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The Real-Time Multi-Objective Vehicle Routing Problem – Case Study: Information Availability and the Quality of the Results

Oren E. Nahum
Tel: +972-52-2899902, e-mail: oren.nahum@live.biu.ac.il
Department of management,
Max ve-Anna Webb street, Bar-Ilan University, Ramat Gan 5290002, Israel
Yuval Hadas*
Tel: +972-4-9590565, Fax: +972-4-9590565, e-mail: yuval.hadas@biu.ac.il
Department of management,
Max ve-Anna Webb street, Bar-Ilan University, Ramat Gan 5290002, Israel
Uriel Spiegel
Tel: +972-3-5318282, Fax: +972-3-7384041, e-mail: Uriel.Spiegel@biu.ac.il
Department of management,
Max ve-Anna Webb street, Bar-Ilan University, Ramat Gan 5290002, Israel
Reuven Cohen
Tel: +972-3-5317103, Fax: +972-3-738-4057, e-mail: reuven@macs.biu.ac.il
Department of mathematics,
Max ve-Anna Webb street, Bar-Ilan University, Ramat Gan 5290002, Israel

**Corresponding author*

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ABSTRACT

Vehicle-routing problems (VRP) have been studied in depth. While traditional VRPs have been thoroughly studied, limited research has to date been devoted to multi-objective, real-time management of vehicles. In this paper a real-time multi-objective VRP is presented and mathematically formulated. Using four case studies, based on two real-world transportation networks (urban and interurban); the result of an improved VEGA algorithms, are tested and compared in various situations. It was shown that the results obtained when information such as customers' demands and travel time, is unknown, are as good as to the results of the algorithm when all information is known in advance.

Key words: Vehicle routing problem; Real-Time; Multi-Objective; Case Study

INTRODUCTION

The Vehicle Routing Problem (VRP) is one of the most important and widely studied combinatorial optimization problems, with many real-world applications in distribution and transportation logistics *(1)*. In the standard VRP (also known as the

Capacitated VRP – CVRP), vehicles start their routes at a depot, call at customers, to whom they deliver goods, and return to the depot, with an objective of finding the lowest-cost set of routes, usually shortest routes. Traditionally, vehicle routing plans are based on deterministic information about demands, vehicle locations and travel times on the roads. What is likely to distinguish most VRPs today from equivalent problems in the past, is that information that is needed to come up with a set of good vehicle routes and schedules is dynamically revealed to the decision maker *(2)*. Until recently, the cost of obtaining real-time traffic information was deemed too high in comparison with the benefits of real time control of the vehicles. Furthermore, some of the information needed for real time routing was impossible to acquire. Advancement of the technology make it possible to operate vehicles using the real-time information about travel times and the vehicles' locations *(3)*.

Moreover, VRPs, which are frequently used to model real cases, are often set up with the single objective of minimizing the cost of the solution, despite the fact that the majority of the problems encountered in industry, particularly in logistics, are multi-objective in nature. In real-life, for instance, there may be several costs associated with a single tour. Moreover, the objectives may not always be limited to cost. In fact, numerous other aspects, such as balancing workloads (time, distance ...), can be taken into account simply by adding new objectives *(4)*.

While traditional VRPs have been thoroughly studied, limited research has to date been devoted to multi-objective, real-time management of vehicles during the actual execution of the distribution schedule, in order to respond to unforeseen events that often occur and may deteriorate the effectiveness of the predefined and static routing decisions. Furthermore, in cases when traveling time is a crucial factor, ignoring travel time fluctuations (due to various factors, such as peak hour traveling time, accidents, weather conditions, etc.) can result in route plans that can take the vehicles into congested urban traffic conditions. Considering time-dependent travel times as well as information regarding demands that arise in real time in solving VRPs can reduce the costs of ignoring the changing environment *(5)*.

The rest of the paper is as follow. Section 2 provides a review on both multi-objective and dynamic VRPs. Section 3 provides a mathematical formulation of the problem described in this paper. Section 4 describes three evolutionary algorithms later used for solving the problem described in section 3. The results of four case studies are described in section 5. Finally, Section 6 concludes the paper.

LITERATURE REVIEW

Multi-Objective VRP

VRPs are frequently used to model real cases. However, they are often set up with the single objective of minimizing the cost of the solution, although the majority of the problems encountered in industry, particularly in logistics, are multi-objective in nature. Multi-objective VRPs are used mainly in three ways: 1) **Extending classic academic problems** – Multi-objective optimization is one possible way to study objectives other than the one initially defined. In this context, the problem definition remains unchanged, and new objectives are added. As an example of such an objective, we can consider the following: (1) *Driver workload* – an extension to VRP in which the balance of tour lengths is considered (to increase the fairness of the solution) *(6-8)*. (2) *Customer Satisfaction* – an objective added to VRP with time windows in order to improve customer satisfaction with regard to delivery dates *(9)*.

2) **Generalizing Classic Problems** – Another way to use multi-objective optimization is to generalize a problem by adding objectives instead of one or several constraints and/or parameters *(10-14)*. 3) **Studying real-life cases** - Multi-objective routing problems are also studied for a specific real-life situation, in which decision makers define several clear objectives that they would like to see optimized *(15-19)*.

Most Common Objectives

The different objectives studied in the literature can be presented and classified according to the component of the problem with which they are associated. The following is a summary of the most common objectives. 1) **Objectives related to the tour**: (a) **Cost**: Minimizing the cost of the solutions generated is the most common objective, usually for economical reasons; however, other motivations are possible. For instance, in *(20, 21)*, it is done to avoid damaging the product being transported. (b) **Makespan**: Minimizing the makespan ensures some fairness in solutions *(16, 22, 23)* (c) **Balance**: Some objectives are designed to even out disparities between the tours *(6, 24)*. 2) **Objectives related to node/arc activity**: Most of the studies dealing with objectives related to node/arc activity involve time windows. Time windows are usually replaced by an objective that minimizes the number of violated constraints *(13)*, the total customer and/or driver's wait time due to earliness or lateness *(10, 11, 25)*, or both *(26, 27)*. 3) **Objectives related to resources**: A common objective is the minimization of the number of vehicles, as in VRP with time windows (usually treated lexicographically) *(28)*. Goods-related objectives are used to take the nature of the goods into account (merchandise is perishable and we want to avoid its deterioration *(20, 21)*).

