 | 1.449 | 0.401 | 1.989 | 1.723 | -0.969 | 99 | 0.358 |
| | 5 | 19.926 | 0.74 | 19.7 | 1.077 | 0.167 | 99 | 0.872 |
| Different | 1 | 131.484 | 6.625 | 128.319 | 9.714 | 0.863 | 99 | 0.413 |
| | 2 | 21.249 | 1.394 | 20.589 | 1.487 | 0.942 | 99 | 0.37 |
| | 3 | 1.044 | 0.259 | 0.607 | 0.424 | 1.793 | 99 | 0.104 |
| | 4 | 0.119 | 0.124 | 0.016 | 0.037 | 0.8 | 99 | 0.445 |
| | 5 | 19.818 | 0.573 | 19.859 | 0.322 | 0.468 | 99 | 0.651 |

**Table 10.69 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As reflected in the results, whether all customers have the same priority or each customer has a different priority, no significant difference was found for any objective function.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.70.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 90.824 | 17.516 | 93.046 | 18.414 | -0.322 | 99 | 0.752 |
| | 2 | 18.474 | 4.281 | 18.437 | 3.5 | -0.002 | 99 | 1 |
| | 3 | 0.78 | 0.509 | 5.412 | 9.818 | -1.532 | 99 | 0.159 |
| | 4 | 29.197 | 18.32 | 23.96 | 15.936 | 0.656 | 99 | 0.531 |
| | 5 | 20.373 | 2.242 | 20.609 | 2.535 | -0.457 | 99 | 0.659 |
| Different | 1 | 94.99 | 18.101 | 99.951 | 3.93 | -0.827 | 99 | 0.432 |
| | 2 | 18.24 | 3.474 | 19.422 | 1.142 | -0.856 | 99 | 0.414 |
| | 3 | 0.876 | 0.292 | 0.913 | 0.298 | -0.394 | 99 | 0.702 |
| | 4 | 0.074 | 0.159 | 0.157 | -0.005 | 0.191 | 99 | 0.853 |
| | 5 | 20.507 | 2.068 | 21.059 | 0.708 | -0.975 | 99 | 0.356 |

**Table 10.70 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As seen from the results, whether all customers have the same priority or each customer has a different priority, no significant difference was found for any of the objective functions.

#### 10.8.4.1 Conclusions

Table 10.71 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | + | | + |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | | + | | + | |
| | 5 | | + | + | + | + |
| Different | 1 | | | + | | + |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | | + | + | + | + |
| | 5 | | + | + | + | |

**Table 10.71 - Best strategy for each of the objective functions**

As seen in Table 10.71, when all customers have the same priority, objective functions 1, travel time, and 2, number of vehicles needed, are obtained by using strategy 1. Objective 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 2 or strategy 4.

When each customer has a different priority, objective 2, number of vehicles is best obtained using strategy 2. Objective functions 1, travel time, 3, tour balance, 4, customers' dissatisfaction and 5, arrival time of the last vehicle needed, are obtained by using strategy 3.

### 10.8.5. Algorithms Comparison

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 3 has to be used. Table 10.72 compares the results obtained for each of the five objectives, using paired t-tests, functions by each one of the three algorithms when applying the 3$^{rd}$ strategy. For each objective function, the best value obtained is highlighted in red.

| Customer's Priority | Objective Function | Algorithm | | |
|---|---|---|---|---|
| | | Imp. VEGA | SPEA2 | VE-ABC |
| The Same | 1 | 84.353 | 86.593 | 93.046 |
| | 2 | 15.607 | 17.766 | 18.437 |
| | 3 | 0.935 | 0.671 | 5.412 |
| | 4 | 30.4 | 15.877 | 23.96 |
| | 5 | 21.203 | 20.791 | 20.609 |
| Different | 1 | 89.941 | 97.33 | 99.951 |
| | 2 | 16.752 | 20.097 | 19.422 |
| | 3 | 0.664 | 0.843 | 0.913 |
| | 4 | 0.107 | 0.04 | 0.157 |
| | 5 | 20.654 | 20.729 | 21.059 |

**Table 10.72 - Comparison of the 5th strategy used in all three algorithms**

As may be seen, whether all customers have the same priority or not, objective 1, travel time, and objective 2, number of vehicles needed are best obtained by using the improved VEGA algorithm. Objective 4, customers' dissatisfaction is best obtained by using the SPEA2 algorithm.

### 10.8.6. Conclusions

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 5 has to be used. From the results obtained, one should use the improved VEGA algorithm, which for most objective functions returns the best solutions. When looking at the different strategies, there is no dominant strategy which provides the best solution for most scenarios.

## 10.9. Case Study 4

In the second case study the test scenario is defined as follows:

1. Network: Israeli transportation network.
2. Dissatisfaction function: It is assumed that all customers don't like the supplier to arrive either early or late. Therefore, the dissatisfaction functions of all customers are in the form of $f_i(t) = 1 - \left( \frac{t - EET_i}{TW_i^S - EET_i} \right)^5$ and $g_i(t) = 1 - \left( \frac{ELT - t}{ELT - TW_i^E} \right)^5$.

The test scenario is solved 100 times. In the first 50 times, it is assumed that all customers have the same priority. Under this assumption, the test scenario is solved 100 times using each of the 5 strategies described earlier. In the next 50 times, it is assumed

that each customer has a priority equal to his demand. Under this assumption, the test scenario is solved 10 times using each of the 5 strategies described earlier.

### 10.9.1. Priority Comparison

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total travel time obtained when all customers have the same priority vs. the travel time obtained when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.73.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 74.415 | 4.496 | 115.336 | 6.93 | -15.094 | 99 | 0.001 |
| | 2 | 93.706 | 5.252 | 133.965 | 6.583 | -16.923 | 99 | 0.001 |
| | 3 | 96.545 | 4.561 | 90.816 | 17.427 | 0.915 | 99 | 0.384 |
| | 4 | 95.298 | 5.347 | 131.458 | 6.299 | -21.316 | 99 | 0 |
| | 5 | 92.853 | 6.377 | 93.129 | 18.483 | -0.041 | 99 | 0.97 |
| SPEA2 | 1 | 103.661 | 6.878 | 92.417 | 6.592 | 3.493 | 99 | 0.006 |
| | 2 | 116.254 | 13.96 | 113.64 | 23.399 | 0.394 | 99 | 0.7 |
| | 3 | 88.067 | 15.175 | 92.171 | 18.351 | -0.633 | 99 | 0.542 |
| | 4 | 117.739 | 16.571 | 117.688 | 16.552 | 0.999 | 99 | 0.344 |
| | 5 | 82.621 | 27.741 | 98.836 | 4.102 | -1.796 | 99 | 0.106 |
| VE-ABC | 1 | 90.767 | 28.797 | 93.073 | 31.142 | -0.153 | 99 | 0.882 |
| | 2 | 134.134 | 4.239 | 134.21 | 7.174 | 0.037 | 99 | 0.97 |
| | 3 | 105.067 | 10.563 | 97.619 | 5.944 | 1.868 | 99 | 0.095 |
| | 4 | 136.467 | 5.37 | 137.561 | 7.623 | -0.291 | 99 | 0.781 |
| | 5 | 96.543 | 17.545 | 100.716 | 13.303 | -0.567 | 99 | 0.584 |

**Table 10.73 – Paired T-Test resuls for comparison of the total travel time for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as seen from the results, for strategy 1, 2 and 4, there exists a significant difference in the solution obtained when all customers have the same priority vs. the solution obtained when each customer has a different priority, which is better when all customers have the same priority. For the SPEA2 algorithm, for strategy 1 there exists a significant difference in the solution obtained when all customers have the same priority vs. the solution obtained when each customer has a different priority, which is better when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the number of vehicles needed when all customers have the same priority vs. the number

of vehicles needed when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.74.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 8.408 | 0.753 | 14.536 | 1.704 | -10.464 | 99 | 0.001 |
| | 2 | 14.85 | 1.835 | 21.898 | 1.328 | -9.391 | 99 | 0.001 |
| | 3 | 17.684 | 0.74 | 18.407 | 4.158 | -0.564 | 99 | 0.585 |
| | 4 | 14.537 | 1.695 | 21.122 | 1.337 | -14.178 | 99 | 0.001 |
| | 5 | 16.933 | 1.464 | 18.546 | 3.404 | -1.191 | 99 | 0.265 |
| SPEA2 | 1 | 13.727 | 1.096 | 12.579 | 1.409 | 2.01 | 99 | 0.077 |
| | 2 | 19.589 | 1.098 | 19.327 | 3.098 | 0.225 | 99 | 0.829 |
| | 3 | 19.1 | 2.125 | 19.914 | 1.658 | -1.431 | 99 | 0.187 |
| | 4 | 19.833 | 1.242 | 19.869 | 1.306 | 1 | 99 | 0.344 |
| | 5 | 18.274 | 3.944 | 19.373 | 1.277 | -0.838 | 99 | 0.425 |
| VE-ABC | 1 | 11.735 | 3.556 | 11.022 | 4.14 | 0.386 | 99 | 0.709 |
| | 2 | 21.788 | 1.127 | 21.837 | 1.022 | -0.288 | 99 | 0.78 |
| | 3 | 20.423 | 1.466 | 19.174 | 2.337 | 1.815 | 99 | 0.103 |
| | 4 | 22.871 | 1.632 | 21.735 | 1.202 | 1.673 | 99 | 0.128 |
| | 5 | 19.35 | 2.701 | 18.778 | 2.36 | 0.372 | 99 | 0.719 |

**Table 10.74 – Paired T-Test resuls for comparison of the number of vehicles needed for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as seen in the results, for strategies 1, 2 and 4, there are significant differences in the number of vehicles needed, which is lower when all customers have the same priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the balance of the tours when all customers have the same priority vs. the number of vehicles needed when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.75.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 1.738 | 0.651 | 1.55 | 0.569 | 0.667 | 99 | 0.519 |
| | 2 | 1.437 | 0.396 | 0.699 | 0.458 | 8.801 | 99 | 0.001 |
| | 3 | 0.757 | 0.136 | 0.666 | 0.524 | -0.24 | 99 | 0.816 |
| | 4 | 1.372 | 0.659 | 1.062 | 0.434 | 1.812 | 99 | 0.104 |
| | 5 | 0.968 | 0.365 | 5.496 | 9.861 | -1.442 | 99 | 0.183 |
| SPEA2 | 1 | 0.774 | 0.185 | 0.897 | 0.143 | -3.021 | 99 | 0.013 |
| | 2 | 1.238 | 0.396 | 1.078 | 0.341 | 1.161 | 99 | 0.275 |
| | 3 | 0.703 | 0.224 | 0.533 | 0.148 | 1.696 | 99 | 0.125 |
| | 4 | 1.037 | 0.455 | 1.029 | 0.4 | 0.999 | 99 | 0.344 |
| | 5 | 0.467 | 0.194 | 0.784 | 0.209 | -3.224 | 99 | 0.012 |
| VE-ABC | 1 | 1.551 | 0.677 | 6.272 | 13.275 | -1.119 | 99 | 0.293 |
| | 2 | 1.019 | 0.457 | 0.655 | 0.313 | 2.149 | 99 | 0.062 |
| | 3 | 2.21 | 5.521 | 0.697 | 0.359 | 0.845 | 99 | 0.421 |
| | 4 | 0.71 | 0.253 | 0.944 | 0.382 | -0.345 | 99 | 0.739 |
| | 5 | 2.929 | 7.77 | 0.85 | 0.46 | 0.868 | 99 | 0.408 |

**Table 10.75 – Paired T-Test results for comparison of the balance of the tours for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as may be seen from the results, for strategy 2, there is a significant difference in the tour balance, which is lower (meaning more balanced) when each customer has a different priority. For the SPEA2 algorithm, for strategies 1 and 5, there is also a significant difference in the tour balance, which is lower when all customers have the same priority. As for the VE-ABC algorithm, for all strategies there is a significant difference in the tour balance.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the total dissatisfaction of the customers when all customers have the same priority vs. the total dissatisfaction of the customers when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.76.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 60.446 | 42.088 | 3.039 | 1.229 | 4.255 | 99 | 0.003 |
| | 2 | 0.767 | 0.696 | 1.431 | 0.411 | -2.982 | 99 | 0.013 |
| | 3 | 1.075 | 0.825 | 29.241 | 18.272 | -4.862 | 99 | 0.002 |
| | 4 | 0.548 | 0.698 | 2.101 | 1.688 | -2.291 | 99 | 0.046 |
| | 5 | 29.526 | 15.461 | 23.993 | 15.882 | 0.69 | 99 | 0.508 |
| SPEA2 | 1 | 433.871 | 383.07 | 0.287 | 0.147 | 3.578 | 99 | 0.005 |
| | 2 | 156.154 | 24.675 | 0.048 | 0.08 | 19.93 | 99 | 0.002 |
| | 3 | 1336.587 | 475.511 | 0.281 | 0.262 | 8.882 | 99 | 0.001 |
| | 4 | 134.412 | 30.199 | 4.074 | 0.905 | 14.051 | 99 | 0.001 |
| | 5 | 1159.944 | 758.856 | 0.224 | 0.125 | 4.832 | 99 | 0.001 |

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VE-ABC | 1 | 6287.886 | 3250.649 | 6.563 | 3.637 | 6.11 | 99 | 0 |
| | 2 | 281.695 | 116.615 | 0.313 | 0.205 | 7.623 | 99 | 0.002 |
| | 3 | 2017.89 | 426.099 | 0.575 | 0.377 | 14.967 | 99 | 0.002 |
| | 4 | 353.173 | 302.496 | 0.245 | 0.108 | 3.689 | 99 | 0.004 |
| | 5 | 1526.441 | 772.201 | 0.451 | 0.489 | 6.248 | 99 | 0.001 |

**Table 10.76 – Paired T-Test results for comparison of the total dissatisfaction of customers for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the improved VEGA algorithm, as seen from the results, for strategy 1, there is a significant difference in total dissatisfaction, which is lower when each customer has a different priority. For strategies 2, 3 and 4, there is a significant difference in total dissatisfaction, which is lower when all customers have the same priority.

