ESSEC

BAR-ILAN UNIVERSITY

Department of Operations Management

Topics in Supply Chain Management

Session 5

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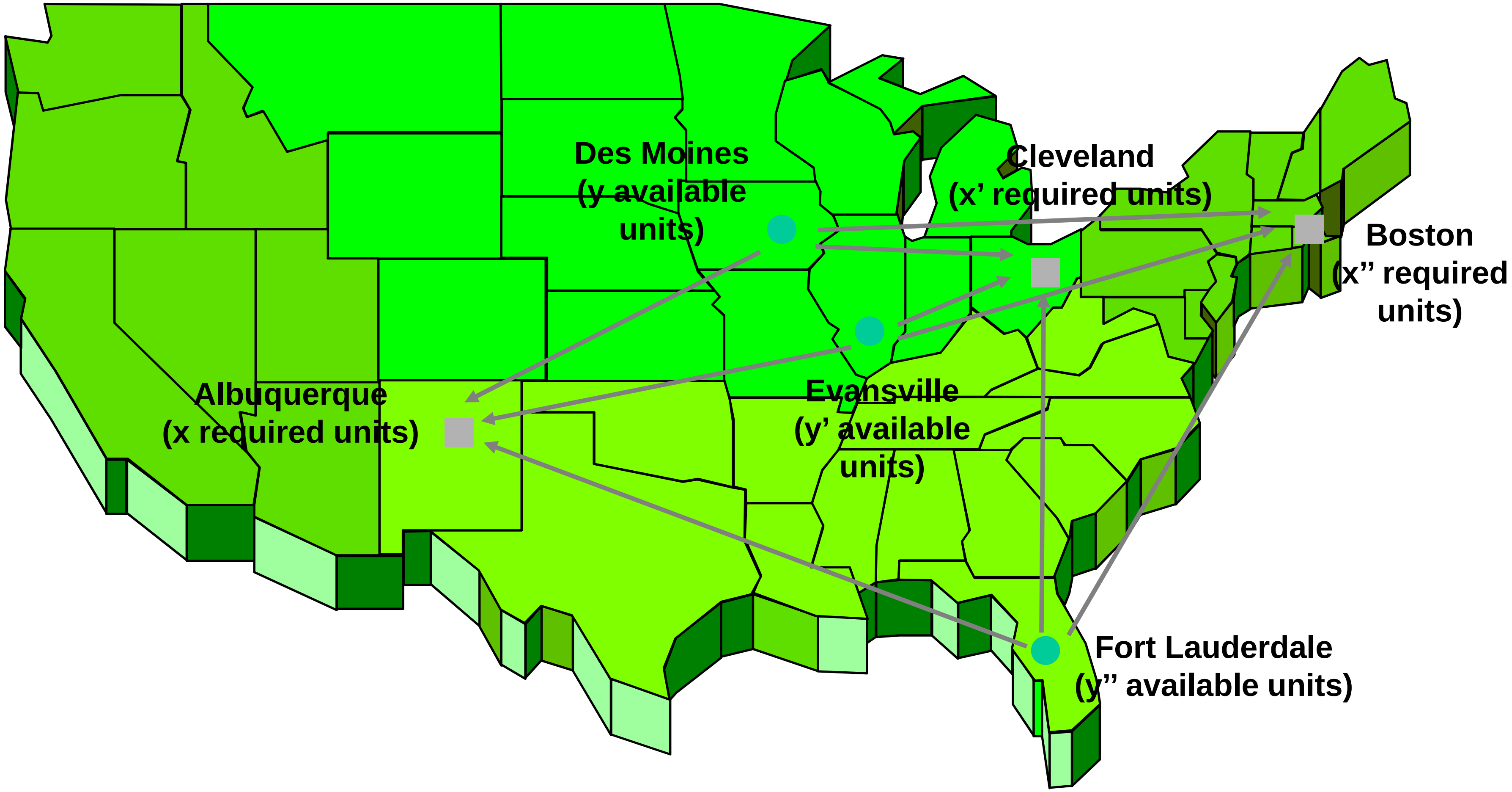
THE DESIGN OF A NETWORK IN A SUPPLY CHAIN

AGENDA

- THE ISSUE
- THE TRANSPORTATION PROBLEM
- WHERE TO LOCATE?
- GEOGRAPHICAL LOCATION AND PRODUCTION SCALE
- THE TRANS-SHIPMENT PROBLEM
- THE DESIGN OF A NETWORK IN A DYNAMIC SUPPLY CHAIN

ISSUE



HOW TO SUPPLY MANY MARKETS FROM MANY FACTORIES AT THE LEAST TOTAL TRANSPORTATION COST?