Multi-Objective Optimization Algorithms

Over the last several years, many techniques have been proposed for solving multi-objective problems. These strategies can be divided into three general categories: 1) **Scalar methods** - The most popular is weighted linear aggregation. For multi-objective VRPs, weighted linear aggregation has been used with specific heuristics *(5, 29)*, local search algorithms *(30)*, and genetic algorithms *(12)*. 2) **Pareto methods -** Pareto methods use the notion of Pareto dominance directly. Pareto methods are used with evolutionary algorithms, local searches, heuristics, and/or exact methods *(31, 32)*. 3) **Methods that belong to neither the first nor the second category** - These non-scalar and non-Pareto methods are based on genetic algorithms, lexicographic strategies, ant colony mechanisms, or specific heuristics *(33, 34)*.

Dynamic VRP

In many real-life applications relevant data changes during the execution of transportation processes and schedules have to be updated dynamically. Thanks to recent advances in information and communication technologies, vehicle fleets can now be managed in real-time. In this context, Dynamic or real-time VRPs (DVRPs), are becoming increasingly important *(2, 35-37)*. The most common source of dynamism in VRP is the online arrival of customer requests during the operation *(38-40)*. In order to consider travel time variations, different approaches have been developed *(5, 41, 42)*. Malandraki and Daskin *(43)* used a step function to represent time-dependent issue and develop a heuristic approach. Stochastic VRP (SVRP) has been proposed to consider such travel time variations *(44-48)*. Due to the difficulties of capturing the variation of travel time in a traffic network, simulation models have been used to generate realistic travel time and applied in different routing strategies.

According to Laporte, Gendreau *(49)*, the tabu search heuristics have proved to be the most successful meta-heuristic approach. A number of researchers have applied the tabu search algorithm on VRP *(50, 51)*.

Finally, some recent work considers dynamically revealed demands for a set of known customers *(52-55)* and vehicle availability *(56-58)*, in which case the source of dynamism is the possible breakdown of vehicles. In the following we use the prefix ‘‘D-’’ to label problems in which new requests appear dynamically.

Problem Formulation

This chapter provides a mathematical formulation to the real-time multi-objective VRP. Since VRP is a hard optimization problem *(59)*, the complexity of the problem will remain the same as CVRP, at least, because of the time dimension and the stochastic properties of the problem.

In the proposed formulation, the following notations are used.

V	Set of nodes, including the depot and the demand nodes
E	Set of edges
N	Number of customers (customer number 0 denotes the depot)
d_i^t	Demand of demand node *i* requested at time *t*.
D_i^t	The total demand of customer i at time t. D_i^t is defined as $\sum_{t=0}^{t_S} d_i^t - \sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1} d_j^t \bar{x}_{ij}^{mt}$, which means that the demand of a customer equals the sum of all customers' demands received between time interval of $t=0$ to time $t=t_S$ minus the demands that already have been served. For customer 0, which is the depot, $d_i^{t_S}=0$ for all t_S.
x_{ij}^{mt}	A decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t \ge t_s$, and 0, otherwise.
$\bar{x}_{ij}^{mt}$	Known decision variable, defined as 1 if vehicle *m* traveled from node *i* to node *j* at time *t*. where $t < t_s$, and 0, otherwise.
t_S	Time of routing plan. The time of routing plan can start at $t_s=0$ and end at $t_S=T$.
C_{ij}^t	A stochastic time-dependent nonnegative cost function, which represents the travel cost from vertex *i* to vertex *j* starting at time *t*.
$\hat{c}_{ij}^t$	Estimated travel cost from vertex *i* to vertex *j* starting at time *t*.
$\bar{c}_{ij}^{\,t}$	Known cost for traveling from node *i* to node *j* at time *t*, where $t < t_s$
M	The maximum number of vehicles available
Q	capacity of a vehicle (all vehicles have the same capacity)
t_S^m	The last departure time of vehicle m from the depot. t_S^m is defined as $t_S^m = \max t \in \{0,...,t_S\}$ which satisfies $\sum_{j=0}^{N} \bar{x}_{0j}^{mt} = 1$, and there is no $\hat{t} \in \{0,...,t_S\}$,

such that $\hat{t} > t$ and $\sum_{j=0}^{N} \overline{x}_{j0}^{m\hat{t}} = 1$. If such t_S^m does not exist, then $t_S^m = t_S$.

$v_i^{t_S}$ Does a customer require a visit at time t_S. $v_i^{t_S}$ is defined as $v_i^{t_S} = \begin{cases} 1 & d_i^{t_S} > 0 \\ 0 & otherwise \end{cases}$.

TW_i^S Start of time window of customer *i*

TW_i^E End of time window of customer *i*

EET_i Endurable earliness time - the earliest service time that customer *i* can endure when a service starts earlier than TW_i^S.

ELT_i Endurable lateness time - the latest service time that customer *i* can endure when a service starts later than TW_i^E .

ST_i Service time at customer *i.*

$\overline{ST}_i$ Known service time at customer *i*.

WT_i Waiting time at customer *i.*

$\overline{WT}_i$ Known waiting time at customer *i*.

Let $G = (V, E)$ be a complete graph, where $V = \{0,...,n\}$ is the nodes set and *E* is the edge set. Each node $i \in V \setminus \{0\}$ represents a customer (where *N* is the number of nodes), having a non-negative demand, whereas node 0 represents the depot. Each edge $e \in E = \{(i,j) : i,j \in V, i < j\}$ is associated with a stochastic time-dependent nonnegative cost, $c_{ij}^{\,t}$, which represents the travel time spent to go from node *i* to node *j* starting at time *t.* The use of the loop edges, (i,i), is not allowed. A fixed fleet of *M* identical vehicles, each of capacity *Q*, is available at the depot.

The travel time cost function, c_{ij}^t, is stochastic in nature, meaning that it may vary from one day to another. The cost function, c_{ij}^t, is associated with a mean, $\mathbb{E}(c_{ij}^t)$ and a standard deviation, $\sigma(c_{ij}^t)$. As an estimation of c_{ij}^t the mean $\mathbb{E}(c_{ij}^t)$ can be used. In this case, the total travel time of the route will not reflect the possibility of arriving at a customer earlier or later than expected, and the changes in travel time it may cause. Therefore, a different estimation of the stochastic cost function, c_{ij}^t, is suggested. Using the concept of relative standard deviation (coefficient of variation) *(60)*, let $\epsilon(t) = t \cdot \max_{t' \le t} \left| \frac{\sigma(c_{ij}^{\,t'})}{\mathbb{E}(c_{ij}^{\,t'})} \right|$, the maximum relative standard deviation for of all time intervals prior and including *t*, multiplied by *t* ,be an impact factor, which defines how much the value of c_{ij}^t is affected by possible changes in travel time (compared to the mean) in previous and future time intervals. The estimation of the stochastic cost function, $\hat{c}_{ij}^t$, is defined as $\hat{c}_{ij}^t = \frac{1}{2\epsilon(t)+1} \sum_{\hat{t}=t-\lfloor \epsilon(t) \rfloor}^{t+\lfloor \epsilon(t) \rfloor} \mathbb{E}(c_{ij}^{\,\hat{t}})$. In this definition $\hat{c}_{ij}^t$ equals to an average of expected values of c_{ij}^t over several time intervals defined by the impact factor, thereby taking into consideration the possibility of being early or late.