For the SPEA2 and the VE-ABC algorithm, in all strategies there is a significant difference in the total dissatisfaction of customers obtained when all customers have the same priority and when each customer has a different priority, which is lower when each customer has a different priority.

For each of our three algorithms, five paired-samples t-tests were conducted to compare the arrival time of the last vehicle when all customers have the same priority vs. the arrival time of the last vehicle when each customer has a different priority, for each of the five strategies. The results are summarized in Table 10.77.

| Algorithm | | Same Priority | | Different Priority | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | Strategy | M | SD | M | SD | | | |
| VEGA | 1 | 20.538 | 0.779 | 21.262 | 1.241 | -1.141 | 99 | 0.281 |
| | 2 | 20.133 | 0.757 | 19.882 | 0.719 | 0.597 | 99 | 0.565 |
| | 3 | 21.032 | 0.859 | 20.301 | 2.197 | 0.958 | 99 | 0.364 |
| | 4 | 19.768 | 0.208 | 19.766 | 0.972 | 0.076 | 99 | 0.941 |
| | 5 | 21.621 | 0.923 | 20.775 | 2.431 | 1.108 | 99 | 0.297 |
| SPEA2 | 1 | 19.916 | 0.279 | 19.461 | 0.126 | 3.464 | 99 | 0.007 |
| | 2 | 19.186 | 1.85 | 19.339 | 2.177 | -0.395 | 99 | 0.702 |
| | 3 | 20.374 | 2.3 | 19.792 | 2.627 | 0.468 | 99 | 0.653 |
| | 4 | 19.253 | 1.806 | 19.195 | 1.756 | 0.999 | 99 | 0.345 |
| | 5 | 19.321 | 3.914 | 20.582 | 0.313 | -1.061 | 99 | 0.318 |
| VE-ABC | 1 | 21.291 | 2.444 | 21.477 | 1.065 | -0.056 | 99 | 0.958 |
| | 2 | 19.779 | 0.717 | 19.775 | 0.521 | 0.049 | 99 | 0.963 |
| | 3 | 22.56 | 1.972 | 21.013 | 0.498 | 2.311 | 99 | 0.045 |
| | 4 | 20.178 | 0.502 | 20.114 | 0.87 | 0.116 | 99 | 0.908 |
| | 5 | 20.938 | 2.3 | 20.926 | 0.743 | -0.066 | 99 | 0.95 |

**Table 10.77 – Paired T-Test results for comparison of the arrival time of the last vehicle for all three algorithms when all customers have the same priority vs. each customer has a different priority**

For the SPEA2 algorithm, as seen in the results, for strategy 1, there is a significant difference in the arrival time of the last vehicle, which is earlier when each customer has a different priority. Similarly, for the VE-ABC algorithm, strategy 3 provides better solutions when each customer has a different priority.

#### 10.9.1.1 Conclusions

For the first objective, travel time, the second objective, number of vehicles needed, and fourth objective function, customers' dissatisfaction, the best solution is obtained when all customers have the same priority, using the VEGA algorithm.

For the fourth objective function, customers' dissatisfaction, when each customer has a different priority, the best solution is obtained using the SPEA2 and VE-ABC algorithms for all strategies.

For the other objectives, no significant differences were found between the results obtained when all customers have the same priority, and the results obtained when each customer has a different priority, for all strategies and algorithms

### 10.9.2. Strategies Comparison – VEGA algorithm

In order to examine the effect of each of the strategies of the algorithm's results several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.78.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 74.389 | 4.596 | 93.694 | 5.392 | -10.82 | 99 | 0.001 |
| | 2 | 8.306 | 0.662 | 14.851 | 1.799 | -10.35 | 99 | 0.002 |
| | 3 | 1.821 | 0.641 | 1.553 | 0.541 | 0.814 | 99 | 0.436 |
| | 4 | 1.936 | 1.255 | 0.01 | 0.064 | 4.48 | 99 | 0.003 |
| | 5 | 20.57 | 0.667 | 19.998 | 0.684 | 1.779 | 99 | 0.109 |
| Different | 1 | 115.354 | 6.965 | 133.939 | 6.65 | -5.318 | 99 | 0 |
| | 2 | 14.509 | 1.761 | 21.814 | 1.335 | -9.9 | 99 | 0.001 |
| | 3 | 1.565 | 0.691 | 0.74 | 0.584 | 2.836 | 99 | 0.019 |
| | 4 | 3.09 | 1.152 | 1.521 | 0.359 | 3.793 | 99 | 0.002 |
| | 5 | 21.16 | 1.315 | 19.856 | 0.836 | 2.779 | 99 | 0.022 |

**Table 10.78 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As shown in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of customers' dissatisfaction. When each customer has a different priority, strategy 2 provides better solutions in terms of tour balance and arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.79.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 74.464 | 4.508 | 96.481 | 4.721 | -9.927 | 99 | 0.001 |
| | 2 | 8.381 | 0.675 | 17.72 | 0.9 | -25.363 | 99 | 0.002 |
| | 3 | 1.819 | 0.695 | 0.77 | 0.293 | 4.569 | 99 | 0.002 |
| | 4 | 1.956 | 1.327 | 0.101 | 0.058 | 4.468 | 99 | 0.003 |
| | 5 | 20.557 | 0.677 | 21.149 | 0.881 | -1.402 | 99 | 0.196 |
| Different | 1 | 115.369 | 7.093 | 90.901 | 17.526 | 3.982 | 99 | 0.004 |
| | 2 | 14.581 | 1.777 | 18.501 | 4.169 | -2.994 | 99 | 0.015 |
| | 3 | 1.503 | 0.604 | 0.76 | 0.532 | 3.445 | 99 | 0.007 |
| | 4 | 3.048 | 1.152 | 29.186 | 18.338 | -4.446 | 99 | 0 |
| | 5 | 21.256 | 1.302 | 20.339 | 2.166 | 1.073 | 99 | 0.308 |

**Table 10.79 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As reflected in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 3 provides better solutions in terms of tour balance and customers' dissatisfaction. If all customers have the same priority, strategy 1 provides better solutions in terms of travel time, but if each customer has a different priority, then better travel time is obtained by using strategy 3.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.80.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 93.803 | 5.308 | 96.484 | 4.649 | -1.771 | 99 | 0.111 |
| | 2 | 14.892 | 1.821 | 17.668 | 0.844 | -4.41 | 99 | 0.002 |
| | 3 | 1.566 | 0.48 | 0.723 | 0.308 | 4.65 | 99 | 0.002 |
| | 4 | 0.082 | 0.001 | 0.022 | 0.06 | -0.726 | 99 | 0.487 |
| | 5 | 19.992 | 0.751 | 21.105 | 0.82 | -3.049 | 99 | 0.014 |

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| Different | 1 | 133.976 | 6.656 | 90.879 | 17.472 | 8.409 | 99 | 0 |
| | 2 | 21.774 | 1.405 | 18.572 | 4.143 | 2.683 | 99 | 0.025 |
| | 3 | 0.785 | 0.518 | 0.854 | 0.537 | -0.131 | 99 | 0.896 |
| | 4 | 1.501 | 0.398 | 29.294 | 18.259 | -4.84 | 99 | 0.002 |
| | 5 | 19.814 | 0.836 | 20.306 | 2.32 | -0.591 | 99 | 0.569 |

**Table 10.80 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As shown in the results, if all customers have the same priority, strategy 2 provides better results in terms of the number of vehicles needed and arrival time at the last customer, while strategy 3 provides better solutions in terms of tour balance. When each customer has a different priority, strategy 2 provides better results in terms of customers' dissatisfaction, while strategy 3 provides better solutions in terms of travel time and number of vehicles needed.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.81.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 93.711 | 5.32 | 95.243 | 5.308 | -0.689 | 99 | 0.511 |
| | 2 | 14.873 | 1.828 | 14.548 | 1.62 | 0.36 | 99 | 0.728 |
| | 3 | 1.532 | 0.516 | 1.377 | 0.549 | 0.328 | 99 | 0.749 |
| | 4 | 0.11 | 0.041 | 0.095 | 0.06 | 0.455 | 99 | 0.66 |
| | 5 | 20.039 | 0.686 | 19.807 | 0.312 | 1.404 | 99 | 0.193 |
| Different | 1 | 134.083 | 6.642 | 131.538 | 6.28 | 0.819 | 99 | 0.432 |
| | 2 | 21.884 | 1.279 | 21.266 | 1.336 | 0.969 | 99 | 0.357 |
| | 3 | 0.828 | 0.462 | 0.972 | 0.303 | -1.864 | 99 | 0.094 |
| | 4 | 1.439 | 0.418 | 2.04 | 1.792 | -0.971 | 99 | 0.359 |
| | 5 | 19.908 | 0.802 | 19.831 | 0.987 | 0.168 | 99 | 0.869 |

**Table 10.81 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As demonstrated in the results, no significant differences were found between the two strategies.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.82.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 96.611 | 4.588 | 92.834 | 6.436 | 1.318 | 99 | 0.22 |
| | 2 | 17.689 | 0.92 | 17.021 | 1.546 | 1.768 | 99 | 0.112 |
| | 3 | 0.755 | 0.246 | 0.929 | 0.469 | -1.345 | 99 | 0.212 |
| | 4 | 0.066 | 0.07 | 0.95 | 0.374 | -5.639 | 99 | 0.001 |
| | 5 | 21.141 | 0.737 | 21.461 | 0.828 | -1.056 | 99 | 0.319 |
| Different | 1 | 90.822 | 17.499 | 93.123 | 18.328 | -0.322 | 99 | 0.755 |
| | 2 | 18.59 | 4.12 | 18.567 | 3.407 | 0.001 | 99 | 0.998 |
| | 3 | 0.784 | 0.675 | 5.356 | 9.818 | -1.535 | 99 | 0.16 |
| | 4 | 29.276 | 18.191 | 23.922 | 15.852 | 0.653 | 99 | 0.53 |
| | 5 | 20.253 | 2.311 | 20.654 | 2.547 | -0.458 | 99 | 0.66 |

**Table 10.82 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

When all customers have the same priority, strategy 3 provides better results in terms of customers' dissatisfaction.

### 10.9.2.1 Conclusions

Table 10.83 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | | | |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | | + | + | + | |
| | 5 | | + | | + | |
| Different | 1 | | | + | | + |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | | + | | + | |
| | 5 | | + | + | + | + |

**Table 10.83 - Best strategy for each of the objective functions**

As can be seen in Table 10.83, when all customers have the same priority, objective functions 1, travel time and 2, number of vehicles needed, are best obtained by using strategy 1. Objective function 3, tour balance, is best obtained by using strategies 3 or 5.

Objective functions 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 2 or strategy 4.

When each customer has a different priority, objective function 1, travel time, is best obtained by using strategy 3 or strategy 5. Objective function 2 is best obtained by using strategy 1. Objective functions 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 2 or strategy 4.