THE TRANSPORTATION PROBLEM

Consider n factories supplying m markets, with n = 3 and m = 4, X_{ij} being the quantity supplied from factory i to market j (i = 1, 2, 3 and j = 1, 2, 3, 4), and C_{ij} the transportation cost per unit from factory i to market j ($) such that:

$$C_{11} = 25 \; ; \; C_{12} = 30 \; ; \; C_{13} = 20 \; ; \; C_{14} = 40 \; ;$$

$$C_{21} = 30 \; ; \; C_{22} = 25 \; ; \; C_{23} = 20 \; ; \; C_{24} = 30 \; ;$$

$$C_{31} = 40 \; ; \; C_{32} = 20 \; ; \; C_{33} = 40 \; ; \; C_{34} = 35.$$

In matrix form, we get:

Transportation cost per unit from F_1 to M_1

Markets **Factories**	**M_1**	**M_2**	**M_3**	**M_4**	**Supply**
F_1	25	30	20	40	**37**
F_2	30	25	20	30	**22**
F_3	40	20	40	35	**32**
Demand	**25**	**20**	**25**	**21**	**91**

Equilibrium constraint : Supply = Demand

LINEAR PROGRAMMING FORMULATION

$$\text{Min } Z = \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}X_{ij}$$

such that:

$$\sum_{j=1}^{4} X_{1j} = 37$$

$$\sum_{j=1}^{4} X_{2j} = 22$$

$$\sum_{j=1}^{4} X_{3j} = 32$$

$$\sum_{i=1}^{3} X_{i1} = 25$$

$$\sum_{i=1}^{3} X_{i2} = 20$$

$$\sum_{i=1}^{3} X_{i3} = 25$$

$$\sum_{i=1}^{3} X_{i4} = 21$$

$$X_{ij} \geq 0,$$

$$\forall\, i = 1, 2, 3, \quad \forall\, j = 1, 2, 3, 4.$$

Resolution Method

STEPPING-STONE ALGORITHM

First step: Find the initial solution

To determine a feasible initial solution, we use the heuristics of the minimum cost (i.e., saturate the constraints using the criterion of the minimum unit cost).

Markets **Factories**	**M_1**	**M_2**	**M_3**	**M_4**	**Supply**
F_1	25 **12**	30	20 **25**	40	**37**
F_2	30 **2**	25 **20**	20	30	**22**
F_3	40 **11**	20	40	35 **21**	**32**
Demand	**25**	**20**	**25**	**21**	**91**

that is:

$X_{11} = 12$; $X_{12} = 0$; $X_{13} = 25$; $X_{14} = 0$;

$X_{21} = 2$; $X_{22} = 20$; $X_{23} = 0$; $X_{24} = 0$;

$X_{31} = 11$; $X_{32} = 0$; $X_{33} = 0$; $X_{34} = 21$.

Remark: the feasible initial solution must be such that the number of crossings effectively used (i.e., strictly positive X_{ij}) is equal to:

$$n + m – 1 = 3 + 4 – 1 = 6,$$

or such that the number of unused crossings (i.e., zero X_{ij}) is equal to :

$$nm – (n + m - 1) = (m – 1)(n – 1) = (4 – 1)(3 – 1) = 6.$$

Initial total transportation cost

$Z_{\text{initial}} = 25 \times 12 + 20 \times 25 + 30 \times 2 + 25 \times 20 + 40 \times 11 + 35 \times 21 =$ **$ 2 535**

Second step: Compute the opportunity cost related to each unused crossing

Process:

A. Put one unit of item on an unused crossing

➔ (for example the crossing 3,2);

B. Make the required adjustments to fulfill the constraints

➔ (withdraw one unit to the quantities on the crossings 2,2 and 3,1, and add one unit to the quantities on the crossing 2,1);

C. Compute the reduced cost associated to this operation

➔ (by weighing each addition and each withdrawal by the corresponding cost).

Computation of the reduced costs

Markets Factories	M_1	M_2	M_3	M_4	Supply
F_1	25 **12**	30	20 **25**	40	**37**
F_2	30 +1 **2**	25 -1 **20**	20	30	**22**
F_3	40 -1 **11**	20 +1	40	35 **21**	**32**
Demand	**25**	**20**	**25**	**21**	**91**

transportation circuit associated to Δ_{32}

Unused crossing

On the whole, we should compute 6 reduced costs (one for each unused crossing), that is:

Δ_{12}= + 1 x 30 – 1 x 25 + 1 x 30 – 1 x 25 = 10 > 0,

Δ_{14}= 40 – 35 + 40 – 25 = 20 > 0 (potential cost related to the use of the unused crossing 1,4),

Δ_{23} = 20 – 20 + 25 – 30 = - 5 < 0 (potential benefit related to the use of the unused crossing 2,3),

Δ_{24} = 30 – 35 + 40 – 30 = 5 > 0,

Δ_{32} = 20 – 25 + 30 – 40 = - 15 < 0,

Δ_{33} = 40 – 20 + 25 – 40 = 5 > 0.