Based on the notation of $\hat{c}_{ij}^{t}$, the real-time multi-objective VRP can be defined as a mixed integer linear programming module.

The objective of the mixed integer programming are:

$$\begin{aligned} \min \quad Z = &\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t_S-1}\left[\max\left(\overline{c}_{ij}^{t}, TW_j^S - t\right) + \overline{ST}_j + \overline{WT}_j\right]\overline{x}_{ij}^{mt} + \\ &+\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\sum_{t=t_S}^{T}\left[\max\left(\hat{c}_{ij}^{t}, TW_j^S - t\right) + ST_j + WT_j\right]x_{ij}^{mt} \end{aligned} \tag{3.1}$$

$$\min \quad Z = \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=0}^{t=t_s-1}\overline{x}_{0j}^{mt} + \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=t_s}^{T}x_{0j}^{mt} \tag{3.2}$$

$$\min \quad Z = \sum_{i=0}^{n}\sigma_i S_i\left(\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=0}^{t_S-1}\left(\left(t+\overline{c}_{ji}^{t}\right)\overline{x}_{ji}^{mt}\right) + \sum_{t=t_S}^{T}\left(\left(t+\hat{c}_{ij}^{t}\right)x_{ji}^{mt}\right)\right)\right) \tag{3.3}$$

$$w_m = \sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=0}^{t_S-1}\left(\overline{c}_{ji}^{t}\overline{x}_{ji}^{mt}\right) + \sum_{t=t_S}^{T}\left(\hat{c}_{ij}^{t}x_{ji}^{mt}\right)\right) \tag{3.4}$$

$$\min\left\{StdDev\right\} = \min \ Z = \left\{\sqrt{\frac{\sum_{m=1}^{M}w_m^2}{M} - \left(\frac{\sum_{m=1}^{M}w_m}{M}\right)^2}\right\} \tag{3.5}$$

$$\min \ Z = Max\left(t : x_{i0}^{mt} = 1 \quad \forall i \in N, m \in M, t \in T\right) \tag{3.6}$$

and the constraints are:

$$x_{ii}^{mt} = 0 \quad \forall i \in N,\ m \in M,\ t \in T \tag{3.7}$$

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}^{mt} + \sum_{t=t_S}^{T}x_{0j}^{mt}\right) \le 1 \quad \forall m \in M \tag{3.8}$$

$$\sum_{j=1}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{0j}{}^{mt} + \sum_{t=t_S}^{T}x_{0j}{}^{mt}\right) = \sum_{i=1}^{N}\sum_{t=t_S}^{T}x_{i0}{}^{mt} \quad \forall m \in M \tag{3.9}$$

$$\sum_{i=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt} + \sum_{t=t_S}^{T}x_{ij}{}^{mt}\right) = v_j^{t_S} \quad \forall j \in N,\ i \ne j \tag{3.10}$$

$$\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ij}{}^{mt} + \sum_{t=t_S}^{T}x_{ij}{}^{mt}\right) = v_i^{t_S} \quad \forall i \in N,\ i \ne j \tag{3.11}$$

$$\sum_{i=0}^{N}\left(d_i^{t_S}\left(\sum_{j=0}^{N}\sum_{t=t_S}^{T}x_{ij}^{mt}\right)\right) \le Q \quad \forall m \in M \tag{3.12}$$

$$\sum_{k=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\left(t\times\overline{x}_{jk}{}^{mt}\right)+\sum_{t=t_S}^{T}\left(t\times x_{jk}{}^{mt}\right)\right)\geq$$
$$\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\begin{array}{l}\sum_{t=t_S^m}^{t_S-1}\left(\left(t+\max\left(\overline{c}_{ij}^{t},TW_j^S-t\right)+\overline{ST}_j+\overline{WT}_j\right)\times\overline{x}_{ij}{}^{mt}\right)+\\+\sum_{t=t_S}^{T}\left(\left(t+\max\left(\hat{c}_{ij}^{t},TW_j^S-t\right)+ST_j+WT_j\right)\times x_{ij}{}^{mt}\right)\end{array}\right)\quad\forall i>0,j>0 \tag{3.13}$$

$$\sum_{i=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{ip}{}^{mt}+\sum_{t=t_S}^{T}x_{ip}{}^{mt}\right)-\sum_{j=0}^{N}\left(\sum_{t=t_S^m}^{t_S-1}\overline{x}_{pj}{}^{mt}+\sum_{t=t_S}^{T}x_{pj}{}^{mt}\right)=0\quad\forall m\in M,\ p\in N,\ p\neq 0 \tag{3.14}$$

$$P\left(\left(\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{m=1}^{M}\left(\sum_{t=t_S^m}^{t_S-1}\overline{c}_{ij}^{t}\overline{x}_{ij}^{mt}+\sum_{t=t_S}^{T}\hat{c}_{ij}^{t}x_{ij}^{mt}\right)\right)\leq c^*\right)\geq\alpha \tag{3.15}$$

$$x_{ij}{}^{mt}\in\{0,1\}\quad\forall m\in V,\ i,j\in N,\ t\in T \tag{3.16}$$

Five objective functions are considered in this model. The first objective is minimizing the total travel time. If by leaving node *i* at time *t* a vehicle reaches node *j* before its time window's start time, meaning $t+\hat{c}_{ij}^{t}<TW_j^S$, then the vehicle has to wait until the beginning of the time window in order to start serving. Otherwise, it starts serving upon arrival. Therefore, the time passed since a vehicle left node *i* towards node *j* and the time it left node *j* can be defined as $\max\left(\hat{c}_{ij}^{t},TW_j^S-t\right)+ST_j+WT_j$. By summing all possible true travel times (for whom the decision variable x_{ij}^{mt} is equal to 1), we get the total travel time, which is to be minimized.

The total travel time can be decomposed into two parts, the known travel time and the unknown travel time. If the planning time, t_s, is not equal to 0, then we are not at the beginning of the day, and some vehicles have already been sent to customers. In this case, the total traveling cost is the sum of the known traveling cost and the unknown traveling cost, as defined in defined in equation (3.1).

The second objective function concerns the total number of vehicles used, which is defined in equation (3.2), as the number of vehicle leaving the depot.