### 10.9.3. Strategies Comparison – SPEA2 algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.84.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 103.74 | 7.027 | 116.27 | 13.976 | -3.304 | 99 | 0.009 |
| | 2 | 13.737 | 1.196 | 19.558 | 1.165 | -11.217 | 99 | 0.001 |
| | 3 | 0.602 | 0.18 | 1.127 | 0.397 | -3.318 | 99 | 0.007 |
| | 4 | 13.247 | 11.73 | 4.878 | 0.697 | 2.309 | 99 | 0.046 |
| | 5 | 19.879 | 0.357 | 19.185 | 2.011 | 1.22 | 99 | 0.252 |
| Different | 1 | 92.435 | 6.563 | 113.575 | 23.527 | -2.554 | 99 | 0.03 |
| | 2 | 12.683 | 1.493 | 19.345 | 3.028 | -6.162 | 99 | 0 |
| | 3 | 0.849 | 0.128 | 0.928 | 0.342 | -0.506 | 99 | 0.626 |
| | 4 | 0.283 | 0.097 | 0.134 | 0.114 | 2.206 | 99 | 0.056 |
| | 5 | 19.465 | 0.169 | 19.391 | 2.098 | 0.13 | 99 | 0.897 |

**Table 10.84 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As demonstrated by the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Furthermore, when all customers have the same priority, strategy 1 provides better solutions in terms of tour balance. When each customer has a different priority, strategy 2 provides better solutions in terms of customers' dissatisfaction.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.85.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 103.785 | 6.869 | 88.164 | 15.221 | 3.331 | 99 | 0.011 |
| | 2 | 13.693 | 1.238 | 19.007 | 2.227 | -6.711 | 99 | 0.001 |
| | 3 | 0.647 | 0.106 | 0.645 | 0.229 | 0.239 | 99 | 0.816 |
| | 4 | 13.416 | 11.74 | 41.159 | 14.539 | -4.03 | 99 | 0.005 |
| | 5 | 20.006 | 0.283 | 20.321 | 2.249 | -0.628 | 99 | 0.544 |
| Different | 1 | 92.281 | 6.469 | 92.004 | 18.398 | 0.04 | 99 | 0.968 |
| | 2 | 12.56 | 1.501 | 20.007 | 1.506 | -10.535 | 99 | 0.001 |
| | 3 | 0.908 | 0.174 | 0.561 | 0.204 | 5.621 | 99 | 0.001 |
| | 4 | 0.35 | 0.137 | 0.337 | 0.301 | -0.034 | 99 | 0.975 |
| | 5 | 19.601 | 0.083 | 19.816 | 2.693 | -0.453 | 99 | 0.66 |

**Table 10.85 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As reflected in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. If all customers have the same priority, strategy 1 also provides better solutions in terms of customers' dissatisfaction, and strategy 3 provides better results in terms of travel time. If each customer has a different priority, strategy 3 provides better solutions in terms of tour balance.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.86.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 116.331 | 14.085 | 88.132 | 15.201 | 7.344 | 99 | 0.002 |
| | 2 | 19.605 | 1.126 | 18.94 | 2.212 | 0.72 | 99 | 0.491 |
| | 3 | 1.171 | 0.49 | 0.765 | 0.353 | 4.001 | 99 | 0.002 |
| | 4 | 4.762 | 0.735 | 41.121 | 14.543 | -7.761 | 99 | 0.001 |
| | 5 | 19.262 | 1.875 | 20.333 | 2.28 | -1.731 | 99 | 0.115 |
| Different | 1 | 113.645 | 23.364 | 92.016 | 18.473 | 2.246 | 99 | 0.051 |
| | 2 | 19.382 | 2.979 | 20.08 | 1.606 | -0.53 | 99 | 0.608 |
| | 3 | 0.952 | 0.187 | 0.559 | 0.092 | 4.671 | 99 | 0.001 |
| | 4 | 0.126 | 0.088 | 0.361 | 0.338 | -1.33 | 99 | 0.217 |
| | 5 | 19.369 | 2.105 | 19.867 | 2.735 | -0.455 | 99 | 0.662 |

**Table 10.86 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same or different priorities**

As can be seen from the results, whether or not all customers have the same priority, strategy 3 provides better solutions in terms of tour balance. If all customers have the

same priority, strategy 2 also provides better solutions in terms of customers' dissatisfaction, and strategy 3 provides better solutions in terms of travel time.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.87.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 116.331 | 14.078 | 117.686 | 16.549 | -0.198 | 99 | 0.848 |
| | 2 | 19.51 | 1.128 | 19.82 | 1.263 | -0.606 | 99 | 0.56 |
| | 3 | 1.213 | 0.471 | 1.043 | 0.427 | 0.49 | 99 | 0.635 |
| | 4 | 4.817 | 0.834 | 4.212 | 0.885 | 1.714 | 99 | 0.119 |
| | 5 | 19.129 | 1.825 | 19.257 | 1.652 | -0.062 | 99 | 0.949 |
| Different | 1 | 113.49 | 23.469 | 117.783 | 16.433 | -0.464 | 99 | 0.653 |
| | 2 | 19.314 | 3.105 | 19.817 | 1.306 | -0.437 | 99 | 0.671 |
| | 3 | 0.959 | 0.379 | 1.215 | 0.436 | -0.703 | 99 | 0.499 |
| | 4 | 0.202 | 0.118 | 4.105 | 0.997 | -13.5 | 99 | 0.002 |
| | 5 | 19.347 | 2.066 | 19.21 | 1.828 | 0.185 | 99 | 0.855 |

**Table 10.87 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As seen from the results, when each customer has a different priority, strategy 2 provides better results in terms of customers' dissatisfaction.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.88.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 87.989 | 15.243 | 82.654 | 27.675 | 0.57 | 99 | 0.584 |
| | 2 | 19.008 | 2.084 | 18.293 | 4.092 | 0.668 | 99 | 0.523 |
| | 3 | 0.706 | 0.38 | 0.454 | 0.041 | 2.146 | 99 | 0.06 |
| | 4 | 41.134 | 14.722 | 35.596 | 23.297 | 0.574 | 99 | 0.581 |
| | 5 | 20.448 | 2.237 | 19.375 | 3.96 | 0.733 | 99 | 0.48 |
| Different | 1 | 92.176 | 18.467 | 98.894 | 4.221 | -1.045 | 99 | 0.321 |
| | 2 | 19.925 | 1.498 | 19.367 | 1.322 | 1.075 | 99 | 0.31 |
| | 3 | 0.508 | 0.199 | 0.659 | 0.178 | -2.753 | 99 | 0.022 |
| | 4 | 0.249 | 0.362 | 0.152 | 0.095 | 0.738 | 99 | 0.482 |
| | 5 | 19.944 | 2.683 | 20.656 | 0.394 | -0.858 | 99 | 0.413 |

**Table 10.88 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As seen from the results, when each customer has a different priority, strategy 3 provides better results in terms of tour balance.

#### 10.9.3.1 Conclusions

Table 10.89 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | | | + | | + |
| | 2 | + | | | | |
| | 3 | | | + | | + |
| | 4 | | + | | + | |
| | 5 | + | + | + | + | + |
| Different | 1 | + | + | + | | + |
| | 2 | + | | | | |
| | 3 | | | + | | |
| | 4 | + | + | + | | + |
| | 5 | + | + | + | + | |

**Table 10.89 - Best strategy for each of the objective functions**

As may be seen from Table 10.89, when all customers have the same priority, objective functions 1, travel time and 3, route balance, are best obtained by using strategy 3 or strategy 5. Objective function 2, number of vehicles needed, is best obtained by using strategy 1. Objective 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 2 or strategy 4.

When each customer has a different priority, objective functions 1, travel time, 3, route balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using strategy 3. Objective function 2, number of vehicles needed, is best obtained by using strategy 1.

### 10.9.4. Strategies Comparison – VE-ABC algorithm

In order to examine the effect of each of the strategies of the algorithm's results, several paired t-tests were used.

The first set of paired t-tests was used to compare the results obtained by using strategies 1 and 2. The results are summarized in Table 10.90.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 2 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 90.786 | 28.831 | 134.21 | 4.302 | -4.668 | 99 | 0.002 |
| | 2 | 11.784 | 3.588 | 21.736 | 1.066 | -8.506 | 99 | 0.002 |
| | 3 | 1.561 | 0.851 | 1.152 | 0.367 | 1.487 | 99 | 0.172 |
| | 4 | 193.456 | 99.895 | 8.659 | 3.533 | 5.858 | 99 | 0.001 |
| | 5 | 21.309 | 2.507 | 19.863 | 0.649 | 1.971 | 99 | 0.078 |
| Different | 1 | 93.092 | 31.195 | 134.188 | 7.129 | -4.176 | 99 | 0.003 |
| | 2 | 10.929 | 4.15 | 21.959 | 1.082 | -8.476 | 99 | 0.002 |
| | 3 | 6.441 | 13.183 | 0.749 | 0.369 | 1.37 | 99 | 0.204 |
| | 4 | 6.601 | 3.777 | 0.374 | 0.185 | 5.521 | 99 | 0 |
| | 5 | 21.479 | 1.066 | 19.8 | 0.648 | 5.031 | 99 | 0.002 |

**Table 10.90 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 2, when all customers have the same or different priorities**

As seen in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of travel time and number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 2 provides better solutions in terms of customers' dissatisfaction. When each customer has a different priority, strategy 2 provides better results in terms of arrival time of the last vehicle.

The second set of paired t-tests was used to compare the results obtained by using strategies 1 and 3. The results are summarized in Table 10.91.

| Customer's Priority | Objective Function | Strategy 1 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 90.69 | 28.898 | 105.09 | 10.594 | -1.637 | 99 | 0.137 |
| | 2 | 11.703 | 3.728 | 20.43 | 1.413 | -7.832 | 99 | 0 |
| | 3 | 1.519 | 0.853 | 2.2 | 5.521 | -0.358 | 99 | 0.729 |
| | 4 | 193.353 | 99.996 | 62.032 | 13.019 | 4.123 | 99 | 0.002 |
| | 5 | 21.323 | 2.514 | 22.604 | 1.891 | -1.529 | 99 | 0.163 |
| Different | 1 | 92.993 | 31.113 | 97.512 | 5.943 | -0.434 | 99 | 0.675 |
| | 2 | 10.977 | 4.13 | 19.218 | 2.418 | -4.942 | 99 | 0 |
| | 3 | 6.396 | 13.217 | 0.735 | 0.24 | 1.356 | 99 | 0.209 |
| | 4 | 6.521 | 3.736 | 0.56 | 0.334 | 5.193 | 99 | 0.003 |
| | 5 | 21.518 | 1.037 | 21.169 | 0.502 | 0.963 | 99 | 0.36 |

**Table 10.91 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 1 and 3, when all customers have the same or different priorities**

As reflected in the results, whether or not all customers have the same priority, strategy 1 provides better solutions in terms of the number of vehicles needed. Similarly, whether all customers have the same priority or not, strategy 3 provides better solutions in terms of customers' dissatisfaction.

The third set of paired t-tests was used to compare the results obtained by using strategies 2 and 3. The results are summarized in Table 10.92.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 3 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 134.2 | 4.209 | 105.135 | 10.68 | 7.455 | 99 | 0.001 |
| | 2 | 21.899 | 0.952 | 20.414 | 1.481 | 2.328 | 99 | 0.045 |
| | 3 | 1.036 | 0.428 | 2.204 | 5.514 | -0.643 | 99 | 0.536 |
| | 4 | 8.613 | 3.575 | 62.056 | 13.183 | -13.19 | 99 | 0.002 |
| | 5 | 19.746 | 0.623 | 22.493 | 1.885 | -4.669 | 99 | 0.001 |
| Different | 1 | 134.123 | 7.216 | 97.671 | 5.969 | 11.506 | 99 | 0 |
| | 2 | 21.861 | 1.194 | 19.186 | 2.424 | 3.2 | 99 | 0.012 |
| | 3 | 0.633 | 0.347 | 0.719 | 0.316 | -0.734 | 99 | 0.481 |
| | 4 | 0.365 | 0.201 | 0.556 | 0.34 | -1.783 | 99 | 0.108 |
| | 5 | 19.783 | 0.489 | 21.186 | 0.529 | -5.301 | 99 | 0 |

**Table 10.92 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 3, when all customers have the same priority**

As shown in the results, whether or not all customers have the same priority, strategy 2 provides better solutions in terms of the arrival time of the last vehicle. Similarly, whether all customers have the same priority or not, strategy 3 provides better solutions in terms of travel time and number of vehicles needed. When all customers have the same priority, strategy 2 provides better results in terms of the number customers' dissatisfaction.

The fourth set of paired t-tests was used to compare the results obtained by using strategies 2 and 4. The results are summarized in Table 10.93.

| Customer's Priority | Objective Function | Strategy 2 | | Strategy 4 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 134.278 | 4.138 | 136.443 | 5.309 | -1.105 | 99 | 0.299 |
| | 2 | 21.733 | 1.006 | 22.839 | 1.646 | -1.677 | 99 | 0.127 |
| | 3 | 1.135 | 0.447 | 0.858 | 0.325 | 1.635 | 99 | 0.137 |
| | 4 | 8.655 | 3.638 | 10.959 | 9.246 | -1.09 | 99 | 0.305 |
| | 5 | 19.846 | 0.572 | 20.161 | 0.606 | -1.202 | 99 | 0.261 |
| Different | 1 | 134.191 | 7.182 | 137.535 | 7.467 | -1.027 | 99 | 0.33 |
| | 2 | 21.982 | 1.134 | 21.658 | 1.273 | 0.391 | 99 | 0.707 |
| | 3 | 0.655 | 0.271 | 0.775 | 0.448 | -2.11 | 99 | 0.065 |
| | 4 | 0.22 | 0.255 | 0.202 | 0.09 | 0.63 | 99 | 0.545 |
| | 5 | 19.734 | 0.604 | 20.041 | 0.892 | -1.098 | 99 | 0.299 |

**Table 10.93 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 2 and 4, when all customers have the same or different priorities**

As reflected in the results, no significant differences were found between the two strategies.