Third step: Improve the initial solution

1 – Determine the maximum potential bénéfit:

$$\text{Min } \{\Delta_{23}, \Delta_{32}\} = \Delta_{32}$$

$$\Delta_{32} = C_{32} – C_{22} + C_{21} – C_{31} = -15.$$

2 – Modify the transportation circuit associated to Δ_{32}:

$$X_{21} + \text{Min } (X_{22}, X_{31}) = 2 + 11 = 13,$$

$$X_{22} – \text{Min } (X_{22}, X_{31}) = 20 – 11 = 9,$$

$$X_{32} + \text{Min } (X_{22}, X_{31}) = 0 + 11 = 11,$$

$$X_{31} – \text{Min } (X_{22}, X_{31}) = 11 – 11 = 0.$$

Improved transportation network

13 = 2 + 11

9 = 20 - 11

Markets **Factories**	**M_1**	**M_2**	**M_3**	**M_4**	**Supply**
F_1	25 **12**	30	20 **25**	40	**37**
F_2	30 **13**	25 **9**	20	30	**22**
F_3	40	20 **11**	40	35 **21**	**32**
Demand	**25**	**20**	**25**	**21**	**91**

0 = 11 - 11

11 = 0 + 11

Total transportation cost after the first iteration :

$ 2 370

(that is, 6,5% less than the initial solution)

Fourth step: Iterate the second and third step

Optimality criterion: all the reduced cost are non-negative (i.e., there exists no potential benefit corresponding to a modification of the transportation network)

OPTIMAL TRANSPORTATION NETWORK

Markets Factories	M_1	M_2	M_3	M_4	Supply
F_1	25 **25**	30	20 **12**	40	**37**
F_2	30	25	20 **13**	30 **9**	**22**
F_3	40	20 **20**	40	35 **12**	**32**
Demand	**25**	**20**	**25**	**21**	**91**

Optimal total transportation cost:

$ 2 215

(that is, 12,6% less than the initial solution)

Degenerate solution

First example: consider the following initial matrix

Markets **Factories**	$\mathbf{M_1}$	$\mathbf{M_2}$	$\mathbf{M_3}$	**Supply**
$\mathbf{F_1}$	3 **35**	6 **25**	7	**60**
$\mathbf{F_2}$	8	5 **30**	7	**30**
$\mathbf{F_3}$	4	9	11 **30**	**30**
Demand	**35**	**55**	**30**	**120**

Number of crossings effectively used:

$$4 < n + m - 1 = 6 - 1 = 5,$$

➔ the initial feasible solution is *degenerate.*

Initial total transportation cost:

$ 735

Resolution process (1/2)

1 – Create an *artificial* effectively used crossing*:*

e.g., crossing (2,3)

Artificial Cell

Markets Factories	M_1	M_2	M_3	Supply
F_1	3 **35**	6 **25**	7	**60**
F_2	8	5 **30**	7 **0**	**30**
F_3	4	9	11 **30**	**30**
Demand	**35**	**55**	**30**	**120**

2 – Compute the remaining reduced costs

Markets Factories	M_1	M_2	M_3	Supply
F_1	3 - **35**	6 + **25**	7	**60**
F_2	8 ⑥	5 - **30**	7 + **0**	**30**
F_3	4 (- 2) +	9 ⓪	11 - **30**	**30**
Demand	**35**	**55**	**30**	**120**

Resolution process (2/2)

Minimum reduced cost: $\Delta_{31} = C_{31} – C_{11} + C_{12} – C_{22} + C_{23} – C_{33} = - 2$

Corresponding improvement:

$$X_{11} + \text{Min} (X_{11},X_{22}, X_{33}) = 35 – 30 = 5,$$

$$X_{12} + \text{Min} (X_{11},X_{22}, X_{33}) = 25 + 30 = 55,$$

$$X_{22} - \text{Min} (X_{11},X_{22}, X_{33}) = 30 – 30 = 0,$$

$$X_{23} - \text{Min} (X_{11},X_{22}, X_{33}) = 0 + 30 = 30,$$

$$X_{33} - \text{Min} (X_{11},X_{22}, X_{33}) = 30 – 30 = 0,$$

$$X_{31} - \text{Min} (X_{11},X_{22}, X_{33}) = 0 + 30 = 30,$$

➔ the solution obtained after the first iteration is also degenerate.

➔Iteration of 1. and 2.

Optimal solution

Marchés **Usines**	**M_1**	**M_2**	**M_3**	**Q$^{\text{tés}}$ Disponibles**
F_1	3 **5**	6 **25**	7 **30**	**60**
F_2	8	5 **30**	7	**30**
F_3	4 **30**	9	11	**30**
Q$^{\text{tés}}$ Requises	**35**	**55**	**30**	**120**

➔ the optimal solution is not degenerate.