VRP with Time Windows (VRPTW) is an important extension of the CVRP in which service at every customer *i* must start within a given time window $[a_i,b_i]$. A vehicle is allowed to arrive before a_i and wait until the customer becomes available, but arrivals after b_i are prohibited. This problem is known as VRPTW with hard time windows. In other cases, both lower and upper bounds of the time window need not be satisfied, but can be violated at a penalty. These are Vehicle Routing Problems with Soft Time Windows *(1)*.

For VRPTW with soft time windows, when a customer is served within his time window, the supplier's service level is satisfactory; otherwise, it is not. Hence, a customer's satisfaction level can be described using a binary variable, which takes 1 if the service time falls within the specified time window, and 0 if it does not.

Time windows may sometimes be violated for economic and operational reasons. However, there exist certain bounds on the violation that a customer can endure. Earliness and lateness are closely related to the quality of service. The response of a customer satisfaction level to a given service time may not be simply "good" or "bad"; instead, it may be between "good" and "bad". For example, the customer might say, "it's all right" to be served within $\left[EET_i, TW_i^S\right]$ or $\left[TW_i^E, ELT_i\right]$. In either case, the service level cannot be described by only two states (0 or 1).

Intuitively, with the concepts of EET_i and ELT_i, the supplier's service level for each customer can be described by a fuzzy membership function:

$$S_i(t) = \begin{cases} 0, & t < EET_i \\ f_i(t), & EET_i \le t < TW_i^S \\ 1, & TW_i^S \le t < TW_i^E \\ g_i(t), & TW_i^E \le t < ELT_i \\ 0, & ELT_i \le t \end{cases} \tag{3.17}$$

Since customer's satisfaction level, as a function of the deviation from the customer's time window, in most cases cannot be described as a linear function, the following function is used to describes customer's satisfaction.

$$f_i(t) = \frac{\sum_{j=1}^{n} \beta_i^j \left(\frac{t - EET_i}{TW_i^S - EET_i} \right)^{\alpha_i}}{\sum_{j=1}^{n} \beta_i^j} \tag{3.18}$$

$$g_i(t) = \frac{\sum_{j=1}^{m} \delta_i^j \left(\frac{ELT - t}{ELT - TW_i^E} \right)^{\gamma_i}}{\sum_{j=1}^{m} \delta_i^j} \tag{3.19}$$

Assuming that each customer has his own satisfaction function, $S_i(t)$, and that the service provider assigns an *importance* factor, σ_i, to each customer that states how important it is to satisfy customer *i* compared to all other customers, the third objective function, maximizing customers' satisfaction, is defined as in equation (3.3).

The fourth objective function, seeks to balance the work, defined as the total travel time the vehicle had in a single day, between vehicles. This is achieved by minimizing the standard deviation of the work of each vehicle at the end of the period, as defined in equations (3.4) and (3.5).

The fifth and last objective function considered is minimizing the arrival time of the last vehicle, as defined in (3.6). Each vehicle starts its route and ends its route at the depot. While the start time of each vehicle is known, the end time is unknown and is due to change, mainly because of the stochastic nature of the travel time.

By minimizing the arrival time of the last vehicle, we guarantee two things: (1) Maximum availability of vehicle for unscheduled deliveries and (2) that there are no too long routes.

Next, the various constraints are defined. Equation (3.7) states that a vehicle cannot drive from one node to itself. Equation (3.8) states that all vehicles start their routes at the depot. Similarly, all vehicles end their routes at the depot, as defined in equation (3.9). Equations (3.10) and (3.11) ensure that all customers require a visit, are visited exactly once, while all other customers are not visited at all.

A demand constraint, stating that the total demand of all customers visited by the same vehicle must be less than or equal to the capacity of the vehicle, is defined by equation (3.12).

If node j is visited after visiting node i, then the departure time, t, from node j is equal to or greater than the departure time from node i plus the travel time from node i to node j at time t plus the service time and waiting time at node j as defined in equation (3.13). Another route continuity constraint, defined by equation (3.14), states that a vehicle, visiting node i, that leaves node p and a vehicle visiting node p, that leaves to node j, is the same vehicle.

Equation (3.15), a chance constraint, states that we are looking for a set of routes, that for a given probability, α, the traveling time will not be higher than c^*. A set of possible traveling times, Z^* can be created by solving the relaxed deterministic linear programming (in which all stochastic functions are randomly replaced with a corresponding values) a large number of times (each time using a different instance of the cost function). From this set, $Z' = \min\left(Z \in Z^*\right)$ can be chosen, which satisfies $P\left(\left(Z \in Z^*\right) \le Z'\right) \ge \alpha$ as c^*.

Finally, the last constraint, defined by equation (3.16), states that the decision variable, x_{ij}^{mt}, is a binary variable.

EVOLUTIONARY ALGORITHMS FOR SOLVING REAL-TIME MULTI-OBJECTIVE VRP

Evolutionary Algorithms

In real-time dynamic problems a solution is given based on known data, as time progresses, new data are added to the problem and the initial solution has to be re-evaluated in order to suite the new data. This is usually done at a pre-defined time intervals. If the time intervals are small enough, thus, at each time interval the amount of information added is limited. Therefore, the new solution will be similar to the previous one. The proposed mixed integer programming can hander real-time information, by re-solving the problem whenever new information becomes available. However, due to the complexity of the process, meta-heuristics are better suitable for this process. Such meta-heuristics are Evolutionary Algorithms (EA) that belong to the Evolutionary Computation field of study concerned with computational methods inspired by the process and mechanisms of biological evolution.

Evolutionary algorithms are well suitable for solving this kind of problems, since the previous solution can be considered as an initial solution for the updated problem, while there is no need to start the calculation of the new routes from the beginning. It should be noted, that a population might altered as a response to new information. For example, if a new request is added to the problem, then the new customer has to bee added each chromosome of the previous population.

Genetic Algorithms

Genetic Algorithms *(61, 62)* are computational models inspired by evolution. These algorithms encode a potential solution to a specific problem on a simple chromosome-like data structure and apply recombination operators to these structures in order to preserve critical information. Genetic algorithms begin with a population of (random) chromosomes. One then evaluates these structures, using a fitness function, and allocated reproductive opportunities, to create a new population, such that chromosomes which represent a better solution are given more chances to 'reproduce'. The new population is further evaluated and tested until termination. In this study, two multi-objective Genetic algorithms are used, VEGA and SPEA2.