The fifth set of paired t-tests was used to compare the results obtained by using strategies 3 and 5. The results are summarized in Table 10.94.

| Customer's Priority | Objective Function | Strategy 3 | | Strategy 5 | | t | df | Sig. |
|---|---|---|---|---|---|---|---|---|
| | | M | SD | M | SD | | | |
| The Same | 1 | 105.076 | 10.62 | 96.584 | 17.507 | 1.394 | 99 | 0.198 |
| | 2 | 20.567 | 1.493 | 19.226 | 2.705 | 1.217 | 99 | 0.257 |
| | 3 | 2.201 | 5.51 | 2.907 | 7.796 | -0.244 | 99 | 0.815 |
| | 4 | 62.051 | 13.2 | 46.852 | 23.676 | 2.279 | 99 | 0.048 |
| | 5 | 22.505 | 1.92 | 20.881 | 2.332 | 2.14 | 99 | 0.063 |
| Different | 1 | 97.505 | 5.881 | 100.742 | 13.307 | -0.871 | 99 | 0.405 |
| | 2 | 19.113 | 2.398 | 18.718 | 2.386 | 0.63 | 99 | 0.543 |
| | 3 | 0.703 | 0.227 | 0.785 | 0.385 | -0.337 | 99 | 0.743 |
| | 4 | 0.406 | 0.28 | 0.463 | 0.435 | -0.14 | 99 | 0.889 |
| | 5 | 21.109 | 0.395 | 20.831 | 0.755 | 0.693 | 99 | 0.506 |

**Table 10.94 – Paired T-Test results for comparison of the results of the different objectives functions when using strategies 3 and 5, when all customers have the same or different priorities**

As may be seen from the results, when each customer has a different priority, strategy 5 provides better results in terms of customers' dissatisfaction.

### 10.9.4.1 Conclusions

Table 10.95 describes which of the five strategies used provides the best value for each of the objective functions, when all customers have the same priority and when each customer has a different priority.

| Customer's Priority | Objective Function | Strategy | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| The Same | 1 | + | | + | | + |
| | 2 | + | | | | |
| | 3 | | + | + | + | + |
| | 4 | | + | | + | |
| | 5 | + | + | + | + | |
| Different | 1 | + | | + | | + |
| | 2 | + | | | | |
| | 3 | + | + | + | + | + |
| | 4 | | + | + | + | + |
| | 5 | | + | | + | |

**Table 10.95 - Best strategy for each of the objective functions**

As seen in Table 10.95, whether all customers have the same priority or not, objective functions 1, travel time, and 2, number of vehicles needed, are best obtained by using strategy 1. Objective functions 3, tour balance, 4, customers' dissatisfaction, and 5, arrival time of the last vehicle, are best obtained by using either strategy 2 or strategy 4.

## 10.9.5. Algorithms Comparison

Since in the real-world information on travel time and customers' demands are not known in advance, strategy 3 has to be used. Table 10.96 compares the results obtained for each of the five objectives, using paired t-tests, functions by each one of the three algorithms when applying the 3rd strategy. For each objective function, the best value obtained is highlighted in red.

| Customer's Priority | Objective Function | Algorithm | | |
|---|---|---|---|---|
| | | Imp. VEGA | SPEA2 | VE-ABC |
| The Same | 1 | 92.834 | 82.654 | 96.584 |
| | 2 | 17.021 | 18.293 | 19.226 |
| | 3 | 0.929 | 0.454 | 2.907 |
| | 4 | 0.95 | 35.596 | 46.852 |
| | 5 | 21.461 | 19.375 | 20.881 |
| Different | 1 | 93.123 | 98.894 | 100.742 |
| | 2 | 18.567 | 19.367 | 18.718 |
| | 3 | 5.356 | 0.659 | 0.785 |
| | 4 | 23.922 | 0.152 | 0.463 |
| | 5 | 20.654 | 20.656 | 20.831 |

**Table 10.96 - Comparison of the 5th strategy used in all three algorithms**

As may be seen, whether all customers have the same priority or not, objective 2, number of vehicles needed is best obtained by using the improved VEGA algorithm. Objective 3, tour balance is best obtained by using the SPEA2 algorithm. When all customers have the same priority, objective 1, travel time, and objective 5, arrival time of last vehicle, are best obtained by using the SPEA2 algorithm. When each customer has a different priority, objective 1, travel time, and objective 5, arrival time of last vehicle, are best obtained by using the improved VEGA algorithm.

## 10.9.6. Conclusions

Since in the real-world, information on travel time and customers' demands are not known in advance, strategy 5 has to be used. From the results obtained, one should use

the SPEA2 algorithm, which returns the best solutions for all objective functions. When looking at the different strategies, there is no dominant strategy which provides the best solution in the majority of scenarios.

## 10.10. Summary

The goal of this chapter is to compare the results of the three algorithms using a case study. The case study is based on two networks (urban and interurban) based on real-world transportation network, including the locations of the depot and the customers and information about travel time between the different customers. The case study is performed using simulation.

In order to perform the case study, simulation was used. The simulation is based on two processes running in parallel, the algorithm process and the simulation process, which exchange information between each other.

The simulation process simulates an entire work today. It does so by handling each of the vehicles, collecting data about travel times and new customers'demands.

In the case study, 5 different strategies of using the evolutionary algorithms were tested, where the fifth strategy represents a situation in which both travel times and customers' demands are unknown (desired real-world situation).

The VEGA algorithm is a well known multi-objective algorithm, but since its development more sophisticated and accurate multi-objective algorithms, such as the SPAE2, towere introduced. Also, the VE-ABC algorithm is a new, slower algorithm, and is therefore able to make far fewer iterations in a given time period, compared to the two other algorithms. It was therefore expected that the best results would be obtained when using the SPEA2 algorithm. However, the case study shows that this is not the case. In an urban network when using a linear dissatisfaction function, it was found that the VEGA algorithms performs best when all customers have the same priority. When each customer has a different priority, using the same network and the same dissatisfaction function, best results were obtained using either the SPEA2 or the VE-ABC algorithms.

In an urban network and a dissatisfaction function that represents customers who don't like that the supplier is either early or late, and in an interurban network with both types of dissatisfaction network, the results of all algorithms were the same.

This shows that the VEGA algorithm when used can provide solutions equal in quality to the the solutions obtained from more sophisticated, more recent algorithms. This is important, since the VEGA algorithm has an advantage in the simplicity of its implementation, running speed compared to other algorithms (and as a result, more iterations in a given time period), and its capacity for modifications.

# 11. Summary

## 11.1. Summary and Conclusions

The Vehicle-Routing Problem (VRP) is a common name for problems involving the construction of a set of routes for a fleet of vehicles. The vehicles start their routes at a depot, call at customers to whom they deliver goods, and return to the depot. The objective function for the vehicle-routing problem is to minimize delivery cost by finding optimal routes, which are usually the shortest delivery routes.

The basic VRP consists of designing a set of delivery or collection routes, such that (1) each route starts and ends at the depot, (2) each customer is called at exactly once and by only one vehicle, (3) the total demand on each route does not exceed the capacity of a single vehicle, and (4) the total routing distance is minimized. It is common to address the basic VRP as the Capacitated Vehicle-Routing Problem (CVRP).

VRP has been solved optimally using Branch-and-Bound algorithms, Set-Covering and Column Generation algorithms, Branch-and-Cut algorithms, Dynamic algorithms and other exact algorithms.

Since VRP is an NP-Hard problem, many heuristics have been developed for solving it. The classic algorithms include, among others, the Savings algorithms, Swap algorithm and the Fisher and Jaikumar algorithm. Meta-heuristics algorithms, such as Simulated Annealing, Tabu Search, Genetic Algorithms, Ant Systems Algorithms and Neural Networks are also used in solving VRPs.

As research developed, extensions to the basic VRP were introduced. The goal was to develop more realistic models, to adapt to the larger number of constraints of the real world. Such extensions include the Split Delivery Vehicle Routing Problems, Vehicle Routing Problems with Time Windows, Multi-Depot Vehicle Routing Problems, Time Dependent Vehicle Routing Problems, Stochastic Vehicle Routing Problems, Mutli-Objective Vehicle Routing problems and Real-Time Vehicle Routing Problems.

VRPs are often used to model real cases. However, they are often set up with the single objective of minimizing the cost of the solution, despite the fact that the majority of the problems encountered in industry, particularly in logistics, are multi-objective in nature. In real-life, for instance, there may be several costs associated with a single tour.

Moreover, the objectives may not always be limited to cost. In fact, numerous other aspects, such as balancing workloads (time, distance ...), can be considered simply by adding new objectives (Jozefowiez et al., 2008).

Traditionally, vehicle routing plans are based on deterministic information about demands, vehicle locations and travel times on the roads. What is likely to distinguish most distribution problems today from equivalent problems in the past, is that information that is needed to come up with a set of good vehicle routes and schedules is dynamically revealed to the decision maker (Psaraftis, 1995). Until recently, the cost of obtaining real-time traffic information was deemed too high to compare with the benefits from the real time control over the vehicles. Furthermore, some of the information needed for the real time routing was impossible to get. The advancement of technology in communication systems, the geographic information system (GIS) and the intelligent transportation system (ITS) make is possible to operate vehicles using the real-time information about the travel times and the vehicles' locations (Ghiani et al., 2003).

While traditional VRPs have been thoroughly studied, limited research has, to date, been devoted to multi-objective real-time management of vehicles during the actual execution of the distribution schedule to respond to unforeseen events that often occur and may deteriorate the effectiveness of the predefined and static routing decisions. Furthermore, in cases when traveling time is a crucial factor, ignoring travel time fluctuations (due to various factors, such as peak hour traveling time, accidents, weather conditions, etc.) can result in route plans that can take the vehicles into congested urban traffic conditions. Considering time-dependent travel times as well as information regarding demands that arise in real time in solving VRPs can reduce the costs of ignoring the changing environment (Haghani & Jung, 2005).

The problem considered in this research is the Real-Time Multi-Objective VRP. The Real-Time Multi-Objective VRP is defined as a vehicle fleet that has to serve customers of fixed demands from a central depot. Customers must be assigned to vehicles, and the vehicles routed so that the a number of objectives are minimized/maximized (Malandraki & Daskin, 1992). The travel time between two customers or a customer and the depot depends on the distance between the points and the time of day, and it also has stochastic properties.

This research attempts to adjust the vehicles' routes at certain times in a planning period. This adjustment considers new information about the travel times, current location of vehicles, and new demand requests (that can be deleted after being served, or added since they arise after the initial service began) and more. This result in a dynamic change in the demand and traveling time information as time changes, which has to be taken into consideration in order to provide optimized real-time operation of vehicles.

According to the vast literature review, the following objectives were addressed: (1) **Minimizing the total traveling time** (e.g. (Malandraki & Daskin, 1992)) - Minimizing the total traveling time can reduce the cost of an organization for, among others, the following seasons: (a) the less time a driver spends driving the less chances there are for being involved in a car accident (b) maintenance has to be performed less often. (2) **Minimizing the number of vehicles** (e.g. (Corberan et al., 2002)) - Since in a real world, the fixed cost of using additional vehicles is much more than the routing operations cost, we can reduce the total cost by minimizing the number of vehicles in service. (3) **Maximizing customers' satisfaction** (e.g. (Sessomboon et al., 1998)) - Customers who are not satisfied with the level of service can switch to a different provider, which results in a reduction of manufacturing and delivery. (4) **Maximizing drivers' satisfaction** (e.g. (Lee & Ueng, 1998)) - In a similar manner, drivers who are not satisfied with their work schedule may feel frustrated, which may damage their work, which in turn may influence customers' satisfaction. (5) **Minimizing the arrival time of the last vehicle** – each vehicle, on its return back to the depot, can be assigned to a new route (meaning more routes with fewer vehicles). Minimizing the arrival time of the last vehicle arriving to the depot, ensures that all other vehicles are present at the depot before the arrival of the last vehicle, and can therefore be assigned to new routes.

The first stage in solving the real-time multi-objective vehicle routing problem was to formulate the problem as a mixed integer linear programming problem on a network. Several assumptions and limitations were considered, such as a system with dynamic conditions (real-time variation in travel times and real-time service requests); all demands have specified service times and service time intervals; soft time windows for service around the desired service times are considered, and more. Next the five objectives were mathematically formulated, as well as the various constraints.

Since VRP is a NP-Hard problem, it cannot be solved to optimality using conventual methods. It is therefore essential to develop an efficient heuristics algorithm for solving the problem. In order to do so, various methods and algorithms for solving dynamic vehicle routing problems as well as for solving multi-objective optimization problems were studied.