Optimal total cost:
$ 645
(that is, 12,25% less than the initial solution)

Degenerate solution

Second example: consider the following initial matrix

Markets / Factories	M_1	M_2	M_3	Supply
F_1	4 **100**	10	6	**100**
F_2	8 **100**	16	6 **200**	**300**
F_3	14	18 **300**	10	**300**
Demand	**200**	**300**	**200**	**700**

Number of crossings effectively used:

4 < n + m – 1 = 6 – 1 = 5,

➔ the initial feasible solution is *degenerate.*

Initial total transportation cost:
$ 7 800

1 – Create an *artificial* effectively used crossing:

e.g., crossing (3,1)

Markets Factories	M_1	M_2	M_3	Supply
F_1	4 **100**	10	6	**100**
F_2	8 **100**	16	6 **200**	**300**
F_3	14 **0**	18 **300**	10	**300**
Demand	**200**	**300**	**200**	**700**

Artificial cell

2 – Compute the remaining reduced costs:

Markets Factories	M_1	M_2	M_3	Supply
F_1	4 **100**	10 ②	6 ④	**100**
F_2	8 + **100**	16 ④	6 - **200**	**300**
F_3	14 - **0**	18 **300**	10 (-2) +	**300**
Demand	**200**	**300**	**200**	**700**

Minimum reduced cost: $\Delta_{33} = C_{33} – C_{31} + C_{21} – C_{23} = -2$

Corresponding improvement:

X_{33} + Min (X_{23}, X_{31}) = 0 + 0 = 0,

X_{31} – Min (X_{23}, X_{31}) = 0 – 0 = 0,

X_{21} + Min (X_{23}, X_{31}) = 100 + 0 = 100,

X_{23} – Min (X_{23}, X_{31}) = 200 – 0 = 200,

➔ the degenerate initial solution is optimal.

Optimal total cost:
$ 7 800

WHERE TO LOCATE?

The market demand of the Williams Company has recently increased significantly. In order to supply its warehouses in Los Angeles and New York, the Williams Company must decide where to locate its new factory. Two options are considered: either New Orleans or Houston. The unit production and distribution costs, the production capacities and sales are reported in the following table.

	Warehouses		Production Capacity	Production Unit Cost
	Los Angeles	New York		
Existing factories:				
Atlanta	$ 8	$ 5	600	$ 6
Tulsa	$ 4	$ 7	900	$ 5
Potential locations:				
New Orleans	$ 5	$ 6	500	$ 4*
Houston	$ 4	$ 6	500	$ 3*
Sales (forecast)	800	1 200	2 000	

*: Estimates.

Which location should be preferred for the new factory?

Example (continued)

Optimal transportation network associated with a location in New Orleans

Destination Source	Los Angeles	New York	Supply
Atlanta	$ 14	$ 11 **600**	**600**
Tulsa	$ 9 **800**	$ 12 **100**	**900**
New Orleans	$ 9	$ 10 **500**	**500**
Demand	**800**	**1 200**	**2 000**

Total distribution cost: $ 20 000

Optimal location

Optimal transportation network associated with a location in Houston

Destination Source	Los Angeles	New York	Supply
Atlanta	$14	$11 **600**	**600**
Tulsa	$9 **800**	$12 **100**	**900**
Houston	$7	$9 **500**	**500**
Demand	**800**	**1 200**	**2 000**

Total distribution cost: $ 19 500

Min [$ 20 000; $ 19 500] = $ 19 500
⇒ the optimal location is Houston.

GEOGRAPHICAL LOCATION AND PRODUCTION SCALE

SunOil, a world company of the petrochimical sector must determine its international location strategy.

One possibility would be to implement one factory close to each market. The benefit would lie in lower transportation cost, and importation taxes. The problem would be that the size of each factory would only depend on the local demand, which might not allow for an efficient exploitation of the economies of scale.

Another possibility would lead to implement larger factories in a limited number of areas. This would allow for an efficient exploitation of the economies of scale but would also increase the transportation cost, and importation taxes, if any.

SunOil would like to optimize its trade-off on these quantitative decision criteria, along with that related to non-quantitative criteria, such as the competitive environment and the political risk.

Data

	Markets Variable costs in k$ (production, inventory, transportation and taxes/million of units)					Fixed costs ($)	Lower production scale	Fixed costs ($)	Higher production scale
Factories	**North America**	**South America**	**Europe**	**Asia**	**Africa**				
North America	81	92	101	130	115	**6 000**	**10**	**9 000**	**20**
South America	117	77	108	98	100	**4 500**	**10**	**6 750**	**20**
Europe	102	105	95	119	111	**6 500**	**10**	**9 750**	**20**
Asia	115	125	90	59	74	**4 100**	**10**	**6 150**	**20**
Africa	142	100	103	105	71	**4 000**	**10**	**6 000**	**20**
Demand	**12**	**8**	**14**	**16**	**7**				

Linear programming problem

We define the following notations:

n = number of potential locations

m = number of markets

D_j = demand/year of market j

K_i = potential production capacity of factory i

f_j = fixed cost/year related to the implementation of factory i

c_{ij} = variable cost of supplying one product unit from factory i to market j

We seek to design a network that minimizes the total cost of fulfillment of the total demand.