VEGA The Vector Evaluated Genetic Algorithm (VEGA), proposed by David Schaffer *(63, 64)*, is normally considered the first implementation of a multi-objective EA (MOEA). In VEGA, for a problem of size *PopSize*, with *NumObj* objectives, *NumObj* sub-populations of size *PopSize*/*NumObj* are created, each uses only one objective functions for fitness assignment. These sub-populations are then shuffled together to obtain a new population of size *PopSize*, on which the GA would apply the crossover and mutation operators in the usual way. Elitism guarantees that best solutions found in each iteration are passed on to the next iteration and not lost. The original VEGA algorithm does not use elitism. In this paper an Improved VEGA algorithms, that uses elitism, is used. In the improved VEGA algorithm, the set of high performance solutions is defined as the set of non-dominated solutions *(65)* obtained in all iterations of the algorithm.

Strength Pareto Evolutionary Algorithm: SPEA2 SPEA2 in an elitist multi-objective EA. It is an improved version of the SPEA algorithm *(66)*, and incorporates a fine-grained fitness assignment scenario, a density estimation technique, and an enhanced archive truncation method. SPEA2 operates with a population (archive) of fixed size, from which promising candidates are drawn as parents of the next generation. The resulting offspring then compete with the old ones for inclusion in the population.

Artificial Bee Colony

Artificial bee colony (ABC) algorithms *(67, 68)* is a new evolutionary meta-heuristic technique inspired by the intelligent behavior of natural honey bees in their search for nectar. The ABC algorithm is an iterative algorithm. It starts by assigning each employed bee (bees that are exploiting a food source) to a randomly generated solution (food source). In each iteration, each employed bee, using a neighborhood operator, finds a new food source near its assigned food source, which replaces the original food source, if it's better. Next, information of the food sources is shared with the onlookers (bees that are waiting in the hive). The onlooker chooses a food source, using proportion selection, then, using a neighborhood operator, each onlooker finds a food source near its selected food source. For each old food source, the best food source among all the food sources near the old one is determined. The employed bee associated with the old food source is assigned to the best food source and abandons the old one if the best food source is better than the old food source. A food source is also abandoned if the quality of the food source has not improved for a predetermined and limited number of successive iterations. The employed bees then become scouts (bees that are searching for new food sources in the neighborhood of the hive) and randomly search for new food source. After a scout finds a new food source, it becomes an employed bee again. After each employed bee is assigned to a food source, another iteration of the ABC algorithm begins. The iterative process is repeated until a stopping condition is met. Since ABC algorithms share common characteristics with GAs, simple modifications made to the basic GAs can be adopted and applied to ABC algorithms. The improved vector evaluated genetic algorithm, described earlier in this chapter, is such an example.

EXPERIMENTAL RESULTS

In a previous work *(69)*, the results of the three algorithms were compared using a case study. The case study is based on two networks that are based on a real-world transportation network, including the locations of the depot, the customers and information about travel time between the different customers. The case study was performed using simulation. It was showed that the VEGA algorithm, when used, can provide solutions equal in quality to the solutions obtained from more sophisticated and more recent algorithms. In this paper, the results of the improved VEGA algorithm are examined in various situations, each with different information available.

Network

Two transportation networks, based on Israel’s road network, each includes real-life information with different characteristics, were generated. The first network is based on the greater Tel-Aviv metropolitan area’s urban road network, and includes 45 customers, based on the store's locations of a large super-market chain store. The second network is based on Israel's interurban road network, and includes 34 customers, based on major cities in Israel. For both networks, using “Google Maps”, the shortest distance and traveling time for different times of the day, between every two customers were found. Based on the data collected, a log-normal travel-time distribution function was calculated for each path *(70)*. Each customer is also associated with a randomly generated time window, which is based on the average traveling time from the depot to the customer. Similarly, each customer is associated with a randomly generated demand, in the range of 10 to 50, similar to the demands used in Solomon's instances.

In each test problem, half of the customers are considered as customers with unknown demands. These are the customers with the latest time window start time. Each unknown demand is revealed to the simulation at least two hours before the beginning of the time window.

Scenarios

In all four case studies, five scenarios for constructing the routing plan are considered.

1. In the first scenario, information about customers’ demands and traveling time, is known in advanced. Using this information, the algorithm runs for a pre-defined period of time, after that, using the TOPSIS *(71)* mechanism, a set of routes is

selected from the set of non-dominated solutions found by the algorithm. All vehicles follow this set of routes.

2. In the second scenario, as in the first, information about customers' demands and traveling time, is known in advanced. Using this information, the algorithm runs for a pre-defined period of time, after that, using the TOPSIS mechanism, a set of routes is selected from the set of non-dominated solutions found by the algorithm. All vehicles follow this set of routes. Vehicles start driving according to this set of routes, while the algorithm continues to run. Whenever a vehicle arrives at a customer, the customer is removed from all solutions evolved by the algorithm. Whenever a vehicle has to leave a customer and drive to the next customer (or the depot), or at pre-defined time intervals, using the TOPSIS mechanism, a new set of routes is selected from the set of non-dominated solutions found by the algorithm. Driving vehicles are then rerouted according to the new set of routes, and new vehicles are assigned as needed. This operation is repeated until all customers have been served, and all vehicles have returned to the depot.
3. The third scenario is similar to the second scenario; however, traveling time is unknown. Traveling time information for the next pre-defined time period is revealed to the algorithm at pre-defined intervals.
4. The fourth scenario is similar to the second scenario; however, demands of some of the customers are known. Information about new customers' demands are revealed to the algorithm, while it run, which, accordingly adds the new customers to the evolved solutions.
5. The fifth scenario is a combination of the third and fourth scenarios, in which neither customers' demands nor traveling times are known in advance.

Case Studies

Four case studies were performed. For the first and second case studies the network used was of the greater Tel-Aviv metropolitan area transportation network. As for the dissatisfaction function, for the first case study it is assumed that the dissatisfaction functions of all customers are linear, meaning $f_i(t) = 1 - \left(\frac{t - EET_i}{TW_i^S - EET_i} \right)^1$ and $g_i(t) = 1 - \left(\frac{ELT - t}{ELT - TW_i^E} \right)^1$. For the second case study it is assumed that all customers dislike it when the supplier is either early or late. Therefore, the dissatisfaction functions of all customers are in the form of $f_i(t) = 1 - \left(\frac{t - EET_i}{TW_i^S - EET_i} \right)^5$ and $g_i(t) = 1 - \left(\frac{ELT - t}{ELT - TW_i^E} \right)^5$.

For the third and fourth case studies the network used was of Israel's transportation network. As for the dissatisfaction function, for the third case study it is assumed that the dissatisfaction functions of all customers are linear, as in the first case study. For the second case study it is assumed that all customers dislike it when the supplier is either early or late, as in the second case study.