Finally, three evolutionary algorithms for solving the real-time multi-objective vehicle routing problem were described.

The first algorithm is an improved version of the vector evaluated genetic algorithm (VEGA). It is based on the concept that for a problem with *NumObj* objectives, *NumObj* sub-populations of size *PopSize*/*NumObj* each would be generated (assuming a total population size of *PopSize*). Each sub-population uses only one of the *NumObj* objective functions for fitness assignment. The proportionate selection operator is used to generate the mating pool. These sub-populations are then shuffled together to obtain a new population of size *PopSize*, on which the GA would apply the crossover and mutation operators in the usual way. In each generation, the set of not-dominated solutions is added to the optimal solutions set, from which non-dominated solutions are removed.

The second algorithm is an implementation of the SPEA2 algorithm. The distinctive feature of SPEA2 lies in the elitism-preserved operation. An external set (archive) is created for storing primarily non-dominated solutions. It is then combined with the current population to form the next archive that is then used to create offspring for the next generation. The size of the archive is fixed. It can be set to be equal to the population size. Therefore, there exist two special situations when filling solutions in the archive. If the number of non-dominated solutions is smaller than the archive size, other dominated solutions taken from the remainder part of the population are filled in. This selection is carried out according to a fitness value, specifically defined for SPEA. In other words, the individual fitness value defined for a solution *x*, is the total of the SPEA-defined strengths of solutions, which dominate *x*, plus a density value.

The second situation happens when the number of non-dominated solutions is over the archive size. In this case, a truncation operator is applied. For that operator, the solution which has the smallest distance to the other solutions will be removed from the set. If solutions have the same minimum distance, the second nearest distance will be considered, and so forth. This is called the *k-th nearest distance rule*.

The third evolutionary algorithm is a combination of the vector evaluated technique and artifical bee colony algorithm. In the ABC algorithm, the colony of artificial bees consists of three groups of bees: (1) employed bees - bees that are currently exploiting a food source; (2) onlookers - bees that are waiting in the hive for the employed bees to share information about the food sources; and (3) scouts - bees that are searching for new food sources in the neighborhood of the hive. The ABC algorithm starts by assigning each employed bee to a randomly generated solution. Next, in each iteration, each employed bee, using a neighborhood operator, finds a new food source near its assigned food source. The nectar amount of the new food source is then evaluated. If the amount of nectar in the new food source is higher than the amount of nectar in the old one, then the older source is replaced by the newer one. Next, the nectar information of the food sources is shared with the onlookers. The onlooker chooses a food source according to the probability proportional to the quality of that food source. Roulette wheel selection is the usual method. Therefore, good food sources, as opposed to bad ones, attract more onlooker bees. Subsequently, using a neighborhood operator, each onlooker finds a food source near its selected food source and calculates its nectar amount. Then, for each old food source, the best food source among all the food sources near the old one is determined. The employed bee associated with the old food source is assigned to the best food source and abandons the old one if the best food source is better than the old food source. A food source is also abandoned by an employed bee if the quality of the food source has not improved in the course of a predetermined and limited number of successive iterations. The employed bees then become scouts and randomly search for new food source. After a scout finds a new food source, it becomes an employed bee again. After each employed bee is assigned to a food source, another iteration of the ABC algorithm begins. The iterative process is repeated until a stopping condition is met.

Next, solutions representation was described. A candidate solution for an instance of the VRP must specify the number of vehicles required, the partition of the demands through all these vehicles; the delivery order for each route as well as waiting time at each customer. Let a node object define an object that has two properties, customer number and waiting time at customer. A solution to the multi-objective real-time VRPs can be encoded using an array of node objects, and based on the permutation representation. A solution contains several routes, each one of them composed by an ordered subset of the

customers. All demands belonging to the problem being solved must be present in one of the routes.

Methods, such as crossover and mutations, which are needed for diversity purposes, were also described. Crossover and mutation are the genetic operators used in the general GAs. In ABCs only neighborhood operators, which are equivalent to GA's mutation operators, are used. Solutions used in a specific problem have their own characteristics, and some particular crossover operators are needed.

A fitness function is a particular type of objective function that is used to summarize, as a single figure of merit, how close a given design solution is to achieving the set aims.

In the field of evolutionary algorithms, at each iteration, the idea is to delete the *n* worst design solutions, and to breed *n* new ones from the best design solutions. Each design solution, therefore, needs to be awarded a figure of merit, to indicate how close it came to meeting the overall specification, and this is generated by applying the fitness function to the test or simulation results obtained from that solution.

In some cases, fitness approximation may be appropriate, especially if (1) fitness computation time of a single solution is extremely high, (2) precise model for fitness computation is missing or (3) the fitness function is uncertain or noisy.

In all three algorithms presented, the fitnesses of all five objective functions have to be calculated. Due to the stochastic nature of travel time, in order to get an accurate value, or accurate fitness functions, simulation has to be used. Simulation is a time-consuming process.

It was shown that it is possible to increase the running time of the algorithm by using an "approximated" fitness function, without influencing the accuracy of the algorithm. A fast algorithm is necessary when coping with real-time problems, which is the final goal of this study.

Usually, when solving a multi-objective optimization problem, the result is a set of non-dominated solutions, from which the decision maker has to choose his preferred alternative. Since the final goal is to create an automated algorithm for solving a real-time multi-objective vehicle routing problem, the TOPSIS method, a mechanism for choosing a preferred solution from a set of non-dominated solutions, has been implemented. It was shown that there is no difference in the quality of the results obtained using the "approximated" or "accurate" methods; however, this does not mean that the same results

exist in both sets, and therefore, it is not guaranteed that the TOPSIS method selects similar results from both sets. It was shown, by means of correlation testing and paired-samples t-tests, that the solutions selected by the TOPSIS methods are similar regardless of the method used for calculating the fitness functions.

Since travel time is more likely to be log-normally distributed, a second set of tests was done, using Solomon's instances. Using 500 generations and a population of 200 chromosomes, the result of the improved VEGA algorithm showed that for problems with a large number of chromosomes (50 and 100 customers) using $w$=100 results with a better solution than when using $w$=1, while for problems with a small number of customers (25 and 50), no significant difference was found. Since it is known that the number of generations used by a genetic algorithm may affect its results, and since in real-time applications, the number of generations is bounded by the time given to the algorithm to come up with a solution, the algorithm was tested againThis time the stopping condition was 30 minutes of running time, instead of the 500 generations. The result showed that in all cases, the result obtained by the algorithm when $w$=1 are better than the results obtained when $w$=100. Moreover, when $w$=1, the algorithm converges to the best solution much faster than when $w$=100.

Another parameter of the algorithm that has to be addressed is the waiting time parameter. Waiting time is the time a vehicle waits after it has finished serving a customer before it starts driving to the next customer. Service time is determined by the algorithm, and can be any value in a pre-determined range. Therefore, the question asked is, What is the best range from which the algorithm should select the waiting time so that the algorithm will converge to the optimal solution in respect to all objective functions, and do it as fast as possible (fewer iterations)?.

In order to find the best waiting time range, a set of tests was done, using Solomon's C101, R101 and RC101 instances for 25, 50 and 100 customers, each solved 10 times. Based on the results of the test instances, for each instance, in order to predict the value of each objective function as a function of the waiting time range, linear regression was used. The results of the linear regression showed that in more than half of the cases, the best results were obtained when the waiting time range was between 0 and 5 minutes. However, half of the functions found by the linear regression (23 out of 45), had a value of $R^2$ lower than 0.75. This means that the value of half of the objective functions

calculated based upon the functions found by the regression, are probably not close to the true value expected. Because of that, averages comparison was done and used as well.

The results of the averages comparison were similar to the results obtained by using the functions found by the linear regression. More than half of the objectives (25 out of 45) are best obtained when the waiting time is in the range of 0 to 5 minutes.

Based on the results obtained by using linear regression and the results obtained by using averages comparison, optimal use of waiting time is within the range of 0 to 5 minutes.

In a traditional VRPTW, a feasible solution must satisfy all time windows. When a customer is served within a specified time window, the supplier's service level is satisfactory or equal to 1; otherwise, it is unsatisfactory, or equal to 0. Time windows may sometimes be violated for economic and operational reasons. However, there are certain limits to this violation (earliness or lateness) that a customer can tolerate. Obviously, the earliness and lateness are closely related with the quality of service of the supplier, and therefore, the service level cannot be described by only two states (0 or 1).

In order to get a feeling for how the service level, also known as customer satisfaction, changes as a function of limits on such violations (earliness or lateness), 38 customers (people), using questionnaires, were asked for their general satisfaction level when a supplier or other service provider arrives 30 minutes to four hours, in 30 minutes intervals, earlier than expected. Similarly, they were asked for their general satisfaction level when a supplier or other service provider arrives later than expected.

Each customer, based upon the results of his questionnaire, was assigned a satisfaction function. From these functions, it seems that most customers are sensitive to suppliers arriving either early or late ( their satisfaction level drops dramatically when the supplier arrives earlier/later than expected).

Finally, the results of the three algorithms were compared using a case study. The case study is based on two networks (urban and interurban) based on a real-world transportation network; this includes the locations of the depot and the customers and information about travel time between the different customers. In order to perform the case study, simulation was used. The simulation is based on two processes running in parallel, the algorithm process and the simulation process, which exchange information between each other.

The simulation process simulates an entire work today. It does so by handling each of the vehicles, collecting data about travel times and new customers'demands.

In the case study, five different strategies for using the evolutionary algorithms were tested, where the fifth strategy represents a situation in which both travel times and customers' demands are unknown (desired real-world situation).

The VEGA algorithm is a well-known, multi-objective algorithm. However, since its development, more sophisticated and accurate multi-objective algorithms, such as the SPAE2, algorithms were introduced. Furthermore, the VE-ABC algorithm is a new algorithm, which is slower, and therefore, is able make far fewer iterations in a given time period, compared to the two other algorithms. It was therefore expected that the best results would be obtained when using the SPEA2 algorithm. However, the case study shows that this is not true. In an urban network, when using a linear dissatisfaction function, it was found that the VEGA algorithm performs best when all customers have the same priority. When each customer has a different priority, using the same network and the same dissatisfaction function, best results were obtained using either the SPEA2 or the VE-ABC algorithms.

In an urban network and a dissatisfaction function that represents customers who don't like when a supplier is either early or late, and in an interurban network with both types of dissatisfaction network, the results of all algorithms were the same.

This shows that the VEGA algorithm when used can provide solutions equal in quality to the solutions obtained from more sophisticated and more recent algorithms. This is important, since the VEGA algorithm has an advantage in its simplicity of implementation and running speed as compared with other algorithms (and as a result, the number of iterations in a given time period), and the capacity for modifications.

## 11.2. Recommendations for Future Research

Although the proposed solution algorithm works well for the real-time multi-objective vehicle routing problem, there are several fruitful avenues for future research- These are described below.

The proposed EAs provide good results for the problems on the generated test network. In this research, among other tests, the Solomon's test instances with 100 customers were used. When the number of customers is increased, calculations of various aspects of the

algorithms, such as finding the set of nondominated solutions, become more complex and more time-consuming. This generates a large search space for the EAs and increases the calculation time significantly.

The dynamic nature of the problem considered in this research required that the algorithms described would converge toward the optimal solution as fast as possible, since in many case the algorithm has less than 30 minutes to come up with a solution. There are several ways that can be used and studied that can reduce calculation time.

**1. Improving Seeding Methods**

Usually the initial population is randomly generated. However, if an initial population with high-quality solutions that are known *a priori* in some ways can be used, the algorithms may provide better solutions more quickly than if the population is randomly generated.

By adopting the construction heuristics that are traditionally used in routing problems, the EAs may start in the regions of the solution space that may be good candidates for locating the optimum. This approach has already been used throughout this research – the initial population was constructed using the Savings algorithm. However, the Savings algorithm is not intended for solving VRP with time windows, and it is certainly not intended for solving multi-objective problems.

**2. Using Parallel Algorithm**

Some research is devoted to developing EAs that can be implemented in parallel machines. There are two ways to implement parallel EAs. The first approach is to evaluate the fitness value of each solution of the population by parallel processors. If there is the same number of processors, $N$, as the number of solutions, the calculation time for the fitness value can be reduced to $1/N$ time compared to the sequential algorithm. The second way of using a parallel algorithm is to allocate a sub-population of solutions to parallel processors which proceed independently for a certain number of generations.

It seems that using the second approach in implementation of parallel EAs is more useful. To implement a parallel EA, a way to redistribute information among the subpopulations has to be studied.

### 3. Comparisons to Other Real-World Data

In the case study, presented in chapter 10, two transportation networks were used. The first is based on Israel's the greater Tel-Aviv metropolitan area urban network and the second on Israel's interurban network. It is interesting to compare the result obtained with the result of other networks from various places around the world. These networks can be constructed using the same method presented in chapter 10, or using other methods for collecting information on the network.