Define the following decision variables:

y_i = 1 if factory i is implemented, 0 in the converse case

x_{ij} = quantity supplied from factory i to market j

The problem is formulated as a mixed-integer programming problem, that is:

$$Min \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ji} x_{ij}$$

such that:

$$\sum_{i=1}^{n} x_{ij} = D_i \,, j = 1, \dots, m \qquad (eq.1)$$

$$\sum_{j=1}^{m} x_{ij} \leq K_i y_i \,, i = 1, \dots, n \qquad (eq.2)$$

$$y_i \in [0,1], i = 1, \dots, n$$

Excel spreadsheet

	A	B	C	D	E	F	G	H	I	J
	Data	Markets Variable costs in k$ (production, inventory, transportation and taxes/million of units)					Fixed costs ($)	Lower production scale	Fixed costs ($)	Higher production scale
	Factories	North America	South America	Europe	Asia	Africa				
4	North America	81	92	101	130	115	6 000	10	9 000	20
5	South America	117	77	108	98	100	4 500	10	6 750	20
6	Europe	102	105	95	119	111	6 500	10	9 750	20
7	Asia	115	125	90	59	74	4 100	10	6 150	20
8	Africa	142	100	103	105	71	4 000	10	6 000	20
9	Demand	12	8	14	16	7	4 000	10	6 000	20
	Decision variables	Markets (k units)					Factories (1 = open)	Factories (1 = open)		
		North America	South America	Europe	Asia	Africa				
14	North America	0	0	0	0	0	0	0		
15	South America	0	0	0	0	0	0	0		
16	Europe	0	0	0	0	0	0	0		
17	Asia	0	0	0	0	0	0	0		
18	Africa	0	0	0	0	0	0	0		
	Constraints									
	Area of production	*Overcapacity*								
22	North America	0								
23	South America	0								
24	Europe	0								
25	Asia	0								
26	Africa	0								
		North America	South America	Europe	Asia	Africa				
28	*Unsupplied demand*	12	8	14	16	7				
	Objective function									
31	Total cost	$ -								

Cell	Formula	Equation	Copied
B28	**=B9-SUM(B14:B18)**	eq.1	B28:F28
B22	**=G14*H4+H14*J4-SUM(B14:F14)**	eq. 2	B22:B26
B31	**=SUMPRODUCT(B14:F18,B4:F8)+ SUMPRODUCT(G14:G18,G4:G8)+ SUMPRODUCT(H14:H18,I4:I8)**	Objective	-

Solver:

1. **Target cell : *$B$31***
2. **Equal to : *Min***
3. **By changing : *$B$14:$H$18***
4. **Constraints ⇒ Add**
5. **$B$14:$H$18≥0 ⇒ Add**
6. **$B$22:$B$26≥0 ⇒ Add**
7. **$B$28:$F$28=0 ⇒ Add**
8. **$G$14:$H$18=binary ⇒ OK**
9. **Solve**

Optimal location and production scale

	A	B	C	D	E	F	G	H		
	Decision variables	Markets (in k units)					Factories (1 = open)	Factories (1 = open)		
		North America	South America	Europe	Asia	Africa				
14	North America	0	0	0	0	0	0	0		
15	South America	**12**	**8**	0	0	0	0	**1**		
16	Europe	0	0	0	0	0	0	0		
17	Asia	0	0	**4**	**16**	0	0	**1**		
18	Africa	0	0	**10**	0	**7**	0	**1**		
	Constraints									
	Area of production	*Overcapacity*								
22	North America	0								
23	South America	0								
24	Europe	0								
25	Asia	0								
26	Africa	**3**								
		North America	South America	Europe	Asia	Africa				
28	*Unsupplied demand*	0	0	0	0	0				
	Objective Function									
31	Total Cost	**$ 23 751**								