All test scenarios were solved 200 times. In the first 100 times, it is assumed that all customers have the same priority. In the next 100 times, it is assumed that each customer has a priority equal to its demand.

Scenarios Comparison

For each case study, the results of the various scenarios were compared, using paired-samples t-tests. The results are summarized in TABLE 1 to TABLE 4.

TABLE 1 PAIRED T-TEST RESULTS FOR COMPARISON OF VARIOUS SCENARIOS USING THE FIRST CASE STUDY

Customer's Priority	Objective Function	Scenario 1		Scenario 2		t	df	Sig.
		M	SD	M	SD			
The Same	1	49.698	1.761	58.195	1.283	-11.11	99	0
	2	10.4	0.699	19.6	2.066	-11.5	99	0
	3	0.683	0.216	0.308	0.078	7.122	99	0
	4	1.997	0.765	0.099	0.102	8.714	99	0
	5	14.398	0.337	12.925	0.258	9.777	99	0
Different	1	49.632	1.633	58.188	2.971	-7.479	99	0
	2	10.2	0.422	21.2	2.781	-12.473	99	0
	3	0.642	0.076	0.224	0.059	12.921	99	0
	4	58.791	15.77	3.122	1.842	10.483	99	0
	5	14.717	0.546	12.827	0.182	11.32	99	0
Customer's Priority	Objective Function	Scenario 1		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	49.698	1.761	55.778	2.121	-6.687	99	0
	2	10.4	0.699	26.6	5.211	-9.191	99	0
	3	0.683	0.216	0.172	0.248	7.035	99	0
	4	1.997	0.765	0.074	0.056	7.726	99	0
	5	14.398	0.337	12.523	0.351	12.249	99	0
Different	1	49.632	1.633	53.755	2.162	-4.802	99	0.001
	2	10.2	0.422	25.8	3.36	-13.601	99	0
	3	0.642	0.076	0.119	0.044	17.22	99	0
	4	58.791	15.77	4.699	3.17	10.721	99	0
	5	14.717	0.546	12.697	0.331	10.764	99	0
Customer's Priority	Objective Function	Scenario 2		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	58.195	1.283	55.778	2.121	2.418	99	0.039
	2	19.6	2.066	26.6	5.211	-4.323	99	0.002
	3	0.308	0.078	0.172	0.248	2.048	99	0.071
	4	0.099	0.102	0.074	0.056	0.598	99	0.564
	5	12.925	0.258	12.523	0.351	3.472	99	0.007
Different	1	58.188	2.971	53.755	2.162	3.505	99	0.007
	2	21.2	2.781	25.8	3.36	-3.052	99	0.014
	3	0.224	0.059	0.119	0.044	3.721	99	0.005
	4	3.122	1.842	4.699	3.17	-1.131	99	0.287
	5	12.827	0.182	12.697	0.331	1.164	99	0.274
Customer's Priority	Objective Function	Scenario 2		Scenario 4		t	df	Sig.
		M	SD	M	SD			
The Same	1	58.195	1.283	60.557	2.857	-3.281	99	0.01
	2	19.6	2.066	21.2	2.7	-1.5	99	0.168
	3	0.308	0.078	0.273	0.11	0.814	99	0.436
	4	0.099	0.102	0.17	0.103	-1.255	99	0.241
	5	12.925	0.258	12.695	0.321	2.053	99	0.07
Different	1	58.188	2.971	53.122	17.288	0.905	99	0.389
	2	21.2	2.781	19.8	6.161	0.593	99	0.568
	3	0.224	0.059	0.246	0.104	-0.562	99	0.588
	4	3.122	1.842	2.762	2.023	0.465	99	0.653
	5	12.827	0.182	12.487	1.197	0.912	99	0.386
Customer's Priority	Objective Function	Scenario 3		Scenario 5		t	df	Sig.
		M	SD	M	SD			
The Same	1	55.778	2.121	56.898	1.861	-1.124	99	0.29
	2	26.6	5.211	29.5	2.321	-1.653	99	0.133
	3	0.172	0.248	0.083	0.016	1.148	99	0.281
	4	0.074	0.056	0.051	0.02	1.233	99	0.249
	5	12.523	0.351	12.283	0.276	1.654	99	0.132
Different	1	53.755	2.162	51.951	4.182	1.285	99	0.231
	2	25.8	3.36	24.2	3.938	1.037	99	0.327
	3	0.119	0.044	0.133	0.04	-0.757	99	0.469
	4	4.699	3.17	7.515	9.936	-0.93	99	0.377
	5	12.697	0.331	12.768	0.542	-0.319	99	0.757