### 4. Other Fitness Functions

The algorithms proposed, and the problems described, can be used with other fitness functions. These functions can represent the existing objectives or can be introduced as part of new objectives. It would be interesting to know if by using other fitness functions, the overall conclusions would be changed.

### 5. Modification of the Vector Evaluated Aproach

Both the improved VEGA algorithm and the VE-ABC algorithm are based on the concept that for a problem with *NumObj* objectives, *NumObj* subpopulations of size *PopSize*/*NumObj* each would be generated (assuming a total population size of *PopSize*). How would the two algorithms perform and what would be the results if the subpopulations are not equal in size? This may enable prioritizing the different objectives considered by the algorithm.

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# תקציר

בעיית ניתוב הרכבים (vehicle routing problem – VRP) הינה קבוצת בעיית העוסקות בבניית מסלולי נסיעה עבור צי של כלי רכב. כל כלי רכב מתחיל את המסלול שלו במחסן, עובר דרך לקוח אחד או יותר, במטרה לספק או לאסוף סחורה, וחוזר חזרה למחסן. המטרה בבעיית ניתוב הרכבים היא מיזעור עלויות ע"י מציאת מסלולים אופטימאליים, לרוב המסלולים הקצרים ביותר.

בעיית ניתוב הרכבים הבסיסית, עוסקת, כאמור, בבניית מסלולי הפצה, כאשר קיימים מספר אילוצים שאותם יש לקיים. (1) כל מסלול מתחיל ומסתיים במחסן, (2) לכל לקוח חייב להגיע כלי רכב אחד בלבד ו-(3) סך כל הדרישה לאספקה של כל הלקוחות השייכים לאותו המסלול אינה יכולה להיות גבוהה מקיבולת של משאית בודדת. המטרה היא כאמור, מציאת אוסף המסלולים, אשר עומדים באילוצים, בעלי מרחק כולל מינימאלי. בעייה זו נהוג לכנות בעיית ניתוב הרכבים עם אילוץ קיבולת (capacitated vehicle routing problem – CVRP).

קיימים מספר אלגוריתמים לקבלת פתרון אופטימאלי עבור בעיית ניתוב הרכבים הבסיסית. אלגוריתמים אלו מבוססים על שיטות כגון branch-and-bound, set-covering ו- column generation, branch-and-cut, תיכנות דינאמי ועוד.

אולם, מכוון שבעיית ניתוב הרכבים היא בעייה NP-קשה, מרב המאמץ מושקע בפיתוח אלגוריתמים היוריסטיים לפתרונה. בין האלגוריתמים ההיוריסטיים הקלאסיים לפתרון בעיית ניתוב הרכבים ניתן למנות את אלגוריתם ה-Savings, אלגוריתם ה-Swap, וכן האלגוריתם של Fisher ושל Jaikumar. כמו כן, פותחו אלגוריתמים מטה-היוריסטיים, המשתמשים בשיטות כגון Simulated Annealing, Tabu Search, אלגוריתמים גנטיים, Ant systems, רשתות עיצביות ועוד.

במקביל לפיתוחם של האלגוריתמים לפתרון בעיית ניתוב הרכבים עם אילוץ קיבולת, הוצגו הרחבות לבעיית ניתוב הרכבים, במטרה להציג מודלים קרובים יותר למציאות, הכוללים אילוצים רבים יותר, הקיימים בעולם האמיתי של הפצה ושינוע. בין ההרחבות ניתן למצוא את ההרחבות הבאות: (1) **בעיית ניתוב רכבים עם מישלוחים מפוצלים** ( split delivery vehicle routing problem) – בעיית ניתוב רכבים שבה האילוץ המחייב שלכל לקוח יגיע כלי רכב יחיד אינו קיים. משתמש מכך, שניתן לפצל משלוח בודד ללקוח, ליותר מכלי רכב אחד. (2) **בעיית ניתוב רכבים עם חלונות זמן** (vehicle routing problem with time windows) – בעיית ניתוב רכבים שבה הלקוח מקצה חלון זמן שבו הספק צריך להגיע ללקוח. חלונות הזמן שמוקצים ע"י הלקוחות מהווים אילוצים נוספים שיש לעמוד בהם. (3) **בעיית ניתוב רכבים עם מספר מחסנים** (multi-depot vehicle routing problem) - בבעיית ניתוב הרכבים הבסיסית, קיים מחסן ממנו יוצאים הרכבים, ואליו הם שבים לאחר שביקרו את הלקוחות. בבעיית ניתוב הרכבים עם מספר מחסנים,

קיימים מספר מחסנים מהם יוצאים הרכבים, ואליהם הם צריכים לשוב. כאשר, אין חובה שכלי הרכב ישוב אל המחסן ממנו הוא יצא. (4) **בעיית ניתוב רכבים תלויית זמן** ( time dependent vehicle routing problem) – בבעיית ניתוב רכבים זו, אנו מחפשים מסלולים בעלי זמן קצר ביותר, כאשר זמני הנסיעה בכבישים השונים משתנים לאורך היום. (5) **בעיית ניתוב רכבים סטוכסטית** (stochastic vehicle routing problem) – בבעיית ניתוב רכבים זו, חלק מהנתונים, כגון דרישות הלקוחות, זמני הנסיעה וכו', הינם סטוכסטיים. (6) **בעיית ניתוב רכבים מרובת פונקציות מטרה** (multi-objective vehicle routing problem) – בניגוד לבעיות ניתוב הרכבים הקודמות, אשר בהן הוגדרה פונקציית מטרה אחת, בבעייה זו אנו מעוניינים להביא לאופטימום מספר פונקציות מטרה בו זמנית, כגון, מסלולים קצרים ביותר, מסלולים בעלי מרחק זהה ככלו האפשר ועוד. (7) **בעיית ניתוב רכבים זמן אמת** (real-time vehicle routing problem) – בבעיית ניתוב רכבים זו, נתונים שונים, כגון דרישות הלקוחות, מתקבלים במהלך יום העבודה. יש צורך להתחשב בנתונים אלו, ולתקן את מסלולי כלי הרכב בהתאם, על מנת לתת מענה ראוי ללקוחות.

בבעיית ניתוב הרכבים משתמשים לעיתים על מנת לפתור בעיות אמיתיות מתחום הלוגיסטיקה בכלל ותובלה ושינוע בפרט. לרוב, במקרים אלו, מוגדרת פונקציית מטרה אחת – מינימום עלויות, זאת למרות שרוב הבעיות הלוגיסטיות, ובפרט בעיות בתחום התובלה והשינוע, בהן אנו נתקלים בעולם האמיתי, הן בעיות מרובות מטרות מטיבען. בעולם האמיתי, למשל, ניתן לשייך מספר עלויות למסלול בודד (לדוגמא, אורך המסלול, זמן הנסיע בו וכו'). יתר על כן, המטרות השונות אינן בהכרח מוגבלות לעלויות. לדוגמא, בתיכנון מסלולי נסיעה ישנן מטרות שונות שלעיתים יש להתחשב בהן, כגון איזון המסלולים השונים (מסלולים בעלי מרחקים שווים, או זמני נסיעה שווים) (Jozefowiez et al., 2008).

לרוב, בעיות ניתוב הרכבים מבוססות על נתונים דטרמיניסטיים לגבי דרישות הלקוחות, מיקומם, זמני הנסיעה אליהם ועוד. ההבדל, כפי שמשתמע ממחקרים עדכניים, בין בעיות ניתוב הרכבים שאיתן אנו מתמודדים היום לבעיות ניתוב רכבים עתידיות, הוא זמינות המידע, אשר מתקבל, אצל מקבל ההחלטות, באופן דינאמי (Psaraftis, 1995). עד לאחרונה, עלות השגת מידע על תנועה בזמן האמת הייתה גבוהה למדי בהשוואה ליתרונות המושגים מהשליטה בזמן אמת על כלי הרכב. יתר על כן, חלק מהמידע הדרוש לניתוב בזמן האמת לא ניתן היה להשיג. בעיקבות התקדמויות טכנולוגיות בתחומי מערכות תקשורת, מערכות מידע גיאוגרפית (GIS) ומערכות תחבורה אינטליגנטית (ITS), ניתן היום להפעיל כלי רכב, בהתייחסות למידע המגיע מהמשתמש בזמן אמת, כגון, מידע על זמני הנסיעה ומיקומים של כלי הרכב (Ghiani et al., 2003).

בעוד שבעיות ניתוב רכבים מסורתיות נחקרו ביסודיות, נכון להיום, מחקר מועט הוקדש לפתרון בעיות ניתוב הרכבים בזמן אמת מרובות מטרות, בדגש לתגובה לאירועים בלתי צפויים המתרחשים לעתים קרובות ואשר עלולים לפגוע ביעילותם של פתרונות מתוכננים מראש המבוססים על נתונים סטטיים. יתר על כן, במקרים שבהם זמן הנסיע הינו גורם מכריע,

התעלמות מתנודות בזמן נסיעה (בשל גורמים שונים, כגון זמני הנסיעה בשעות השיא, תאונות, תנאי מזג אוויר, וכו ') יכולה לגרום לכך שהמסלולים המתוכננים יובילו את כלי הרכב לאזורים עירונים עם תנועה צפופה. התחשבות בשינוי התנועה לאורך היום וכן במידע על דרישות הלקוחות המגיע לאורך היום בזמן אמת, יאפשר לבנות מסלולי נסיעה אשר מפחיתים את העלויות הנובעות משינויים אלו (Haghani & Jung, 2005).

בעיית ניתוב הרכבים המטופלת במחקר זה הינה בעיית ניתוב הרכבים זמן-אמת מרובת מטרות (real-time multi-objective vehicle routing problem). בבעיית ניתות הרכבים זמן-אמת מרובת מטרות צי של כלי רכב נדרש לספק סחורה, ממחסן מרכזי, לקבוצה קבועה של לקוחות. לכל כלי רכב מוקצה מסלול נסיעה, העובר דרך לקוח אחד או יותר, כאשר אין שני כלי רכב המגיעים לאותו הלקוח, וזאת במטרה להביא למינימום / מקסימום מספר פונקציות מטרה (Malandraki & Daskin, 1992). זמני הנסיעה, אשר הינם סטוכסטיים מטיבעם, בין המחסן ללקוחות או בין שני לקוחות תלויים הן במרחק בניהם והן בשעת היום.

במחקר זה נעשה נסיון לכוון מחדש את כלי הרכב, ע"י שינוי המסלולים בנקודות זמן נתונות (זמני היציאה מהמחסן וכן זמני היציאה מהלקוח ללקוח הבא). שינוי מסלולי הנסיעה מתבסס על מידע חדש שמגיע למערכת לגבי זמני הנסיעה ברשת הכבישים, מיקומם של כלי הרכב, ודרישות חדשות של הלקוחות (דרישות ניתן למחוק מהמערכת לאחר שסופקו, או להוסף למערכת דרישות שהתקבלו לאחר בניית המסלולים) ועוד. יש צורך להתחשב בשינויים דינאמיים אלו, בדרישות הלקוחות וכן בזמני הנסיעה, על מנת לספק מסלולים אופטימאליים בזמן אמת.

לצורך המחקר, ובהתבסס על סקירת ספרות נרחבת שנערכה, נבחרו מספר פונקציות מטרה. (1) **זמן נסיעה כולל מינימאלי** (לדוגמא (Malandraki & Daskin, 1992)) – שימוש במסלולים בעלי זמן נסיעה כולל מינימאלי יכול לצמצם עלויות שונות של האירגון ממספר סיבות, כגון, (א) ככול שנהג נמצא זמן מועט יותר על הכביש, הסיכוי שלו להיות מעורב בתאונת דרכים הוא נמוך יותר. (ב) כלי רכב שנמצא זמן מועט יותר על הכביש דורש טיפולי תחזוקה ומעקב בתדירות נמוכה יותר. (2) **הקטנת מספר כלי הרכב הנדרש לצורכי ההפצה** (לדוגמא (Corberan et al., 2002)) – היות ובמציאות העלויות הקבועות הנובעות מרכישה והוספה של כלי רכב נוסף לצי כלי הרכב של האירגון גבוהה בהרבה מהעלויות התיפעוליות של אותו כלי הרכב, אנו יכולים להקטין את העלויות הכוללות של האירגון ע"י הקטנה של צי כלי הרכב שלו. (3) **הגדלת שביעות רצון הלקוחות** (לדוגמא (Sessomboon et al., 1998)) – לקוחות אשר אינם שביעי רצון מרמת השרות שהם מקבלים מהספק יכולים להחליף אותו. אמנם, לספק יהיו פחות הוצאות, אך במקביל יקטנו גם הכנסותיו (פחות מכירות), ולכן אנו מעוניינים להגדיל את שביעות רצון הלקוחות שלנו. (4) **הגדלת שביעות הרצון של הנהגים (או קבלת מסלולים זהים ככל האפשר)** (לדוגמא (Lee & Ueng, 1998)) – בעיית ניתוב הרכבים עוסקת, כאמור, בבניית מסלולי נסיעה אופטימליים עבור צי של כלי רכב. כתוצאה מפונקציית המטרה והאילוצים השונים, המסלולים המתקבלים אינם בהכרח זהים מבחינת אורכם, זמני הנסיעה בהם וכו'. נהגים אשר, לדוגמא, מקבלים מסלולים

ארוכים, יכולים להרגיש חוסר שביעות רצון בהשוואה לנהגים אחרים שקיבלו מסלולים קצרים יותר. נהגים אלו, יכולים להרגיש ממורמרים, באופן כזה שעלול לפגוע באופן עבודתם, וכתוצאה אף בשביעות רצון הלקוחות. (5) **הקדמת זמן החזרה של כלי הרכב האחרון למחסן** – כל כלי רכב, בעת שהוא חוזר למחסן, יכול לצאת לנסיעה במסלול הפצה חדש (יותר מסלולים עם פחות כלי רכב). הקדמת זמן החזרה של כלי הרכב האחרון למחסן מבטיחה לנו שכל כלי הרכב יגיעו למחסן לפני זמן זה, וכך יגדילו את זמינותם לנסיעות חדשות מהמחסן.