THE TRANS-SHIPMENT PROBLEM

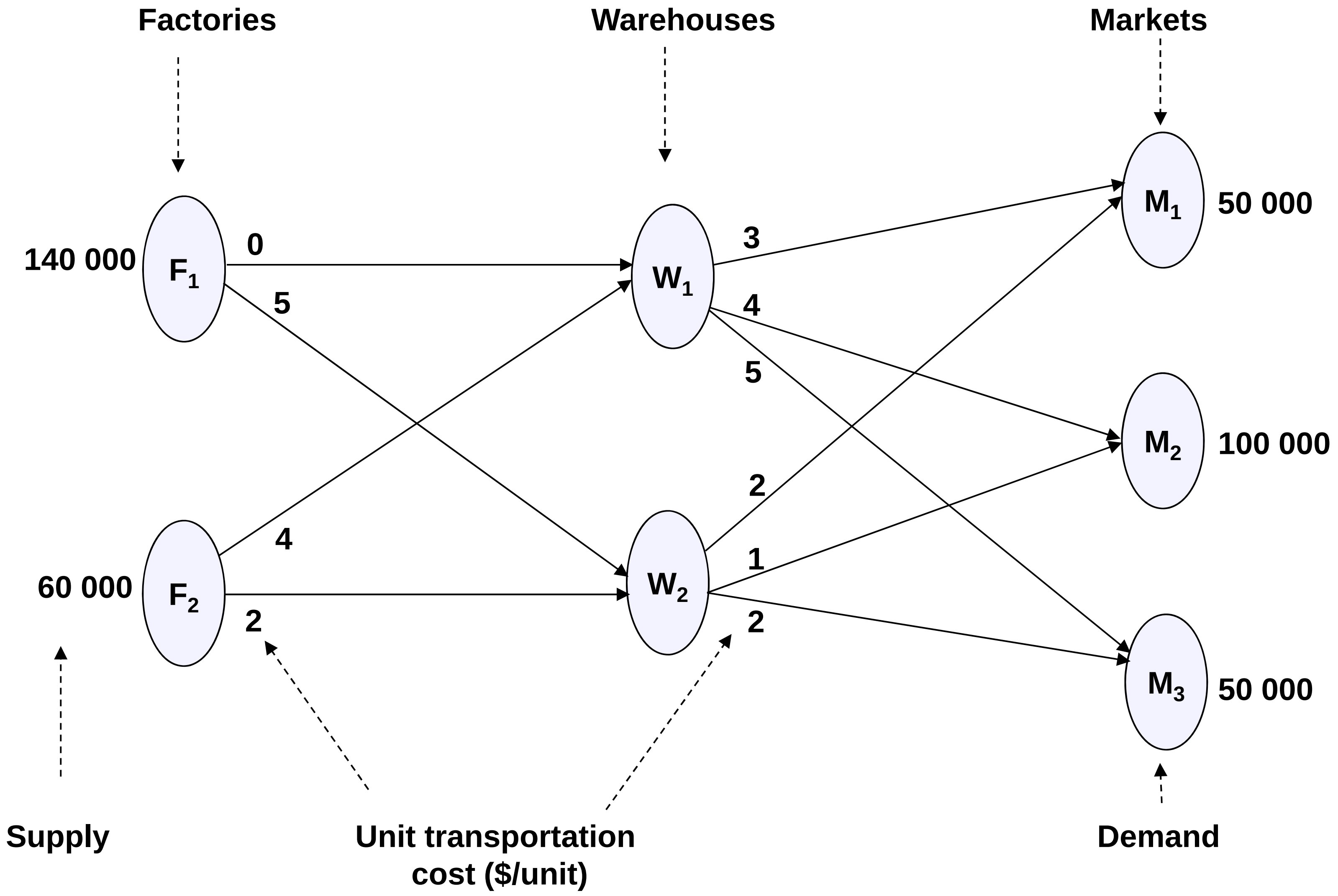
➔ Here, we take into account i*ntermediate* locations between factories and markets, that is, warehouses.

➔ Potential use of multiples crossings, that is:

- *From one factory to another,*
- *From one market to another,*
- *From a factory to a market via a warehouse* (*cross-docking*),
- *From one warehouse to another,*
- *From a factory directly to a market.*

Example

Consider 2 factories, 2 warehouses et 3 markets. The factories are supposed to have similar unit production costs. Also, the warehouses are supposed to have similar unit inventory holding costs.



Heuristic resolution procedure

The objective here is to determine a trans-shipment strategy specifying the flow of products going from the factories to the markets via the warehouses.

To determine an efficient trans-shipment strategy, it is possible to use one of the two following heuristic procedures.

Heuristic procedure 1.

For each market, choose the cheapest warehouse-market path. In this respect, M_1, M_2 and M_3 must be supplied by W_2. Therefore, for this trans-shipment point, it is then sufficient to choose the cheapest factory, which gives 60 000 units from factory F_2 and 140 000 units from factory F_1. The total cost corresponding to this procedure is:

$$2 \text{ x } 50\,000 + 1 \text{ x } 100\,000 + 2 \text{ x } 50\,000 + 2 \text{ x } 60\,000 + 5 \text{ x } 140\,000 = \$\ 1\,120\,000.$$

Heuristic procedure 2.

For each market, choose the path crossing the cheapest trans-shipment point. For market M_1, compare the cost of the paths $F_1 \rightarrow W_1 \rightarrow M_1$, $F_1 \rightarrow W_2 \rightarrow M_1$, $F_2 \rightarrow W_1 \rightarrow M_1$ et $F_2 \rightarrow W_2 \rightarrow M_1$. The cheapest is $F_1 \rightarrow W_1 \rightarrow M_1$, which leads to choose W_1 for M_1. A similar reasoning leads to choose W_2 for M_2 and W_2 for M_3. The best trans-shipment organization leads then to supply 50 000 units from factory F_1 to warehouse W_1, 60 000 units from factory F_2 to warehouse W_2 and 90 000 units from factory F_1 to warehouse W_2. The total cost corresponding to this procedure is $ 920 000.

However, neither of these two procedures is optimal. The use of linear programming is then necessary.