TABLE 2 PAIRED T-TEST RESULTS FOR COMPARISON OF VARIOUS SCENARIOS USING THE SECOND CASE STUDY

Customer's Priority	Objective Function	Scenario 1		Scenario 2		t	df	Sig.
		M	SD	M	SD			
The Same	1	49.47	2.094	57.876	2.812	-9.145	99	0
	2	9.6	0.699	20.9	3.178	-11.421	99	0
	3	0.649	0.079	0.294	0.182	5.79	99	0
	4	192.45	51.91	17.525	11.081	10.109	99	0
	5	14.777	0.425	12.815	0.305	10.399	99	0
Different	1	48.864	1.842	58.295	2.33	-8.232	99	0
	2	9.8	0.632	19	3.162	-8.575	99	0
	3	0.636	0.092	0.339	0.126	6.346	99	0
	4	5.119	1.17	0.658	0.517	10.083	99	0
	5	14.422	0.471	12.945	0.241	7.787	99	0
Customer's Priority	Objective Function	Scenario 1		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	49.47	2.094	53.656	2.299	-4.772	99	0.001
	2	9.6	0.699	24.1	2.685	-16.383	99	0
	3	0.649	0.079	0.14	0.06	16.021	99	0
	4	192.45	51.91	25.859	17.036	9.466	99	0
	5	14.777	0.425	12.913	0.347	15.839	99	0
Different	1	48.864	1.842	56.287	2.019	-12.01	99	0
	2	9.8	0.632	28.6	2.716	-25.324	99	0
	3	0.636	0.092	0.09	0.025	17.587	99	0
	4	5.119	1.17	0.387	0.244	12.119	99	0
	5	14.422	0.471	12.377	0.228	12.004	99	0
Customer's Priority	Objective Function	Scenario 2		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	57.876	2.812	53.656	2.299	3.51	99	0.007
	2	20.9	3.178	24.1	2.685	-2.569	99	0.03
	3	0.294	0.182	0.14	0.06	2.569	99	0.03
	4	17.525	11.081	25.859	17.036	-1.621	99	0.14
	5	12.815	0.305	12.913	0.347	-0.686	99	0.51
Different	1	58.295	2.33	56.287	2.019	1.75	99	0.114
	2	19	3.162	28.6	2.716	-6.499	99	0
	3	0.339	0.126	0.09	0.025	6.516	99	0
	4	0.658	0.517	0.387	0.244	1.342	99	0.213
	5	12.945	0.241	12.377	0.228	4.295	99	0.002
Customer's Priority	Objective Function	Scenario 2		Scenario 4		t	df	Sig.
		M	SD	M	SD			
The Same	1	57.876	2.812	57.76	2.22	0.106	99	0.918
	2	20.9	3.178	20.7	2.869	0.242	99	0.814
	3	0.294	0.182	0.259	0.099	0.895	99	0.394
	4	17.525	11.081	14.809	8.867	0.984	99	0.351
	5	12.815	0.305	12.797	0.3	0.288	99	0.78
Different	1	58.295	2.33	57.678	2.128	0.673	99	0.518
	2	19	3.162	19.6	1.776	-0.557	99	0.591
	3	0.339	0.126	0.284	0.087	1.245	99	0.245
	4	0.658	0.517	0.345	0.211	1.914	99	0.088
	5	12.945	0.241	12.825	0.214	1.095	99	0.302
Customer's Priority	Objective Function	Scenario 3		Scenario 5		t	df	Sig.
		M	SD	M	SD			
The Same	1	53.656	2.299	53.71	3.216	-0.036	99	0.972
	2	24.1	2.685	24.7	3.653	-0.37	99	0.72
	3	0.14	0.06	0.116	0.036	1.027	99	0.331
	4	25.859	17.036	31.494	19.388	-0.564	99	0.586
	5	12.913	0.347	12.793	0.525	0.532	99	0.608
Different	1	56.287	2.019	55.454	6.084	0.405	99	0.695
	2	28.6	2.716	29.3	4.762	-0.44	99	0.671
	3	0.09	0.025	0.082	0.027	0.828	99	0.429
	4	0.387	0.244	0.183	0.096	3.276	99	0.01
	5	12.377	0.228	12.337	0.221	0.639	99	0.539

TABLE 3 PAIRED T-TEST RESULTS FOR COMPARISON OF VARIOUS SCENARIOS USING THE THIRD CASE STUDY

Customer's Priority	Objective Function	Scenario 1		Scenario 2		t	df	Sig.
		M	SD	M	SD			
The Same	1	74.938	5.827	96.058	3.907	-7.58	99	0

	2	8.6	1.075	14.8	1.398	-10.463	99	0
	3	2.287	1.117	1.45	0.624	2.475	99	0.035
	4	93.083	33.203	5.557	5.703	8.557	99	0
	5	21.313	0.845	20.023	0.627	4.19	99	0.002
Different	1	72.844	4.031	90.48	23.474	-2.345	99	0.044
	2	8.1	0.876	14.3	2.83	-6.016	99	0
	3	1.773	0.717	1.367	0.827	1.104	99	0.298
	4	4.699	2.047	0.098	0.108	7.179	99	0
	5	21.282	1.02	19.317	1.989	2.868	99	0.019
Customer's Priority	Objective Function	Scenario 1		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	74.938	5.827	86.686	8.994	-2.946	99	0.016
	2	8.6	1.075	15.9	2.514	-6.661	99	0
	3	2.287	1.117	0.97	0.531	4.61	99	0.001
	4	93.083	33.203	30.421	15.686	5.641	99	0
	5	21.313	0.845	21.68	0.728	-0.957	99	0.363
Different	1	72.844	4.031	95.328	7.654	-7.574	99	0
	2	8.1	0.876	17.9	1.197	-25.21	99	0
	3	1.773	0.717	0.892	0.49	4.147	99	0.002
	4	4.699	2.047	0.181	0.153	7.061	99	0
	5	21.282	1.02	21.115	0.673	0.413	99	0.689
Customer's Priority	Objective Function	Scenario 2		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	96.058	3.907	86.686	8.994	3.515	99	0.007
	2	14.8	1.398	15.9	2.514	-1.16	99	0.276
	3	1.45	0.624	0.97	0.531	2.407	99	0.039
	4	5.557	5.703	30.421	15.686	-5.493	99	0
	5	20.023	0.627	21.68	0.728	-5.281	99	0.001
Different	1	90.48	23.474	95.328	7.654	-0.61	99	0.557
	2	14.3	2.83	17.9	1.197	-3.553	99	0.006
	3	1.367	0.827	0.892	0.49	1.871	99	0.094
	4	0.098	0.108	0.181	0.153	-2.624	99	0.028
	5	19.317	1.989	21.115	0.673	-2.731	99	0.023
Customer's Priority	Objective Function	Scenario 2		Scenario 4		t	df	Sig.
		M	SD	M	SD			
The Same	1	96.058	3.907	95.465	7.798	0.21	99	0.838
	2	14.8	1.398	15.1	1.853	-0.519	99	0.616
	3	1.45	0.624	1.552	0.946	-0.464	99	0.654
	4	5.557	5.703	5.328	7.44	0.068	99	0.948
	5	20.023	0.627	19.965	1.16	0.182	99	0.86
Different	1	90.48	23.474	93.88	6.561	-0.431	99	0.677
	2	14.3	2.83	14.4	1.578	-0.093	99	0.928
	3	1.367	0.827	1.397	0.291	-0.105	99	0.918
	4	0.098	0.108	0.117	0.063	-0.441	99	0.669
	5	19.317	1.989	20.165	0.943	-1.146	99	0.282
Customer's Priority	Objective Function	Scenario 3		Scenario 5		t	df	Sig.
		M	SD	M	SD			
The Same	1	86.686	8.994	84.327	8.001	0.648	99	0.533
	2	15.9	2.514	15.6	1.776	0.26	99	0.801
	3	0.97	0.531	0.993	0.567	-0.099	99	0.924
	4	30.421	15.686	30.458	14.653	-0.006	99	0.995
	5	21.68	0.728	21.297	1.493	0.646	99	0.534
Different	1	95.328	7.654	90.003	16.18	1.059	99	0.317
	2	17.9	1.197	16.7	2.908	1.177	99	0.269
	3	0.892	0.49	0.683	0.245	1.127	99	0.289
	4	0.181	0.153	0.179	0.19	0.02	99	0.985
	5	21.115	0.673	20.723	1.799	0.639	99	0.539