השלב הראשון בפתרון בעיית ניתוב רכבים מרובת מטרות בזמן אמת הינו בניית מודל מתמטי המתאר את פונקציות המטרה והאילוצים השונים של הבעייה, כבעיית תיכנות לינארי בשלמים על גבי רשתות. בבניית המודל המתמטי הונחו מספר הנחות, כגון, שהמערכת הינה מערכת דינאמית (בעלת שינויים בזמני הנסיעה לאורך היום ודרישות לקוחות המשתנים באופן דינאמי לאורך יום העבודה), לכל דרישת לקוח משוייך חלון זמן שבו הלקוח דורש לקבל את דרישתו, חלונות הזמן הם חלונות זמן רכים, כלומר, אפשר לסטות מהם, כאשר לסטייה יש משמעות בדמות כנס או פגיעה בשביעות רצון הלקוח.

מכוון שבעיית ניתוב הרכבים היא בעייה NP-קשה, לא ניתן לפתור אותה בצורה אופטימאלית בעזרת שיטות קונבציונאליות לפתרון בעיות אופטימיזצייה. לכן, יש צורך בפיתוח אלגוריתם היוריסטי יעיל לפתרון בעייה. בכדי לעשות זאת, בוצעה סקירה נרחבת של בעיות ניתוב רכבים דינאמיים והאלגוריתמים לפתרונם וכן של שיטות לפתרון בעיות מרובות מטרות. על סמך סקירה זו הוחלט שהפתרון יתבסס על אלגוריתמים אבולוציוניים (אלגוריתמים לפתרון בעיות אופטימיזצייה שאופן פעולתם מבוסס על פעולות מתחום הביולוגייה, כגון אבולוצייה וגנטיקה).

לצורך פיתרון בעיית ניתוב הרכבים מרובת מטרות זמן אמת, הוצגו שלושה אלגוריתמים אבולוציוניים.

האלגוריתם הראשון הינו גירסה משופרת של האלגוריתם הגנטי VEGA ( vector evaluated genetic algorithm). אלגוריתם זה מבוסס על גישה האומרת שעבור בעייה בעלת *NumObj* פונקציות מטרה, בונים *NumObj* תת-אוכלוסיות בגודל *PopSize*/*NumObj* (כאשר גודל האוכלוסייה המקורית הוא *PopSize*). כל תת-אוכלוסייה כוללת את הפתרונות הטובים ביותר בהתבסס על ערכי פונקציית ה-fitness (פונקצייה המציינת עד כמה פתרון הינו טוב ביחס לפונקציית המטרה) של פונקציית מטרה אחת בלבד, כאשר כל תת-אוכלוסייה מבוססת על פונקציית מטרה אחרת. כל תת-האוכלוסיות מאוחדות לאוכלוסייה אחת בגודל *PopSize*, אשר עוברת תהליך עירבול. אלגוריתם VEGA משתמש באוכלוסייה זאת על מנת לבנות את הדור הבא של הפתרונות (באופן הרגיל שבו אלגוריתם גנטי עושה זאת). בניגוד לאלגוריתם VEGA הרגיל, האלגוריתם המשופר שומר את קבוצת הפתרונות האופטימאליים, שהינם קבוצת פתרונות לא דומיננטיים. קבוצת הפתרונות הלא דומינטיים בכל דור מתווספת לקבוצת הפתרונות

האופטימאליים, אשר עוברת תהליך "ניקוי" להסרת פתרונות שהפכו לדומיננטיים בעקבות הוספת הפתרונות החדשים.

אלגוריתם VEGA הינו האלגוריתם הגנטי הראשון שפותח לפתרון בעיות מרובות מטרות. אלגוריתם מתקדם יותר, אשר, תאורטית, אמור לתת תוצאות קרובות יותר לתוצאות האופטימאליות הינו אלגוריתם SPEA2. אלגוריתם SPEA2 משתמש בקבוצה, הנקראת ארכיב, על מנת לשמור קבוצה ראשונית של פתרונות לא דומיננטיים. הפתרונות הנשמרים בארכיב מצטרפים לפתרונות של האוכלוסייה הנוכחית, לייצירת ארכיב חדש, אשר משמש לייצירת הדור הבא של הפתרונות. גודלו של הארכיב קבוע, ולרוב זהה לגודלה של האוכלוסייה. היות וגודל הארכיב קבוע, ישנן שתי אפשריות לאופן מילויו. הראשונה, כאשר קבוצת הפתרונות הלא דומיננטיים קטנה מגודל הארכיב. במקרה זה, בנוסף לקבוצת הפתרונות הלא דומיננטיים, פתרונות דומינטיים מתוך האוכלוסייה מתווספים לארכיב, בהתאם לכמות החסרה. תוספת זו מתבצעת בהתאם לערך פונקציית ה-fitness של כל אחת מהפתרונות כפי שהיא מוגדרת ע"י אלגוריתם SPEA2, שהינה מספר הפתרונות שהינם דומיננטיים ביחס לפתרון בתוספת פונקציית צפיפות (במידה ולשני פתרונות יש מספר זהה של פתרונות שהינם דומיננטיים ביחס אליהם). השנייה, כאשר קבוצת הפתרונות הלא דומיננטיים גדולה מגודל הארכיב. במקרה זה, פתרונות בעלי המרחק הקטן ביותר משאר הפתרונות מוסרים מהארכיב. עבור פתרונות בעלי מרחק זהה, נשתמש במרחק השני הקטן ביותר, וכך הלאה.

האלגוריתם האבולוציוני השלישי, הוא שילוב של שיטת ה-vector evaluated (כפי שיושמה באלגוריתם VEGA המשופר) ומטודולוגייה הנקראת "מושבות דבורים מלאכותיות" ( artificial bee colony). באלגוריתמים המבוססים על מטודולוגיית "מושבות דברים מלאכותיות", ישנם שלושה סוגים של סוכנים המכונים "דבורים": (1) דבורים עובדות – אלו הן דבורים אשר מנצלות כרגע מקור מזון (מקור מזון הוא כינוי לפתרון במטודולוגייה זו). (2) דבורים צופות – אלו הן דבורים אשר נמצאות בכוורת וממתינות לדבורים העובדות שיחלקו עימן מידע לגבי מקורות המזון. (3) דבורים סיירות – דבורים אשר מחפשות מקורות מזון חדשים בקירבת הכוורת. השלב הראשון באלגוריתמים המבוססים על "מושבות דבורים מלאכותיות", הוא הקצאת מקור מזון (פיתרון) אקראי לכל אחת מהדבורים העובדות. לאחר שלב זה, מתחיל תהליך איטרטיבי. בכל איטרצייה, כל דבורה עובדת מוצאת מקור מזון (פיתרון) חדש הקרוב למקור המזון (הפיתרון) הנוכחי שלה. פעולה זו מתבצעת ע"י הפעלת פונקציות שכנויות שונות על מקור המזון (הפיתרון) הנוכחי השייך לכל אחת מהדבורים, אשר מבצעות מניפולציות שונות עליו, ובכך משנות אותו. אח"כ כמות הצוף הנמצאת במקור המזון מוערכת. הכוונה היא למציאת ערך מיספרי (כמות הצוף) המבטת את טיב הפיתרון, בדומה לערך פונקציית fitness באלגוריתמים גנטיים. אם כמות הצוף במקור המזון החדש גבוהה מכמות הצוף במקור המזון המקורי, מקור המזון של הדבורה מוחלף במקור המזון החדש (הפיתרון הישן מוחלף בפיתרון חדש טוב יותר). בשלב הבא, המיידע על כמות המזון בכל אחד ממקורות המזון מועבר לדבורים הצופות. כל דבורה צופה בוחרת במקור

מזון בהסתברות יחסית לאיכות המזון (ככל שכמות הצוף בכל מקור מזון גבוהה יותר, משמע הפיתרון טוב יותר, והסיכוי של מקור המזון הזה להיבחר גבוה יותר). לכן, מקורות מזון טובים, בניגוד לאלו הרעים, ימשכו אליהם יותר דבורים צופות. בהמשך, בעזרת שימוש בפונקציות השכנויות השונות, כל דבורה צופה מוצאת מקור מזון קרוב למקור המזון אותו היא בחרה, כשכמות הצוף באותו מקור מזון חדש מוערכת גם כן. בעזרת המידע שנאסף ע"י הדבורים הצופות, עבור כל מקור מזון, מקור המזון הטוב ביותר מבין כל מקורות המזון הקרובים אליו נבחר. מידע זה מועבר לדבורים העובדות, כך שכל מקור מזון שהשייך לדבורה עובדת נבחן מול מקור המזון הקרוב אליו הטוב ביותר שנמצא. אם מקור מזון זה יותר טוב ממקור המזון המקורי, מקור המזון של הדבורה העובדת מוחלף במקור המזון החדש. מקור מזון גם כן מוחלף במקור מזון חדש ואקראי במידה ולא ניתן היה למצוא עבורו מקור מזון טוב יותר במשך מספר איטרציות עוקבות קבוע מראש (דבורה עובדת הופכת לדבורה סיירת). התהליך האיטרטיבי ממשיך עד שתנאי עצירה מתממש, כשברוב המקרים, תנאי העצירה מוגדר כמספר איטרציות קבוע מראש. בדומה לאלגוריתם VEGA המשופר, גם אלגוריתם זה שומר את קבוצת הפתרונות האופטימאליים, שהינם קבוצת פתרונות לא דומיננטיים. קבוצת הפתרונות הלא דומינטיים בכל איטרצייה מתווספת לקבוצת הפתרונות האופטימאליים, אשר עוברת תהליך "ניקוי" להסרת פתרונות שהפכו לדומינטיים בעקבות הוספת הפתרונות החדשים.

בנוסף לתאור האלגוריתמים השונים, אופן יצוג הפתרונות הוצג גם כן. פתרון לבעיית ניתוב הרכבים צריך לציין את מספר כלי הרכב הנדרשים, את סך דרישת הלקוחות עבור כל אחד מכלי הרכב, את הלקוחות השייכים לכל אחד מהמסלולים ואת סדר המעבר בניהם וכן את זמן ההמתנה אצל כל אחד מהלקוחות. נגדיר אובייקט לקוח כאובייקט בעל שני מאפיינים, מספר הלקוח וזמן ההמתנה אצל הלקוח. פתרון לבעיית ניתוב רכבים מרובת מטרות זמן אמת, ניתן לייצג ע"י שימוש במערך של אובייקטי לקוח. במערך זה, המחסן מיוצג ע"י אובייקט לקוח בעל מס' לקוח אפס. מכוון שכל אחד מהמסלולים מסתיים במחסן, קיום של איבר במערך המייצג מחסן משמוע שהאיבר הבא במערך שייך כבר למסלול חדש. סדר הלקחות במערך, קובע גם את סדר המעבר בניהם בכל אחד מהמסלולים. כמו כן, על סמך האיברים במערך, ניתן לחשב את סך הדרישה בכל אחד מהמסלולים.

פעולות, כגון שיחלוף ומוטציות, הנדרשות לצורכי גיוון הפתרונות, גם כן מתוארות. שיחלוף ומוטציות הינן פעולות המבוצעות ע"י אלגוריתמים גנטיים. באלגוריתמים מבוססים "מושבות דבורים מלאכותיות", משתמשים בפעולות מוטצייה בלבד בכדי לשנות את הפתרונות. פעולות אלו ניקראות באלגוריתמים אלו, פונקציות שכנויות. את פעולות השיחלוף והמוטצייה יש להתאים לבעייה הנפתרת, וזאת על מנת למקסם את יעילותם של הפעולות, לגרום לאלגוריתם להתכנס לכיוון הפיתרון האופטימלי במהירות רבה יותר, וחשוב מכל, להימנע מקבלת פתרונות לא תקינים (פיתרונות לא פיזביליים, שאינם עומדים באילוצי הבעייה).