LINEAR PROGRAMMING FORMULATION

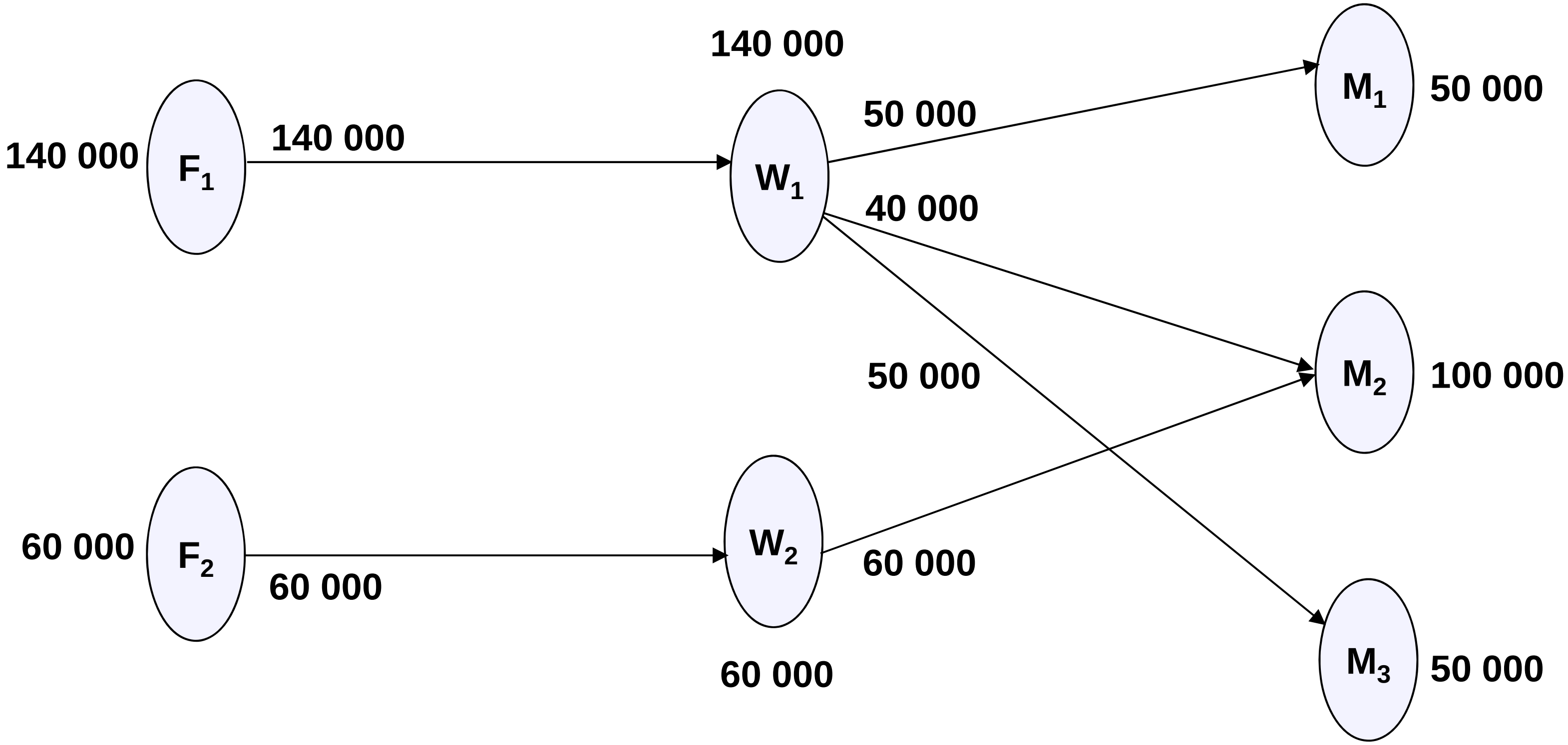
$$\text{Min } Z = 0X_{F1W1} + 5X_{F1W2} + 4X_{F2W1} + 2X_{F2W2}$$
$$+ 3X_{W1M1} + 4X_{W1M2} + 5X_{W1M3}$$
$$+ 2X_{W2M1} + 1X_{W2M2} + 2X_{W2M3}$$

such that:

$$X_{F1W1} + X_{F1W2} <= 140\ 000,$$
$$X_{F2W1} + X_{F2W2} <= 60\ 000,$$
$$X_{F1W1} + X_{F2W1} = X_{W1M1} + X_{W1M2} + X_{W1M3}$$
$$X_{F1W2} + X_{F2W2} = X_{W2M1} + X_{W2M2} + X_{W2M3}$$
$$X_{W1M1} + X_{W2M1} = 50\ 000,$$
$$X_{W1M2} + X_{W2M2} = 100\ 000,$$
$$X_{W1M3} + X_{W2M3} = 50\ 000.$$

$$X_{ij} \geq 0,$$
$$i = F_1, F_2, W_1, W_2,$$
$$j = W_1, W_2, M_1, M_2, M_3, i \neq j.$$

Optimal trans-shipment network



Total cost of the optimal trans-shipment network:

$ 740 000

(that is, 20 % less than the second heuristics)

Remark: the storage capacity of the warehouses is determined *ex post*.