TABLE 4 PAIRED T-TEST RESULTS FOR COMPARISON OF VARIOUS SCENARIOS USING THE FOURTH CASE STUDY

Customer's Priority	Objective Function	Scenario 1		Scenario 2		t	df	Sig.
		M	SD	M	SD			
The Same	1	74.46	4.592	93.755	5.331	-10.819	99	0
	2	8.4	0.699	14.8	1.874	-10.352	99	0
	3	1.759	0.713	1.517	0.474	0.813	99	0.437
	4	1.857	1.295	0.024	0.021	4.478	99	0.002
	5	20.585	0.701	20.072	0.691	1.778	99	0.109
Different	1	115.318	7.01	134.01	6.632	-5.317	99	0
	2	14.6	1.713	21.8	1.317	-9.9	99	0

	3	1.539	0.619	0.74	0.531	2.836	99	0.02
	4	3.02	1.227	1.484	0.337	3.791	99	0.004
	5	21.248	1.312	19.855	0.765	2.777	99	0.022
Customer's Priority	Objective Function	Scenario 1		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	74.46	4.592	96.514	4.645	-9.927	99	0
	2	8.4	0.699	17.7	0.823	-25.364	99	0
	3	1.759	0.713	0.717	0.226	4.568	99	0.001
	4	1.857	1.295	0.031	0.025	4.469	99	0.002
	5	20.585	0.701	21.053	0.79	-1.401	99	0.195
Different	1	115.318	7.01	90.81	17.495	3.983	99	0.003
	2	14.6	1.713	18.5	4.197	-2.992	99	0.015
	3	1.539	0.619	0.765	0.603	3.445	99	0.007
	4	3.02	1.227	29.215	18.281	-4.448	99	0.002
	5	21.248	1.312	20.307	2.26	1.075	99	0.31
Customer's Priority	Objective Function	Scenario 2		Scenario 3		t	df	Sig.
		M	SD	M	SD			
The Same	1	93.755	5.331	96.514	4.645	-1.771	99	0.11
	2	14.8	1.874	17.7	0.823	-4.411	99	0.002
	3	1.517	0.474	0.717	0.226	4.648	99	0.001
	4	0.024	0.021	0.031	0.025	-0.726	99	0.486
	5	20.072	0.691	21.053	0.79	-3.048	99	0.014
Different	1	134.01	6.632	90.81	17.495	8.41	99	0
	2	21.8	1.317	18.5	4.197	2.684	99	0.025
	3	0.74	0.531	0.765	0.603	-0.133	99	0.897
	4	1.484	0.337	29.215	18.281	-4.839	99	0.001
	5	19.855	0.765	20.307	2.26	-0.592	99	0.568
Customer's Priority	Objective Function	Scenario 2		Scenario 4		t	df	Sig.
		M	SD	M	SD			
The Same	1	93.755	5.331	95.264	5.315	-0.687	99	0.51
	2	14.8	1.874	14.5	1.65	0.361	99	0.726
	3	1.517	0.474	1.422	0.582	0.327	99	0.751
	4	0.024	0.021	0.019	0.019	0.454	99	0.66
	5	20.072	0.691	19.807	0.261	1.404	99	0.194
Different	1	134.01	6.632	131.50	6.364	0.819	99	0.434
	2	21.8	1.317	21.2	1.317	0.97	99	0.357
	3	0.74	0.531	1.039	0.34	-1.865	99	0.095
	4	1.484	0.337	2.042	1.735	-0.969	99	0.358
	5	19.855	0.765	19.783	0.987	0.168	99	0.871
Customer's Priority	Objective Function	Scenario 3		Scenario 5		t	df	Sig.
		M	SD	M	SD			
The Same	1	96.514	4.645	92.903	6.464	1.319	99	0.22
	2	17.7	0.823	17	1.491	1.769	99	0.111
	3	0.717	0.226	0.904	0.383	-1.345	99	0.211
	4	0.031	0.025	0.908	0.473	-5.64	99	0
	5	21.053	0.79	21.552	0.882	-1.055	99	0.319
Different	1	90.81	17.495	93.136	18.42	-0.323	99	0.754
	2	18.5	4.197	18.5	3.44	0	99	1
	3	0.765	0.603	5.45	9.762	-1.533	99	0.16
	4	29.215	18.281	23.909	15.933	0.655	99	0.529
	5	20.307	2.26	20.698	2.458	-0.458	99	0.658

Usually, the more information we have, the more accurate is the solution an algorithm can provide. However, for the real-time multi-objective VRP, the results obtained from the four case studies show that even when information such as customer's demands and travel time are missing, the results of the algorithm are as good to the results of the algorithm when all information is known in advance. Similar results were obtained for both the SPEA2 and VE-ABC algorithms *(72)*.

CONCLUSIONS AND SUMMARY

The problem considered in this research is the Real-Time Multi-Objective VRP. The Real-Time Multi-Objective VRP is defined as a vehicle fleet that has to serve unknown number customers of fixed demands from a central depot. Customers must be assigned to vehicles, and the vehicles routed so that a number of objectives are minimized/maximized. The travel time between two customers or a customer and the depot depends on the distance between the points and the time of day, and it also has stochastic properties.

This research attempts to adjust the vehicles' routes at certain times in a planning period. This adjustment considers new information about the travel times, current location of vehicles, and new demand requests (that can be deleted after being served, or added since they arise after the initial service began) and more. This result in a dynamic change in the demand and traveling time information as time changes, which has to be taken into consideration in order to provide optimized real-time operation of vehicles.

According to the vast literature review, five objectives were addressed: (1) Minimizing the total traveling time, (2) Minimizing the number of vehicles, (3) Maximizing customers' satisfaction, (4) Maximizing drivers' satisfaction, and (5) Minimizing the arrival time of the last vehicle.

The problem was formulated as a mixed integer programming problem on a network and three evolutionary algorithms for solving it were described: (1) an improved version of the VEGA algorithm, (2) the SPEA2 algorithm, and (3) VE-ABC algorithm.

Usually, the more information we have, the more accurate is the solution an algorithm can provide. However, for the real-time multi-objective VRP, using the improved VEGA algorithm, it was shown that the results obtained from the four case studies, even when information such as customer's demands and travel time are missing, are as good as to the results of the algorithm when all information is known in advance. Similar results were obtained for both the SPEA2 and VE-ABC algorithms, however they were not presented in this paper.

Although the proposed solution algorithm works well for the real-time multi-objective vehicle routing problem, there are several fruitful avenues for future research, such as: (1) using parallel algorithms, (2) modifications to the improved VEGA algorithm, in which the sub-populations for each objective function, are not equal in size, (3) using other fitness functions, and (4) comparison to other real-world networks.

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