פונקציית fitness הינה סוג מיוחד של פונקציית מטרה, שתפקידה הינו לסכם, כערך מיספרי יחיד, עד כמה קרוב פיתרון נתון למטרה בודדת או אוסף של מטרות.

בתחום האלגוריתמים האבולוציוניים, בכל איטרצייה, אנו מעוניינים לבנות $n$ פתרונות חדשים, המבוססים על הפתרונות הטובים ביותר הקיימים לנו כעט, על מנת שיחליפו את $n$ הפתרונות הגרועים ביותר מהדור הקודם. לכן, עבור כל אחד מהפיתרונות יש צורך לחשב את ערך פונקציית ה-fitness שלו.

ישנם מקרים, שבהם במקום לחשב את ערך ה-fitness המדוייק של פיתרון נתון, אפשר להסתפק בערך מקורב. נהוג להשתמש בקרוב של ערך ה-fitness במספר מצבים: (1) זמן החישוב של ערך ה-fitness המדוייק הוא ארוך יחסית. (2) מודל מדוייק לחישוב ערך ה-fitness אינו קיים. (3) קיים קושי בשיערוך ערך ה-fitness עקב קיום "רעש" או סטוכסטיות במודל.

בכל שלושת האלגוריתמים שהוצגו, יש צורך בחישוב ערך ה-fitness עבור כל אחת מפונקציות המטרה. היות וארבע פונקציות מטרה תלויות בזמן הנסיעה, אשר הינו סטוכסטי, ומשתנה לאורך היום, בכדי לחשב את ערך ה-fitness שלהם באופן מדוייק, יש להשתמש בסימולצייה. סימולצייה הינו תהליך הדורש זמן מיחשוב רב.

במחקר הוצג שניתן לקצר את זמני הריצה של האלגוריתמים ע"י שימוש בערכי fitness מקורבים, במקום לחשב את ערכם המדוייק, שכאמור הינו תהליך הדורש זמן מיחשוב רב, וזאת מבלי להשפיע על איכות התוצאות המתקבלות מהאלגוריתמים. הצורך באלגוריתמים מהירים נובע מכך שאנו בסופו של דבר, נידרשים לתת פתרונות בזמן אמת.

לרוב, כאשר פותרים בעיות מרובות פונקציות מטרה, הפתרון המתקבל הוא קבוצה של פתרונות לא דומיננטיים, כשמקבל ההחלטות צריך לבחור את הפתרון המועדף עליו מקבוצה זו. מכוון שהמטרה הסופית של עבודה זו היא יצירת מערכת אוטומטית אשר פותרת את בעיית ניתוב הרכבים מרובת המטרות זמן אמת באופן עצמאי, שיטת ה-TOPSIS, מנגנון לבחירת פתרון מועדף מקבוצה של פתרונות לא דומיננטיים, יושמה. במחקר, כאמור, הוצג שניתן לקצר את זמני הריצה של האלגוריתמים ע"י שימוש בערכי fitness מקורבים, וזאת מבלי להשפיע על איכות התוצאות המתקבלות מהאלגוריתמים. אולם, אין זה אומר, שאוסף הפתרונות המתקבל בכל אחד מהשיטות הינו זהה, ולכן אין זה מחייב שבשימוש בשיטת ה-TOPSIS, יתקבלו תוצאות זהות. בעזרת מבחני קורלצייה ומבחני T תלויים נמצא שהתוצאות המתקבלות לאחר שימוש בשיטת ה-TOPSIS, בין אם בשימוש בערכי fitness מדוייקים או מקורבים, הינן זהות בשני המקרים.

מעבר לכך, הזמן הנדרש לקבלת התשובה בעזרת שיטת ה-TOPSIS, הן כאשר ה-fitness מחושבים בצורה מדוייקת או מקורבת, נבדק גם כן. הבדיקה נערכה בעזרת אלגוריתם VEGA המשופר, כשגודל האוכלוסייה הוא 200 פריטים וזמן הריצה של האלגוריתם הוא 30 דקות. התוצאות הראו, שללא קשר לגודל הבעייה (לצורך הבדיקה השתמשו בסט הרשתות של Solomon, בעלי 25, 50 ו-100 לקוחות), הזמן הנדרש לקבלת הפתרון הינו קצר משמעותית כאשר משתמשים בערכי fitness מקורבים.

פרמטר נוסף של האלגוריתם שדרש בחינה הינו פרמטר זמן ההמתנה. זמן ההמתנה הוא הזמן שכלי רכב ממתין לאחר שסיים לספק סחורה לקוח, ולפני שהוא מתחיל את דרכו ללקוח הבא. זמן ההמתנה נקבע על ידי האלגוריתם, ויכול להיות כל ערך בטווח שנקבע מראש. לכן, השאלה שנשאלת הוא, מהו הטווח הטוב ביותר שממנו האלגוריתם צריך לבחור את זמן ההמתנה, כך שהאלגוריתם יתכנס לפתרון האופטימלי, ביחס לכל הפונקציות המטרה, במהירות המרבית (פחות איטרציות).

בכדי למצוא את טווח זמן ההמתנה הטובה ביותר, נערכה סדרה של בדיקות מבוססת על סט הרשתות C101, R101 ו-RC101 של Solomon, בעלי 25, 50 ו-100 לקוחות. התוצאות שהתקבלו שימשו כקלט עבור ריגרסייה לינארית, שמטרתה לחזות את הערך של כל אתחת מפונקציית המטרה. תוצאות הרגרסיה לינארית הראו שעבור יותר ממחצית המקרים, התוצאות הטובות ביותר הושגו כאשר טווח זמן ההמתנה היה בין 0 ל-5 דקות. עם זאת, עבור יותר מחצית מפונקציות (23 מתוך 45) ערכו של $R^2$ הינו הנמוך מ-0.75. כלומר, ערכם של מחצית מהפונקציות המטרה שחושבו בהתבסס על הפונקציות הרגרסיה, הינם, ככל הנראה, רחוקים מערכם האמיתי. מסיבה זו, נערכה בנוסף השוואת הממוצעים.

תוצאות השוואת הממוצעים היו דומות לתוצאות שהתקבלו מהרגרסיה הלינארית. ערכם של יותר ממחצית מפונקציות המטרה (25 מתוך 45) הינו מיטבי כאשר זמן ההמתנה הוא בטווח של 0 עד 5 דקות.

המסקנה המתקבלת, בהסתמך על התוצאות שהתקבלו הן מהריגרסייה הלינארית והן מהשוואת ממוצעים, היא שהפתרונות הטובים ביותר מתקבלים כאשר זמן המתנה הוא בטווח של 0 עד 5 דקות.

בבעיית ניתוב הרכבים עם חלונות זמן מסורתית, פתרון ישים חייב לספק את כל חלונות הזמן. כאשר הספק מגיע ללקוח בטוח חלון זמן של הלקוח, רמת השירות של הספק הינה משביעת רצון או שווה ל 1, אחרת, היא אינה מספקת, או שווה ל-0. אילוץ זה הינו בעייתי, היות ולעיתים הספק אינו מגיע בטווח חלון הזמן שהוגדר לו ע"י הלקוח מסיבות כלכליות ותפעוליות שונות. עם זאת, גם במקרים אלו, קיים גבול מסויים (של הקדמה או איחור) שלקוח יכול לסבול. שביעות הרצון של הלקוח, קשורה, אם כך, קשורה קשר הדוק עם איכות השירות של הספק, ולפיכך, רמת השירות אינה יכולה להיות מתוארת על ידי שני ערכים בלבד (0 או 1).

על מנת לקבל תחושה כיצד רמת השירות, השקולה לשביעות הרצון של הלוח, משתנה כפונקציה של סטייה מטווח חלון הזמן של הלקוח (הקדמה או איחור), 38 לקוחות התבקשו להעריך את שביעות הרצון הכוללת שלהם, בעזרת שאלונים, כאשר ספק או נותן שירותים אחר מגיע כ-30 דקות עד ארבעה שעות, במרווחים של 30 דקות, מוקדם מצפוי. ובאופן דומה, הם התבקשו

להעריך את שביעות הרצון הכוללת שלהם, כאשר ספק או נותן שירותים אחר מגיע כ-30 דקות עד ארבעה שעות, במרווחים של 30 דקות, מאוחר מצפוי.

לכל לקוח, בהתבסס על תוצאות השאלון שלו, חושבה פונקציית שביעות רצון. מהפונקציות האלה, ניתן לראות כי רוב הלקוחות הינם רגישים להקדמות או איחורים של ספקים, וכן שרמת שביעות הרצון שלהם יורדת באופן דרמטי כאשר הספק מגיע מוקדם או מאוחר מהצפוי.

בשלב האחרון של העבודה, התוצאות של כל אחד משלושה האלגוריתמים הושוו באמצעות ניתוח מקרה (case study). ניתוח המקרה מבוסס על שתי רשתות (עירונית ובינעירונית) המבוססות על רשת תחבורה מהעולם האמיתי, הכולל את המיקומים של המחסן, הלקוחות וכן מידע על זמני הנסיעה בין הלקוחות השונים. ניתוח המקרה נעשה באמצעות סימולציה.

הסימולציה מבוססת על שני תהליכים רצים במקביל, תהליך האלגוריתם ותהליך הסימולציה, אשר בין שני התהליכים מתקיים שיתוף מידע.

תהליך הסימולציה מדמה יום עבודה מלא, הנעשה באמצעות טיפול בכל אחד מכלי הרכב, איסוף נתונים על זמני הנסיעה ודרישות של לקוחות חדשים.

בניתוח המקרה נעשה שימוש בחמש אסטרטגיות ריצה שונות עבור כל אחד המאלגוריתמים האבולוציוניים, כשהאסטרטגיה החמישית מייצגת מצב שבו הן זמני הנסיעה והן דרישות לקוחות אינם ידועים (מצב הזהה למצב בעולם האמיתי).

אלגוריתם VEGA הינו אלגוריתם ידוע לפתרון בעיות אופטימיזצייה מרובות מטרות. אולם, מאז פיתוחו פותחו אלגוריתמים מתוחכמים ומדויקים יותר לפתרון אלו, כגון SPAE2, אלגוריתמים הוכנסו. כמו כן, אלגוריתם VE-ABC הוא אלגוריתם חדש, אך איטי יותר, ולכן הוא מסוגל לעשות הרבה פחות איטרציות בתקופת זמן נתונה, בהשוואה לשני האלגוריתמים האחרים. לכן, היה צפוי שהתוצאות הטובות ביותר יתקבלו ע"י שימוש באלגוריתם SPEA2. עם זאת, המחקר מראה שאין זה כך. ברשת עירונית, בעת שימוש בפונקצית חוסר שביעות רצון לינארית, נמצא כי כאשר לכל הלקוחות מקבלים את אותה העדיפות, הפתרון הטוב ביותר התקבל ע"י שימוש באלגוריתם VEGA המשופר. לעומת זאת, כאשר לכל לקוח יש עדיפות שונה (השקולה לגודל הדרישה שלו), תוך שימוש באותה הרשת ואותה פונקציית חוסר שביעות רצון, התוצאה הטובה ביותר הושגה באמצעות שימוש באלגוריתמים SPEA2 ו-VE-ABC.

ברשת עירונית כשפונקציית חוסר שביעות רצון מציינת שהלקוחות לא אוהבים שהספק מקדים או מאחר, וכן ברשת בינעירונית עם שני סוגי פונקציות חוסר שביעות הרצון, הפתרון שהתקבל משלושת האלגוריתמים היה זהה.

מתוצאות אלו ניתן לראות, שאלגוריתם VEGA המשופר, על אף היותו ישן ופחות טוב (כלומר מספק פתרונות רחוקים יותר מהפתרון האופטימאלי) ביחס לאלגוריתמים חדשים יותר, מספק פתרונות באיכות שווה לפתרונות המתקבלים מאלגוריתמים מתוחכמים וחדשים יותר. על סמך

תוצאות אלו, האפשרות להשתמש באלגוריתם VEGA ועדיין לקבל פתרונות השקולים באיכותם לפתרונות המתקבלים מאלגוריתמים חדשים מהווה יתרון, היות לאלגוריתם VEGA (הן המקורי והן המשופר) יש יתרון בפשטות היישום ומהירות ריצה לעומת האלגוריתמים האחרים (וכתוצאה מכך, מספר החזרות בתקופת זמן נתונה), וכן ביכולת התאמתו לשינויים בנתונים ובאילוצים.

# תוכן עיניינים

עבודה זו נעשת בהדרכתם של פרופ' אוריאל שפיגל ופרופ' ראובן כהן מהמחלקות לניהול ומתמטיקה
של אוניברסיטת בר-אילן

# פתרון בעיית ניתוב רכבים זמן-אמת מרובת מטרות

**חיבור לשם קבלת התואר "דוקטור לפילוסופייה"**

**מאת:**
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**המחלקה לניהול**

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