THE DESIGN OF A NETWORK IN A DYNAMIC SUPPLY CHAIN

Integration in a *dynamic setting* of the following activities:

- *Production,*
- *Inventory,*
- *Transportation,*
- *Distribution.*

Example

Consider 3 factories and 4 markets over 2 time periods (t_1 and t_2).

Production capacity and demand

	Production capacity (units/period)	Demand (units)	
		t_1	t_2
	-	M_1 : 700	M_1 : 1 000
	F_1 : 850	M_2 : 100	M_2 : 250
	F_2 : 400	M_3 : 300	M_3 : 200
	F_3 : 250	M_4 : 300	M_4 : 150
Total	1 500	1 400	1 600

Transportation costs ($/unit)

Warehouses Factories	M_1	M_2	M_3	M_4
F_1	64	50	77	14
F_2	37	20	48	24
F_3	25	14	15	48

Production costs and sales price

Production costs ($/unit)	Sales Price ($/unit)
-	M_1 : 1 200
F_1 : 1 000	M_2 : 1 150
F_2 : 1 100	M_3 : 1 000
F_3 : 950	M_4 : 1 100

Inventory holding cost/unit : $ 5

Late delivery penalty/unit : $ 1 000

Unit profits Matrix

1200 – 1000 – 64 = 136

		M_1		M_2		M_3		M_4		Supply
		t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	
F_1	t_1	136	131	100	95	- 77	- 82	86	81	850
	t_2	- 864	136	- 900	100	-1077	- 77	- 914	86	850
F_2	t_1	63	58	30	25	- 148	- 153	- 24	-29	400
	t_2	- 937	63	-970	30	- 1148	-148	- 1024	-24	400
F_3	t_1	225	220	186	181	35	30	102	97	250
	t_2	- 775	225	- 814	186	- 965	35	- 898	102	250
Demand		700	1 000	100	250	300	200	300	150	3 000

1150 – 1100 – 20 – 5 = 25

1100 – 950 – 48 – 1000 = – 898

We seek to maximize the total profit rather than to minimize the total cost

➔ We then compute the reduced profits instead of the reduced costs

Optimal supply chain network

		M_1		M_2		M_3		M_4		Supply
		t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2	
F_1	t_1	550						300		850
	t_2		700						150	850
F_2	t_1	150	100	100		50				400
	t_2		150		250					400
F_3	t_1					250				250
	t_2		50				200			250
Demand		700	1 000	100	250	300	200	300	150	3 000

Total profit:
$ 263 500

Optimal production, inventory and distribution

	Sold production		Inventory between $t_1$and t_2
	t_1	t_2	
F_1	850	850	0
F_2	300	400	100
F_3	250	250	0
Total	1 400	1 500	100

➔ **Maximum utilization of the production capacity,**

➔ **Maximum coverage of the market.**

Remark

Given the existence of negative unit profits, it might be more profitable:

- Not to use the whole production capacity,
- And/or not to supply all the markets.

To check these assumptions, we introduce a fictitious factory and a fictitious market that will respectively serve to:

- Produce (at no cost) for the unprofitable markets, and
- To absorb (at no cost) the production of inefficient factories.

Unit profits matrix

Fictitious market

		M_1		M_2		M_3		M_4		M_F	Supply
		t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2		
F_1	t_1	136	131	100	95	- 77	- 82	86	81	0	850
	t_2	- 864	136	- 900	100	-1077	- 77	- 914	86	0	850
F_2	t_1	63	58	30	25	- 148	- 153	- 24	-29	0	400
	t_2	- 937	63	-970	30	- 1148	-148	- 1024	-24	0	400
F_3	t_1	225	220	186	181	35	30	102	97	0	250
	t_2	- 775	225	- 814	186	- 965	35	- 898	102	0	250
F_F		0	0	0	0	0	0	0	0	0	3000
Demand		700	1 000	100	250	300	200	300	150	3000	6000

Fictitious factory

Absorption capacity
sufficiently large to
absorb all the production

Production capacity
sufficiently large to supply
all the markets

Optimal supply chain network

		M_1		M_2		M_3		M_4		M_F	Supply
		t_1	t_2	t_1	t_2	t_1	t_2	t_1	t_2		
F_1	t_1	450		100				300			850
	t_2		700						150		850
F_2	t_1									400	400
	t_2		50		250					100	400
F_3	t_1	250									250
	t_2		250								250
F_F						300	200			2500	3000
Demand		700	1000	100	250	300	200	300	150	3000	3000

Total profit:
328 250 $
(= 1,25 x **263 500 $**)

Optimal Production, inventory and distribution

Optimal policy for production and inventory

	Utilization rate of the production capacity		Inventory between $t_1$and t_2
	t_1	t_2	
F_1	100%	100%	0
F_2	0%	75%	0
F_3	100%	100%	0
Total	≅ 74%	94 %	0

➔ *Reduce or reallocate part of the production capacity of F_2*

Optimal distribution network

	Market coverage rate	
	t_1	t_2
M_1	100%	100%
M_2	100%	100%
M_3	0%	0%
M_4	100%	100%
Total	≅ 79%	87,5%

➔ *Leave M_3*